Optimal deep-space heliocentric transfers with an electric sail and an electric thruster

The 6th International Symposium on Space Sailing (ISSS23)

New York, 05–09 June 2023

Lorenzo Niccolai

Department of Civil and Industrial Engineering – Aerospace Division

Università di Pisa

DICI
Dipartimento di Ingegneria Civile e Industriale

Outline

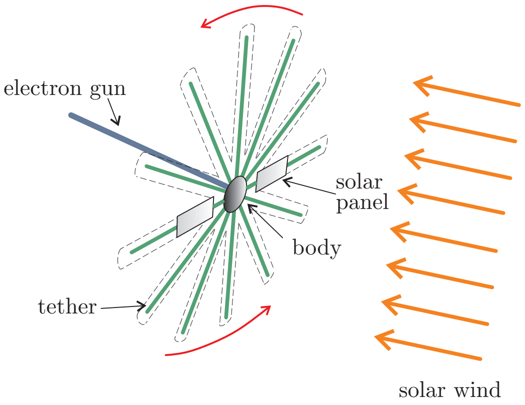
1. Introduction
2. Mathematical model
   - System dynamics
   - Thrust models
   - Optimal control problem formulation
3. Numerical simulations
   - Simulation parameters
   - Earth-Mars scenario
   - Earth-Venus scenario
4. Conclusions and further developments

Introduction

Introduction: the E-sail concept

Original concept

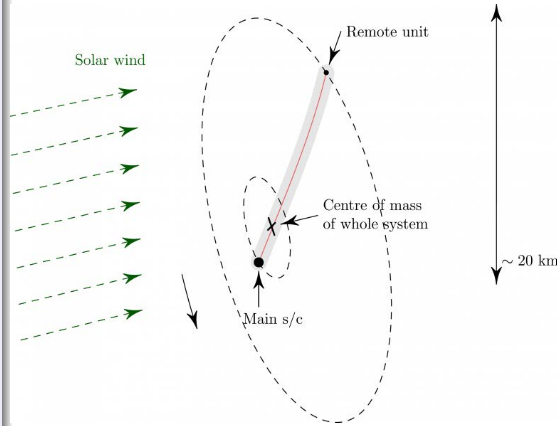
- The electric solar wind sail (E-sail) generates thrust from the electrostatic interaction between **solar wind ions** and **charged tethers** (Janunhen, 2004).
- The first E-sail design consisted of a very large grid (tens of km) with thousands of tethers: **huge problems** with **deployment** and **attitude control**.



Introduction: current E-sail designs

Current E-sail designs

- Currently, E-sails composed of **one or few spinning tethers** are considered more realistic.
- A **multi-asteroid touring** mission with CubeSats equipped with single-tether E-sails has been proposed (Slavinskis et al., 2018).
- Remote unit should host **FEEP thrusters** for attitude control.



Introduction: aim of the work

Motivation of the work

- The thrust generated by an E-sail with a limited number of tethers has a small magnitude.
- The thrust direction is constrained to lie within a cone with half-angle 20 degrees centered along the outward radial direction (Huo et al., 2018).

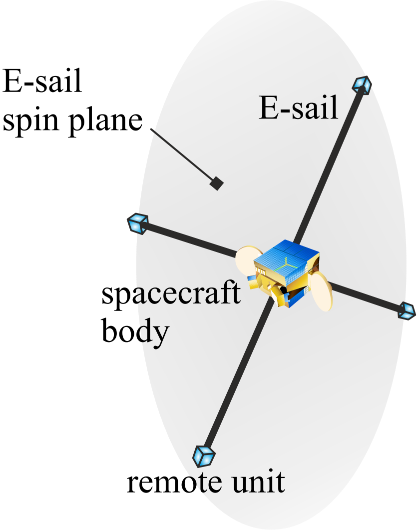
Aim of the work

- This work assumes that a spacecraft is equipped with two propulsive systems
  - a **small E-sail** (thrust $\propto$ inverse Sun-spacecraft distance);
  - an **electric thruster** (such as a FEEP) powered by onboard solar panels (thrust $\propto$ power $\propto$ inverse square Sun-spacecraft distance).
- An optimal control problem is formulated to **test the effectiveness of the combination**.

