University of Glasgow | James Watt School of Engineering

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Space and Exploration Technology Group

Adaptive Sliding Mode Control for Asteroid Hovering with Solar Sailing

Application to 433 Eros

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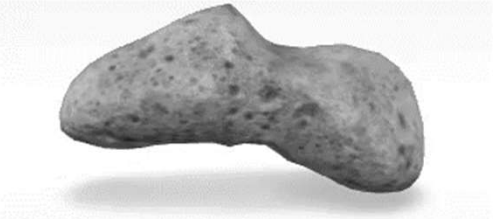
Introduction

Why do we explore asteroids?

- origin of the solar system, planets and life
- space resources extraction
- planetary defence
- …

NEAR-Shoemaker

Hayabusa


433 Eros


25143 Itokawa

Image credit: NASA asteroids, comets & meteors

Will solar sailing make a difference? YES! It’s propellant-free!

- It takes great advantage in single/multiple asteroids rendezvous missions.

Introduction

Sail operation in proximity of asteroids needs to be studied:

- to maximise scientific return of rendezvous missions
- to support energy-consuming mapping operations, such as hovering
- …

Difficulties:

- **Underactuated**: control force constrained in both magnitude and direction.
- **Non-affine**: attitude angles as input is not linear.
- **Complex gravity**: non-spherical perturbation hard to model prior to a mission.

Our idea:

- ✓ To reduce one DOF to be controlled. → **Underactuated**
- ✓ To control the derivatives of attitude angles. → **Non-affine**
- ✓ To use robust control with adaptive estimation. → **Complex gravity**

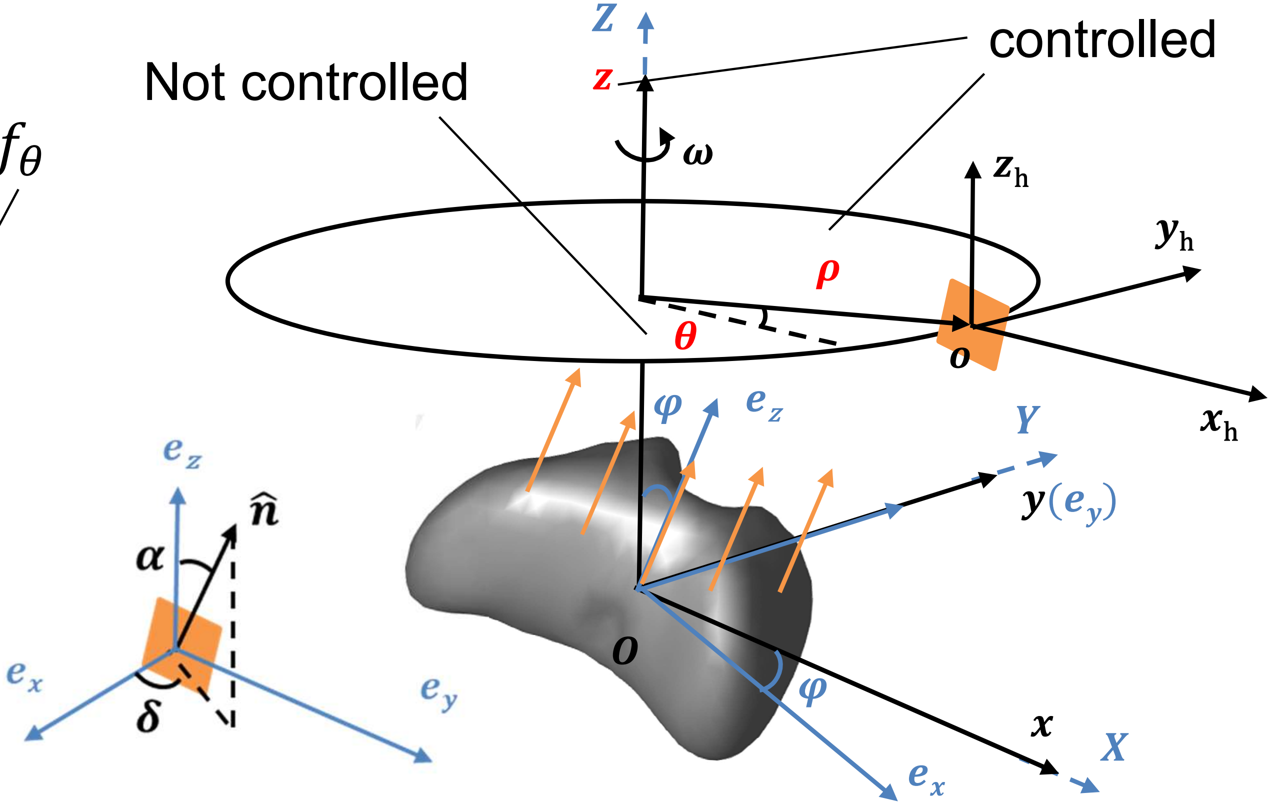
Dynamics

Dynamics in cylindrical coordinates $\chi = [\rho, \theta, z]^T$ in asteroid-fixed frame

