New York City College of Technology.
Department of Mathematics.
MAT 1372 –Final Examination Review Problems-Fall 2007
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1. The probability that an unbalanced coin lands on head is $1/5$. If this coin is tossed twice and each toss is independent answer the following:
   a. List the sample space.
   b. Construct a discrete probability distribution using X as the number of heads of the two-coin toss.
   c. Find $P(X<1)$, $E(X)$ and $E(1/(X+1))$.

2. a. A random variable X is uniformly distributed between 2 and 12. Find $P[X>9]$.

   b. An amateur magician claims that she can read people's minds. To prove this she tells a person to think of one of the colors red, yellow and green and she then predicts what the color is. In nine such mind-reading sessions, find the probability that she is correct at least seven times.

3. X is a normal distribution with standard deviation σ and mean μ. Find
   a. $P[x-\mu<2\sigma]$, do not assign values to σ and μ.
   b. $P\left[X^2\le 1\right]$ if $\sigma=8$ and $\mu=3$.

4. The mean grade on an examination is 72 with standard deviation 8. If the distribution is normal, find the cutoff score of the top 5% the grades.

5. A coin that is claimed to be fair $(p=1/2)$ is flipped 80 times and 45 heads are observed. Can we accept the claim at the 5% significance level?

6. Find the correlation coefficient and the regression line of the points:

x	-3	0	1	2	5	7	8.5	9	9.9	11	11.3
y	7	-11	9	2	0	3	11	9.1	-7	11	-5

7. An entertainer claims that there are equal numbers of red, white and blue balls in a container. In a random sample of 12 balls, there are 3 red, 5 white and 4 blue balls. Use a chi-square goodness of fit test to check the claim at the 5% significance level.

8. The average amount of tar in a random sample of 25 cigarettes is 12mg with standard deviation of 3mg. The cigarette company claims that the average amount of tar is 11 mg. Can we reject this claim at the 10% significance level?

9. If X is a random variable with $E(X) = 2$ and $E(X^2) = 9$, use Chebyshev's inequality to find a lower bound for $P(-1 < X < 5)$.

10. Based on experience, an instructor at New York City College Of Technology knows that the score of students on her final examination is randomly distributed with a mean value of 76.45. Give an upper bound for the probability that a student will get an A on the final examination.

11. If the professor in problem 10 also knows that the variance of a student's score is 9, give a lower bound on the probability that a student's score will be in the range of a C to a B minus.

12. (Challenge Problem) Give an example of three events that are pairwise independent, but are not independent.

Solutions.

1. a. $\{hh, ht, th, tt\}$. Note: the events are not equally likely.

   b.

x	$P(x)$
0	0.64
1	0.32
2	0.04

   c. 16/25, 2/5 and 61/75

2. a. $f(x) = 0.1$ for $x \in [2,10]$ and zero elsewhere. $P[X > 9] = 0.3$.

   b. $163/3^9$.

3. a. 0.98

   b. 0.093

4. 85.1

5. $H_0 : \mu = np = 40, H_1 : \mu \neq 40$, $\sigma = \sqrt{np(1-p)} = 2\sqrt{5}$.

   Using the 2- tail test and a normal approximation at the 5% level.

   $z = 1.12 \in (-1.96, 1.96)$, we do not reject the claim.

6. $\hat{y} = 5.517 + 0.035\hat{x}$.

7. $\chi^2 = 0.5$, we do not reject the claim at the 5% level.

8. $H_0 : \mu = 11mg, H_1 : \mu > 11mg$, $t = 1.67 > 1.32$. Yes we can reject this claim at the 10% level.

9. $P(|x-2| \leq 3) = 1 - P(|x-2| \geq 3) \geq 1 - \frac{5}{9} = \frac{4}{9}$

10. Let X be the student's score, by Markov's inequality: $P(X \geq 93) \leq \frac{76.45}{93} \approx 0.82$

11. $P(70 \leq X \leq 82.9) = P(|X - 76.45| \leq 6.45)$

$$= 1 - P(|X - 76.45| > 6.45)$$

$$\geq 1 - \frac{9}{(6.45)^2} \approx 0.22$$ (By Chebyshev's inequality)

12. Consider a balanced four sided dice. On the first three faces label as a_1, a_2 and a_3 respectively. On the remaining face label with the three events a_1, a_2 a_3. Note: $P(a_i) = 0.5$, $i = 1, 2$ and 3. $P(a_i \cap a_j) = 0.25 = P(a_i)P(a_j)$, where $i \neq j$. This shows pairwise independence.
$P(a_1 \cap a_2 \cap a_3) = 0.25 \neq P(a_1)P(a_2)P(a_3) = 0.125$ which shows that the three events are not independent.