Introduction: compatibility of E-sail and electric thruster

Compatibility issues?

- Different combinations are possible:
    1. a single small electric thruster placed in the **spacecraft body**;
    2. two or more very small thrusters located in the **remote units**.
- Option 1 should not generate interactions between the thruster and one or few spinning tethers.
- Option 2 has already been suggested for **FEEP-based attitude control**.



Mathematical model

Spacecraft dynamics

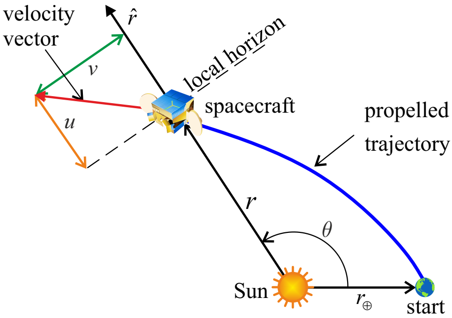
Nomenclature

$r \triangleq$ Sun-spacecraft distance; $\theta \triangleq$ polar angle; $\{u, v\} \triangleq$ radial and circumferential velocity components;
$m \triangleq$ dimensionless mass $a_{ES} \triangleq$ E-sail propulsive acceleration; $a_T \triangleq$ electric thruster propulsive acceleration; $r_\oplus \triangleq 1$ au.

2D Dynamical equations

A heliocentric polar frame $\mathcal{T}(r, \theta)$ is used

$$\dot{r} = u$$
$$\dot{\theta} = \frac{v}{r}$$
$$\dot{u} = \frac{v^2}{r} - \frac{\mu_\odot}{r^2} + a_{ES_r} + a_{T_r}$$
$$\dot{v} = -\frac{uv}{r} + a_{ES_\theta} + a_{T_\theta}$$
$$\dot{m} = -\dot{m}_{\text{ex}}$$



E-sail thrust model

Nomenclature

$a_{c_0} \triangleq$ initial characteristic acceleration; $\hat{\boldsymbol{r}} \triangleq$ radial unit vector; $\hat{\boldsymbol{n}} \triangleq$ unit vector normal to the sail spinning plane; $\alpha \triangleq$ E-sail cone angle; $\tau \in [0, 1] \triangleq$ E-sail switching parameter; subscript $0 \triangleq$ initial value.

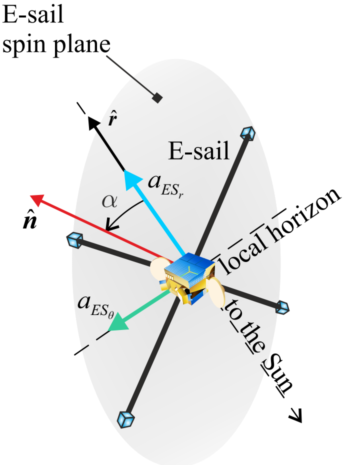
E-sail thrust model (Huo et al., 2018)

- Propulsive acceleration components

$$a_{ES_r} = \tau \frac{a_{c_0}}{2m} \left(\frac{r_\oplus}{r} \right) \left(1 + \cos^2 \alpha\right)$$

$$a_{ES_\theta} = \tau \frac{a_{c_0}}{2m} \left(\frac{r_\oplus}{r} \right) \cos \alpha \sin \alpha$$

- **Initial characteristic acceleration** is calculated at $t_0 \triangleq 0$ $(m = 1)$ at Sun-Earth distance $(r = r_\oplus)$ for a Sun-facing E-sail (i.e., $\alpha = 0$).