$$\begin{cases} \ddot{\rho} = \rho\left(\omega + \dot{\theta}\right)^2 + g_\rho + f_\rho \\ \rho\ddot{\theta} = -2\dot{\rho}\left(\omega + \dot{\theta}\right) + g_\theta + f_\theta \\ \ddot{z} = g_z + f_z \end{cases}$$

$$\boldsymbol{g} = \left[g_\rho, g_\theta, g_z\right]^T = \mathbf{C}_a^h \nabla U$$

$$\boldsymbol{f} = \left[f_\rho, f_\theta, f_z\right]^T = \mathbf{C}_I^h \mathbf{C}_E^I \boldsymbol{a}_{SRP}$$

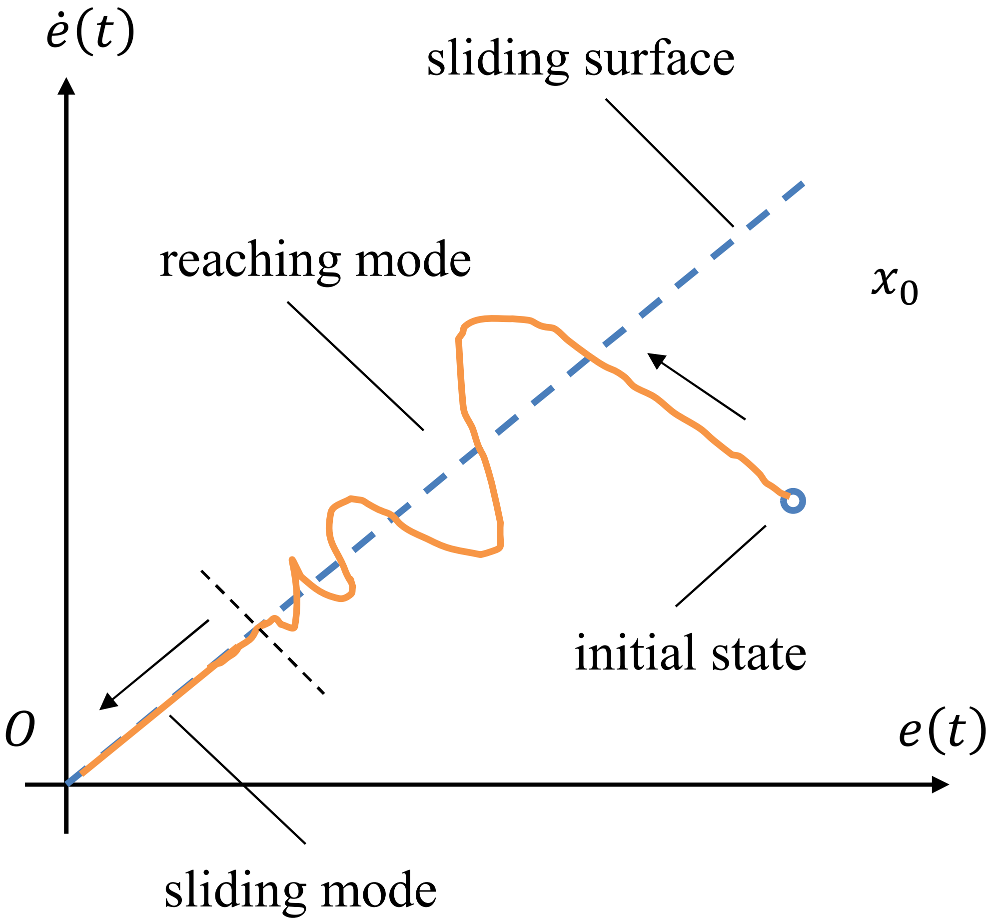


Objective of control:

- To track constant ρ and z, leaving θ aside.

$$E \xrightarrow{C_y(\varphi)} I \xrightarrow{C_z(\omega t)} a \xrightarrow{C_z(\theta)} h$$

Sliding mode control



Tracking error

$$\boldsymbol{e} = \boldsymbol{\chi} - \boldsymbol{\chi_d} = \begin{bmatrix} \rho - \rho_d \\ z - z_d \end{bmatrix}$$

