New York City College Of Technology
Final Examination-Review MAT 1475H
Fall 2008
By Prof. S. Singh

1. Evaluate the following integrals:

   a. $\int \frac{\left(1-\sqrt{x}\right)^2}{\sqrt{x}}\,dx$ b. $\int_0^1 x\sqrt{5x^2+4}\,dx$

2. a. Find y' and simplify: $xy^2+\cos(y)=\sqrt{\pi}$

   b. show that $y' = -\frac{x^2}{1+x^2}$, when $\tan(x+y)=x$

3. a. Evaluate: $\lim_{x\to 0}\left(x\cos\left(x+\frac{1}{x^2}\right)\right)$

   .

   b. Evaluate: $\lim_{n\to\infty}\left(\sin(\alpha\pi n!)\right)$, for $\alpha\in\mathbb{Q}$

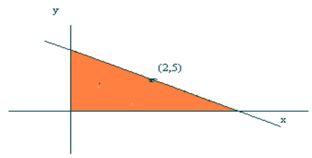
4. Evaluate: $\lim_{n\to\infty}\sum_{i=1}^{n}\frac{i^{0.5}}{n^{1.5}}$

5. Sketch the graph of $f(x)=\sqrt{x}(x+1)$. Indicate the intervals where the function increases, decreases, is concave up and concave down. Label any maxima, minima and inflection point(s). Give the values of all intercepts.

6. Find the area of the region enclosed between the curves $y=x^2+2$ and $y=2+\sqrt{x}$. Sketch the region.

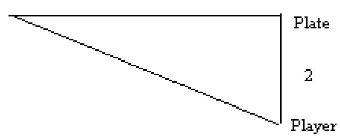
7. Find the area of the region enclosed between the curves $y=x-3$, $y=\sqrt{x-1}$ and the x-axis. Sketch the region.

8. A triangle is formed by a line through $(2,5)$ and the coordinate axes. Find the minimal area of such a triangle.



9. Show that $f(x) = x^3 - 3x + 1$ has a root in the interval $(0,1)$. Use Newton's method to find the first three iterates of the root with an initial guess of 0.5. Give a value of x for which Newton's method fails in this problem.

10. A baseball player stands 2 feet from home plate and watches a pitch fly by. In the diagram, x is the distance from the ball to home plate and θ is the angle indicating the player's gaze. Find the rate $\frac{d\theta}{dt}$ at which his eyes must move to watch a fastball with $\frac{dx}{dt} = -102$ ft/s as it crosses home plate at $x = 0$.
(Suggested reading: *Keep Your Eye On the Ball, Robert G. Watts and A. Terry Bahill*)



Solutions

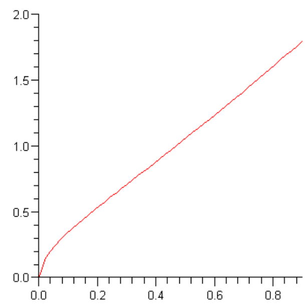
1. a. $2\sqrt{x} - 2x + \frac{2}{3}x^{3/2} + c$ b. $\frac{19}{15}$

2. a. $y' = \frac{y^2}{\sin y - 2xy}$

3. a. 0 b. 0

4. $\frac{2}{3}$

5. $f:[0,\infty)\to[0,\infty)$, $f(x)$ is increasing on $[0,\infty)$, concave down on $\left(0,\frac{1}{3}\right)$ and concave up on $\left(\frac{1}{3},\infty\right)$. There is an inflection point at $\left(\frac{1}{3},\frac{2\sqrt{3}}{3}\right)$ and an intercept at $(0,0)$.



6. $\frac{10}{3}$

7. Area= $\int_0^1\left(2+\sqrt{x}-x^2-2\right)dx=\frac{1}{3}$

8. Minimum Area: 20 square units.

9. $f(0)$ and $f(1)$ have opposite signs. Use the intermediate value theorem to justify the root in $(0,1)$.

$x_0=0.5,\quad x_1=1/3,\quad x_2=0.347222,\quad x_3=0.347296$

10. $\frac{d\theta}{dt}=-51$ rad/s