Electric engine thrust model

Nomenclature

$a_{T_0} \triangleq$ initial maximum propulsive acceleration; $\hat{\boldsymbol{a}}_T \triangleq$ thruster acceleration unit vector; $\phi \triangleq$ thrust angle; $\kappa \in [0, 1] \triangleq$ power feeding parameter; $g \triangleq$ standard gravity; $I_{sp} \triangleq$ specific impulse; subscript $0 \triangleq$ initial value.

Electric engine thrust model

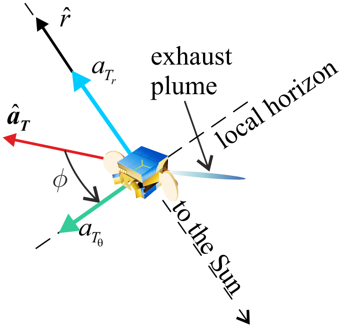
- Propulsive acceleration components

$$a_{T_r} = \kappa \frac{a_{T_0}}{m} \left(\frac{r_\oplus}{r}\right)^2 \sin\phi$$

$$a_{T_\theta} = \kappa \frac{a_{T_0}}{m} \left(\frac{r_\oplus}{r}\right)^2 \cos\phi$$

- **Initial maximum propulsive acceleration** is calculated at $t_0 \triangleq 0$ $(m = 1)$ at $r = r_\oplus$.
- Dimensionless mass flow rate

$$\dot{m}_{\text{ex}} = \kappa \frac{a_{T_0}}{g I_{sp}} \left(\frac{r_\oplus}{r}\right)^2$$



Optimal control problem formulation (1/3)

Cost function

- The dimensionless **cost function** to be **maximized** at final time (subscript f) is:

$$J = \gamma m_f - (1-\gamma) t_f / T_\oplus$$

with $T_\oplus \triangleq 1\,\text{year}$.

- γ is a **trade-off parameter** between two competing requirements:
  - minimize the **flight time**;
  - minimize the **propellant consumption**.

Adjoint variables

- A set of adjoint (costate) variables $\{\lambda_r, \lambda_\theta, \lambda_u, \lambda_v, \lambda_m\}$ is added to the set of physical state variables $\{r, \theta, u, v, m\}$
- Each adjoint variable λ_i is associated with a state variable i.

Optimal control problem formulation (2/3)

Hamiltonian function

- The **Hamiltonian function** is defined as follows:

$$\mathcal{H} \triangleq \lambda_r \dot{r} + \lambda_\theta \dot{\theta} + \lambda_u \dot{u} + \lambda_v \dot{v} + \lambda_m \dot{m}$$

- The time history of adjoint variables is given by **Euler-Lagrange equations**:

$$\lambda_i = -\frac{\partial \mathcal{H}}{\partial i} \qquad \text{with} \quad i \in \{r,\, \theta,\, u,\, v,\, m\}$$

Boundary and transversality conditions (BCs and TCs)

- A **circle-to-circle**, ephemeris-free, interplanetary transfer is analyzed.
  - Departure (t_0)

$$t_0 = 0 \quad , \quad r(t_0) = r_\oplus$$
$$\theta(t_0) = 0 \quad , \quad u(t_0) = 0$$
$$v(t_0) = \sqrt{\frac{\mu_\odot}{r_\oplus}} \, , \, m(t_0) = 1$$

  - Arrival (t_f)

$$r(t_f) = r_f \quad , \quad u(t_f) = 0$$
$$v(t_f) = \sqrt{\frac{\mu_\odot}{r_f}} \, , \, \lambda_\theta(t_f) = 0$$
$$\lambda_m(t_f) = \gamma \, , \, \mathcal{H}(t_f) = \frac{1-\gamma}{T_\oplus}$$

Optimal control problem formulation (3/3)