Sliding surface

$$\boldsymbol{s} = \dot{\boldsymbol{e}} + \mathbf{k}\boldsymbol{e}$$

Non-singular terminal sliding surface

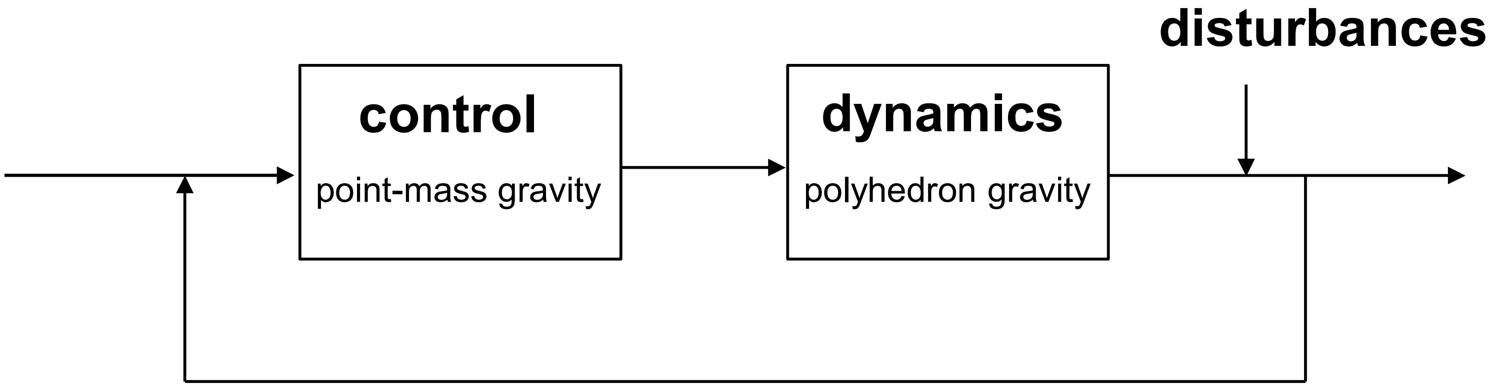
$$\boldsymbol{\sigma} = \boldsymbol{s} + k_0 \dot{\boldsymbol{s}}^{\frac{p}{q}}$$

$$\boldsymbol{\sigma} = \mathbf{0} \rightarrow \boldsymbol{s} = \mathbf{0} \rightarrow \boldsymbol{e} = \mathbf{0}$$

- $\mathbf{k}$, k_0 – *positive numbers*
- p, q – *positive odd number,* $1 < p/q < 2$

Control

Gravity used is different in control and dynamics!



disturbances

control

point-mass gravity

dynamics

polyhedron gravity

polyhedron gravity = mass point gravity + disturbances

irregular-shape perturbation / high-order harmonics

Control

Objective of control:

- **To control $\dot{\boldsymbol{u}} = [\dot{\alpha}, \dot{\delta}]^T$ instead of $\boldsymbol{u} = [\alpha, \delta]^T$.**

Further differentiate the dynamics

$$\dddot{\chi} = \boldsymbol{h}(\chi, \dot{\chi}) + \dot{\mathbf{C}}_I^{o}\mathbf{C}_E^{I}\boldsymbol{a}_{SRP} + \mathbf{C}_I^{o}\mathbf{C}_E^{I}\mathbf{B}(\boldsymbol{u})\dot{\boldsymbol{u}}$$

$$\ddot{\chi} = \begin{cases} \underline{\rho(\omega+\dot{\theta})^2 + g_\rho} + f_\rho \\ \underline{g_z} + f_z \end{cases}$$

$$\boldsymbol{h} = \boldsymbol{h}_0 + \boldsymbol{d}$$

known portion

unknown disturbance (by irregular gravity)

Assumption: $||\boldsymbol{d}|| \leq \boldsymbol{D}$

Control

Differentiate $\boldsymbol{s}$ twice and $\boldsymbol{\sigma}$ once:

$$\ddot{\boldsymbol{s}} = \boldsymbol{h} + \dot{\mathbf{C}}_I^o \mathbf{C}_E^I \boldsymbol{a}_{SRP} + \mathbf{C}_I^o \mathbf{C}_E^I \mathbf{B}(\boldsymbol{u})\dot{\boldsymbol{u}} - \dddot{\boldsymbol{\chi}}_{\boldsymbol{d}} + \mathbf{k}\ddot{\boldsymbol{e}}$$

$$\dot{\boldsymbol{\sigma}} = k_0 \frac{p}{q} \operatorname{diag}\left(\dot{\boldsymbol{s}}^{\frac{p}{q}-1}\right)\left(\frac{q}{kp}\dot{\boldsymbol{s}}^{2-\frac{p}{q}} + \ddot{\boldsymbol{s}}\right)$$

Design reaching law as:

$$\dot{\boldsymbol{\sigma}} = \operatorname{diag}\left(\dot{\boldsymbol{s}}^{\frac{p}{q}-1}\right)\left(\underbrace{-\varepsilon_1 \boldsymbol{\sigma}}_{\text{rapidity term}} \underbrace{- \varepsilon_2 \operatorname{sign}(\boldsymbol{\sigma})}_{\text{robustness term}}\right)$$

Control law obtained:

$$\dot{\boldsymbol{u}} = \left(\mathbf{C}_I^o \mathbf{C}_E^I \mathbf{B}\right)^{-1}\left[\dddot{\boldsymbol{\chi}}_d - \boldsymbol{h} - \dot{\mathbf{C}}_I^o \mathbf{C}_E^I \boldsymbol{a}_{SRP} - k\ddot{\boldsymbol{e}} - \frac{q}{kp}\dot{\boldsymbol{s}}^{2-\frac{p}{q}} - \varepsilon_1 \sigma - \varepsilon_2 \operatorname{sign}(\boldsymbol{\sigma})\right]$$

Control

Objective of control:

- **To be robust against gravity disturbances.**

Adaptive estimation on boundary of gravity disturbances:

$$\dot{\hat{\boldsymbol{D}}} = \gamma \frac{k_0 p}{q} \mathrm{diag}\left(\dot{\boldsymbol{s}}^{\frac{p}{q}-1}\right)|\boldsymbol{\sigma}|$$

$$\dot{\boldsymbol{u}} = (\mathbf{C}_I^O \mathbf{C}_E^I \mathbf{B})^{-1}\left[\dddot{\chi}_d - \underline{\boldsymbol{h}} - \dot{\mathbf{C}}_I^O \mathbf{C}_E^I \boldsymbol{a}_{SRP} - k\ddot{\boldsymbol{e}} - \frac{q}{kp}\dot{\boldsymbol{s}}^{2-\frac{p}{q}} - \varepsilon_1\sigma - \varepsilon_2 \mathrm{sign}(\boldsymbol{\sigma})\right]$$

$$\Rightarrow \boxed{\dot{\boldsymbol{u}} = (\mathbf{C}_I^O \mathbf{C}_E^I \mathbf{B})^{-1}\left[\dddot{\chi}_d - \underline{\boldsymbol{h}_0 - \hat{\boldsymbol{D}}} - \dot{\mathbf{C}}_I^O \mathbf{C}_E^I \boldsymbol{a}_{SRP} - k\ddot{\boldsymbol{e}} - \frac{q}{kp}\dot{\boldsymbol{s}}^{2-\frac{p}{q}} - \varepsilon_1\sigma - \varepsilon_2 \mathrm{sign}(\boldsymbol{\sigma})\right]}$$

Results

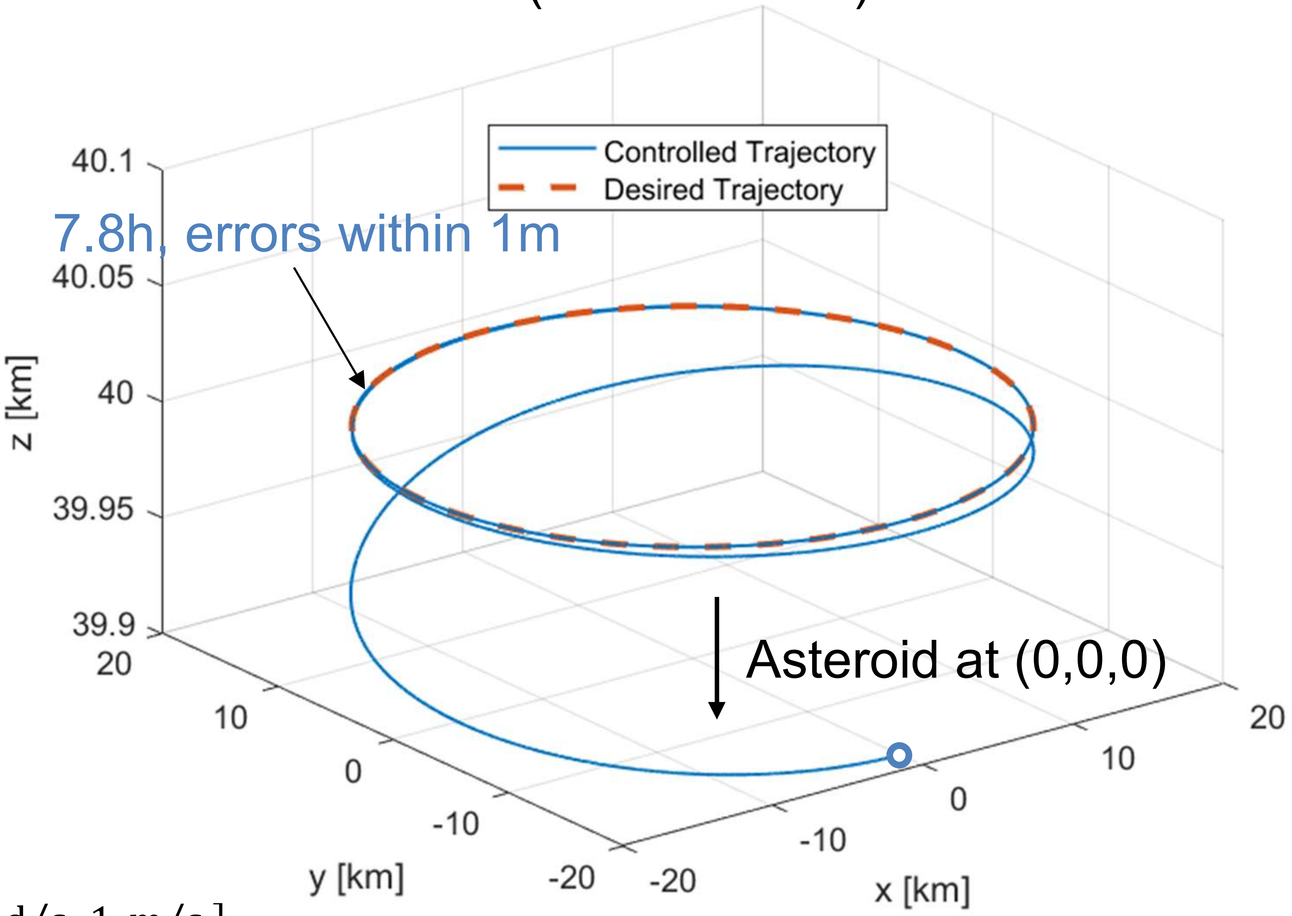
Constants	Value
Eros Gravitational Constant μ	4.4602×10^4 km^3/s^2
Eros dimension	34.4 × 11.2 × 11.2 km
Eros Spin Rate ω	3.3117×10^{-4} rad/s
Eros Heliocentric Distance R	1.6917×10^6 km
Solar Incidence Angle φ	0 deg
Sail Lightness Number β	0.2