Pontryagin's maximum principle

- The control variables are selected so to **maximize** the Hamiltonian $\forall t \geq t_0$
  - Optimal values of **E-sail control variables** $\{\tau^\star, \alpha^\star\}$:

$$\tau^\star = \frac{1}{2} + \frac{1}{2}\mathsf{sign}\left(1 + \frac{3\lambda_u}{\sqrt{\lambda_u^2 + \lambda_v^2}}\right)$$

$$\alpha^\star = \frac{1}{2}\arctan\left(\frac{\lambda_v}{\lambda_u}\right)$$

  - Optimal values of **electric thruster control variables** $\{\kappa^\star, \phi^\star\}$:

$$\kappa^\star = \frac{1}{2} + \frac{1}{2}\mathsf{sign}\left(\lambda_u \sin\phi^\star + \lambda_v \cos\phi^\star - \lambda_m \frac{m}{g I_{sp}}\right)$$

$$\sin\phi^\star = \frac{\lambda_u}{\sqrt{\lambda_u^2 + \lambda_v^2}} \qquad \cos\phi^\star = \frac{\lambda_v}{\sqrt{\lambda_u^2 + \lambda_v^2}}$$

  - Thrust angle ϕ must belong to the **feasible range** $[\phi_{\min}, \phi_{\max}]$.

Numerical simulations

Simulation parameters

- **E-sail parameters** used in the simulations (Slavinskis et al., 2018):

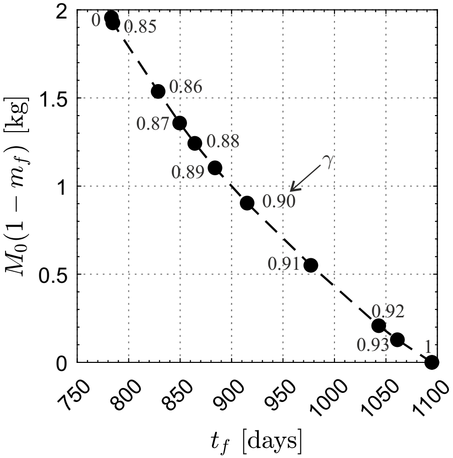
Quantity	Value	Measurement unit
Total tether length	20	km
Tether voltage	20	kV
Initial spacecraft mass	20	kg
Initial characteristic acc. a_{c_0}	0.307	mm/s^2

- **FEEP Electric thruster parameters** used in the simulations (Grimaud et al., 2019):

Quantity	Value	Measurement unit
Initial nominal thrust	1.0	mN
Specific impulse I_{sp}	2150	s
Initial spacecraft mass	20	kg
Initial propulsive acc. a_{T_0}	0.05	mm/s^2
Thrust cone half-angle	30	deg

Earth-Mars scenario: Pareto front

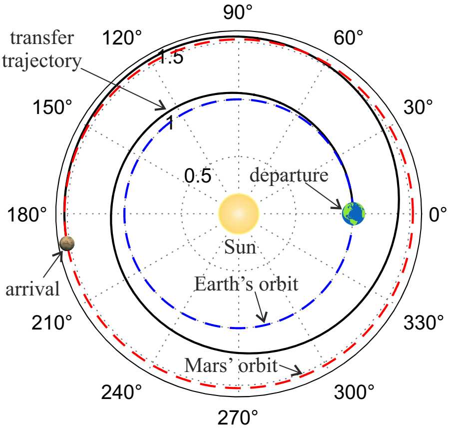
- **Earth-Mars** transfer: $r_f = 1.524\,\mathrm{au}$, $\phi \in [-30,\,30]\,\mathrm{deg}$.
- **Pareto front**: optimal flight times and propellant consumptions obtained with different values of γ.

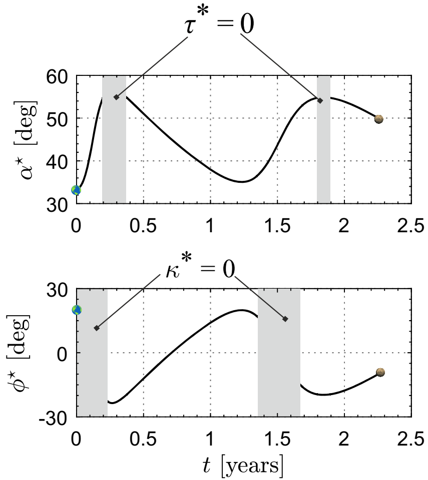


Remarks

- E-sail+FEEP combination is capable of **significantly reducing the transfer time**.
- Consuming 1 kg of propellant reduces the flight time of about 200 days (18%).