Initial conditions:

$$[\rho, \theta, z]^T = [18.1 \text{ km}, -\pi/2\,, 39.9\ \text{ km}]$$

$$[\dot{\rho}, \dot{\theta}, \dot{z}]^T = [-1\ \text{ m/s}\,, -3.3117 \times 10^{-4}\ \text{ rad/s}\,, 1\ \text{ m/s}]$$

Controlled trajectory to $\rho_d = 18 \text{ km}$, $z_d = 40 \text{ km}$ (z-axis dilated)

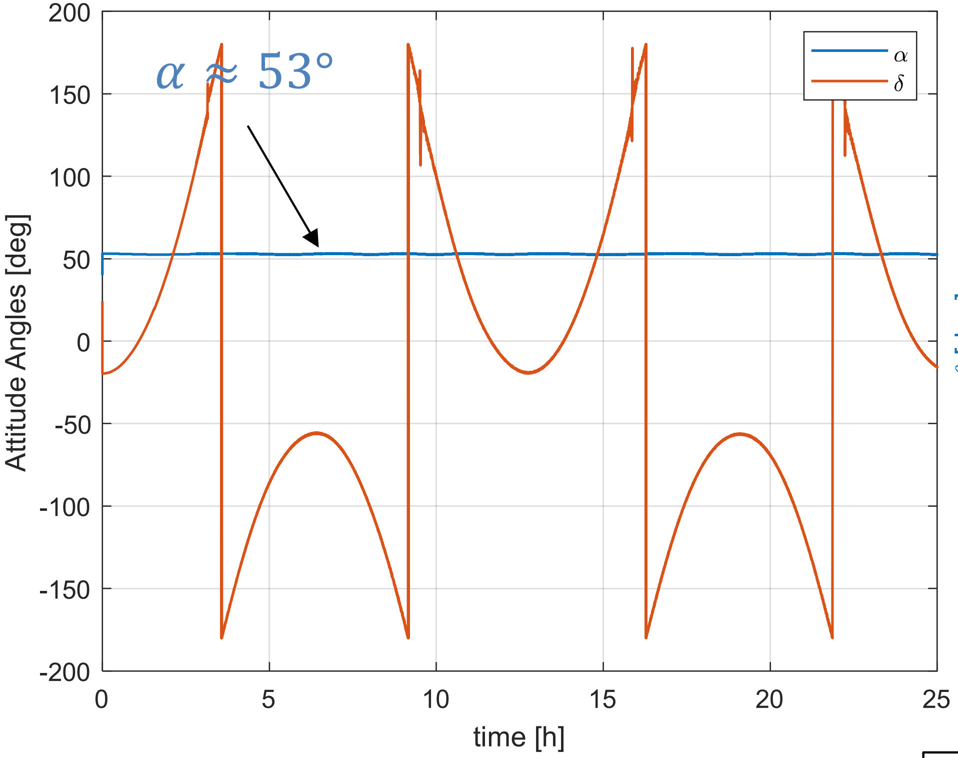


7.8h, errors within 1m