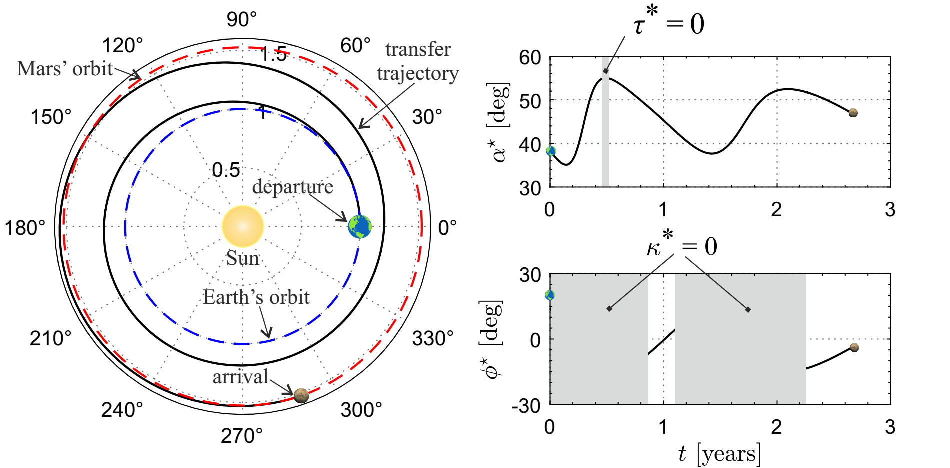
Earth-Mars scenario: example ($\gamma = 0.86$)





- The electric thruster is switched on for most of the trajectory.
- Flight time 829 days, propellant consumption 1.54 kg.

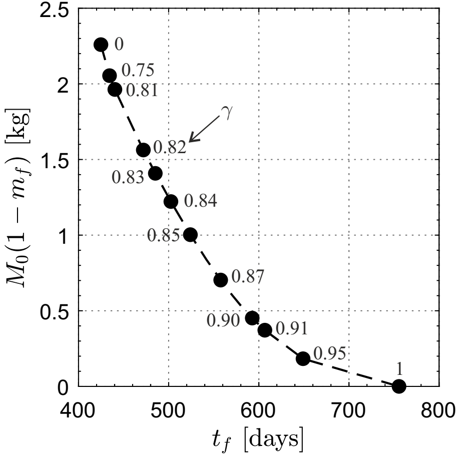
Earth-Mars scenario: example ($\gamma = 0.91$)



- The electric thruster is switched on for shorter firing times.
- Flight time 977 days, propellant consumption 0.55 kg.