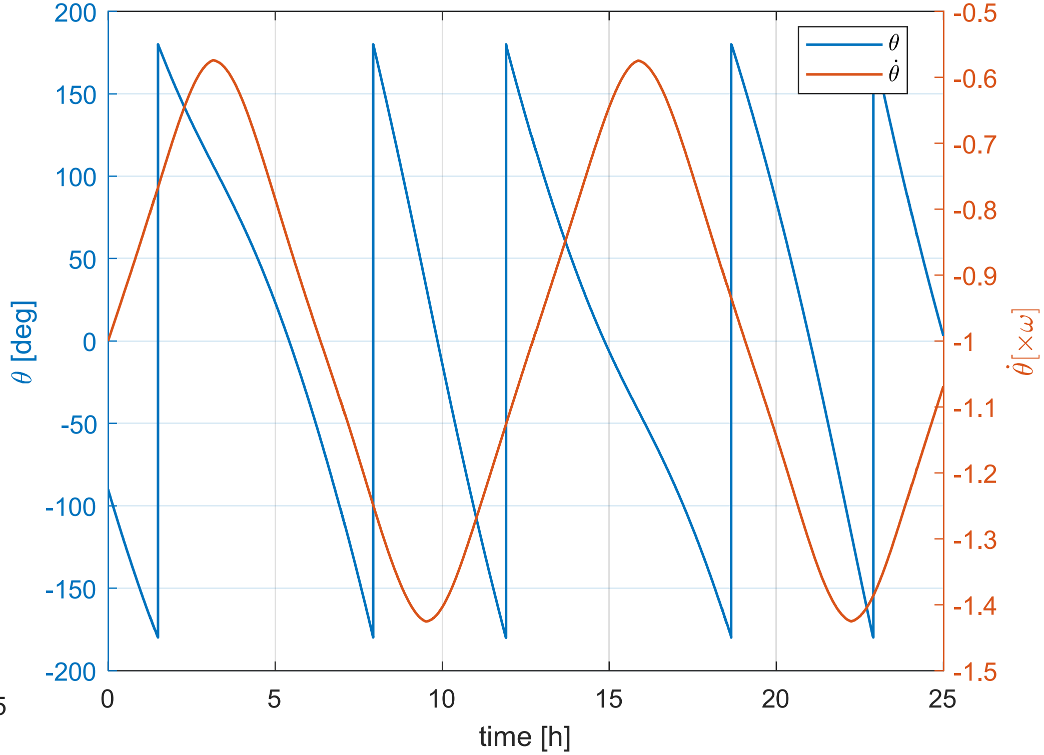
Asteroid at (0,0,0)

Results

Control of attitude angles
α (red), δ (blue)



Uncontrolled polar angle θ (blue)
and polar angular velocity $\dot{\theta}$ (red)

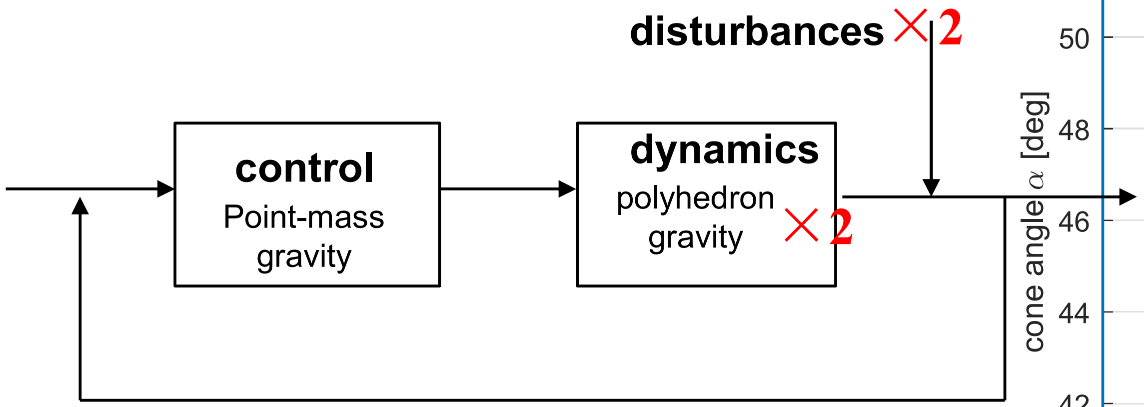


$$\rho\ddot{\theta} = \underbrace{\boxed{-2\dot{\rho}\left(\omega + \dot{\theta}\right)}}_{0} + g_\theta + \underbrace{\boxed{f_\theta}}_{\text{dominant term}}$$