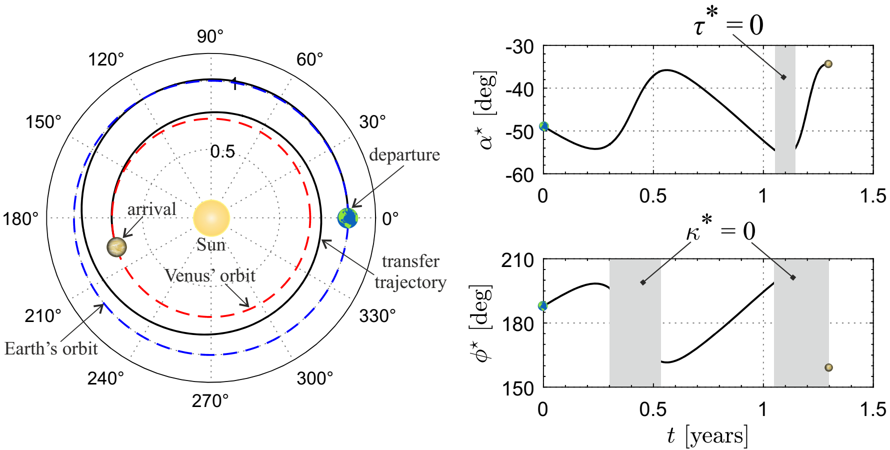
Earth-Venus scenario: Pareto front

- **Earth-Venus** transfer: $r_f = 0.723\,\text{au}$, $\phi \in [150,\ 210]\,\text{deg}$
- **Pareto front**: optimal flight times and propellant consumptions obtained with different values of γ.



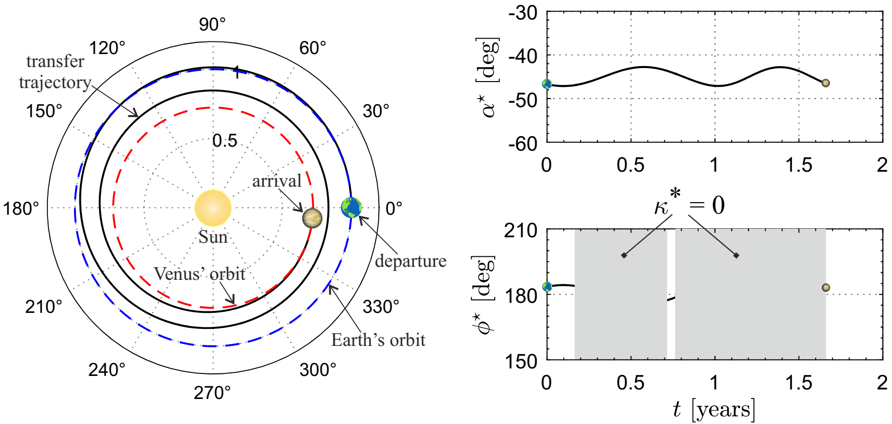
- E-sail+FEEP combination is capable of **significantly reducing the transfer time**.
- Consuming $1\,\text{kg}$ of propellant reduces the flight time of about 232 days (31%).

Earth-Venus scenario: example ($\gamma = 0.82$)



- The electric thruster is switched for most of the trajectory.
- Flight time 472 days, propellant consumption 1.56 kg.

Earth-Venus scenario: example ($\gamma = 0.91$)



- The electric thruster is switched on for shorter firing times.
- Control angle variations are slow.
- Flight time 607 days, propellant consumption 0.37 kg.

Conclusions and further developments

Conclusions

- Current technological trends suggest that a nano- or micro-satellite equipped with a **small electric sail** with **a limited number of tethers** is a realistic near-term scenario.
- At the same time, **small electric thrusters** are currently commercially available or undergoing space qualification tests.
- The **combination** of a small **electric sail** and an **electric thruster** (as a FEEP) could **significantly increase the flexibility** of the propulsion system.
- A **trade-off** between the competing requirements of short flight time and small propellant consumption is made by tuning a suitable trade-off parameter.
- Numerical simulations highlight that the **transfer times** towards inner and outer solar system could be **significantly shortened**, even with **small propellant conumptions**.

Further developments

- The discussed optimization method could be generalized to **three-dimensional transfers**, also keeping into account planetary eccentricities and inclinations.
- Further analysis could consider **different mission scenarios**, as:
  - flyby of planets or asteroids;
  - mission towards outer regions of the solar system.
- The control variables related to the thruster and the electric sail **may not be independently selected**, so other scenarios could be considered:
  - the electric sail could be kept in a Sun-facing configuration;
  - the electric thruster could be not steerable, so its thrust direction would only depend on the spacecraft attitude;
  - constraints on the thrust angle ϕ could be related to the instantaneous value of α.

Thank you for your attention!

Lorenzo Niccolai

Department of Civil and Industrial Engineering – Aerospace Division

University of Pisa, Pisa (PI), Italy

lorenzo.niccolai@unipi.it