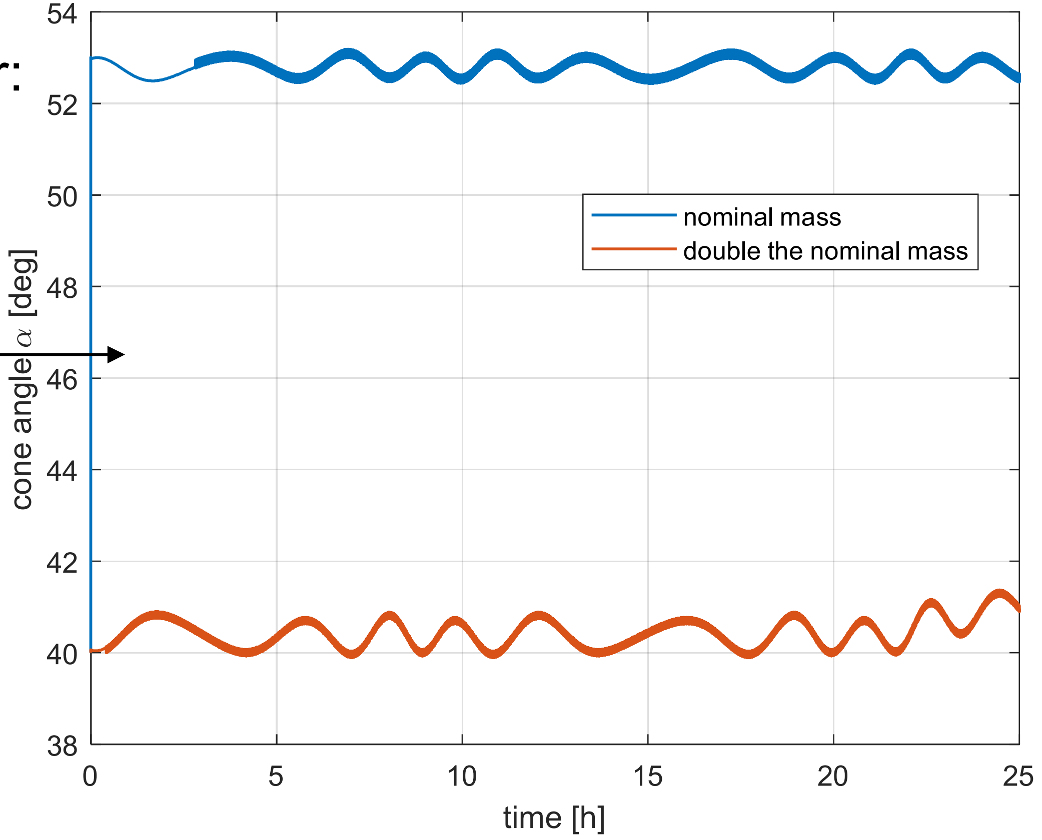
Results

Robustness

Knowledge of asteroid mass with large error:



Response to cone angle with nominal asteroid mass (blue) and double nominal mass (red)



Results

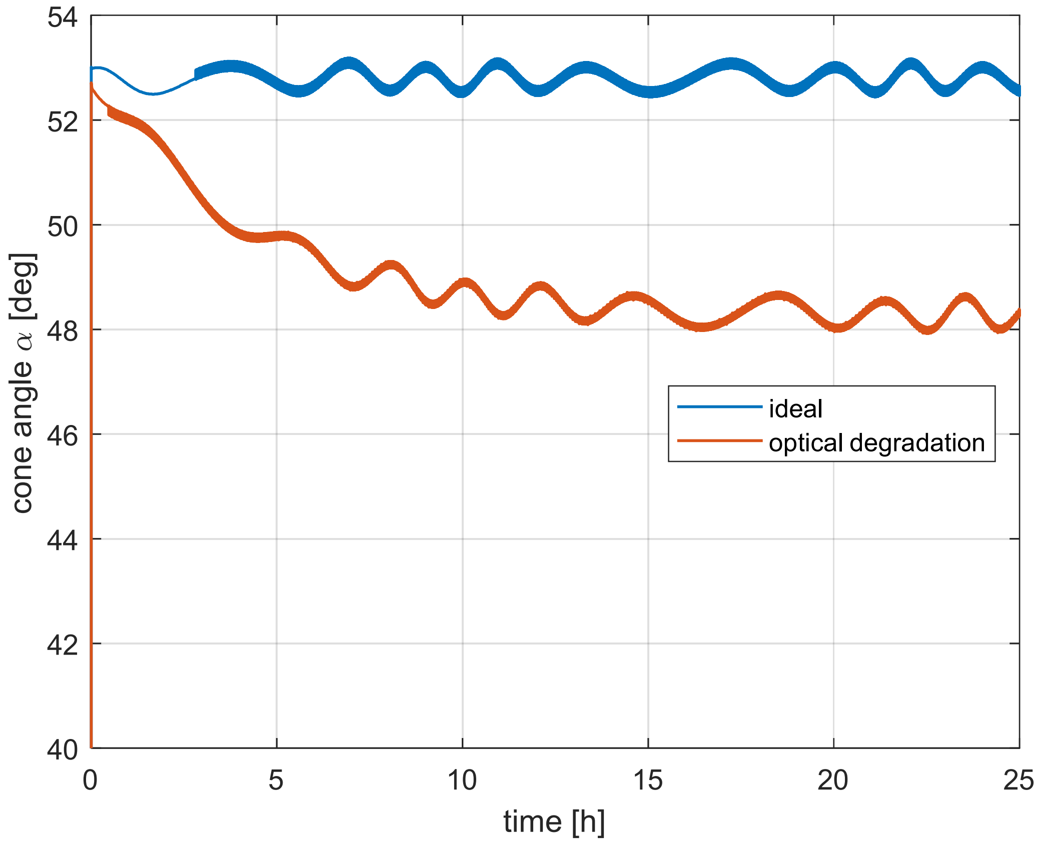
Robustness

Optical degradation:

$$\beta(t) = 0.05e^{-t/13500} + 0.15$$

$$\beta\text{: } 0.2 \xrightarrow{15h} 0.15$$

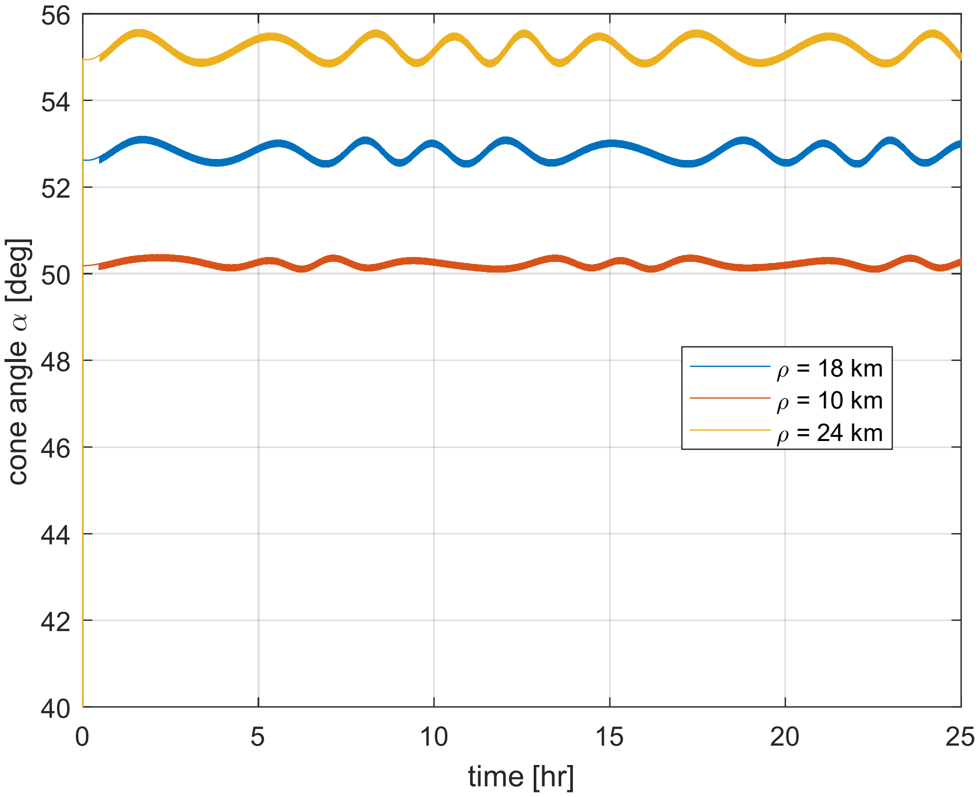
Response to cone angle with ideal sail (blue) and degraded sail (red)

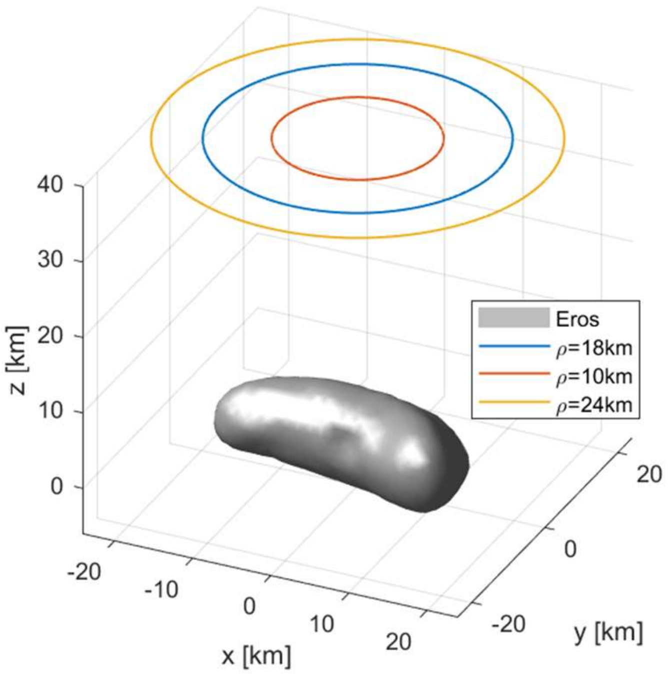


Effect of Hovering Radii

Response to cone angle with different radii

z = 40 km, ρ = 18 km (blue), ρ = 10 km (red), ρ= 24 km (yellow)





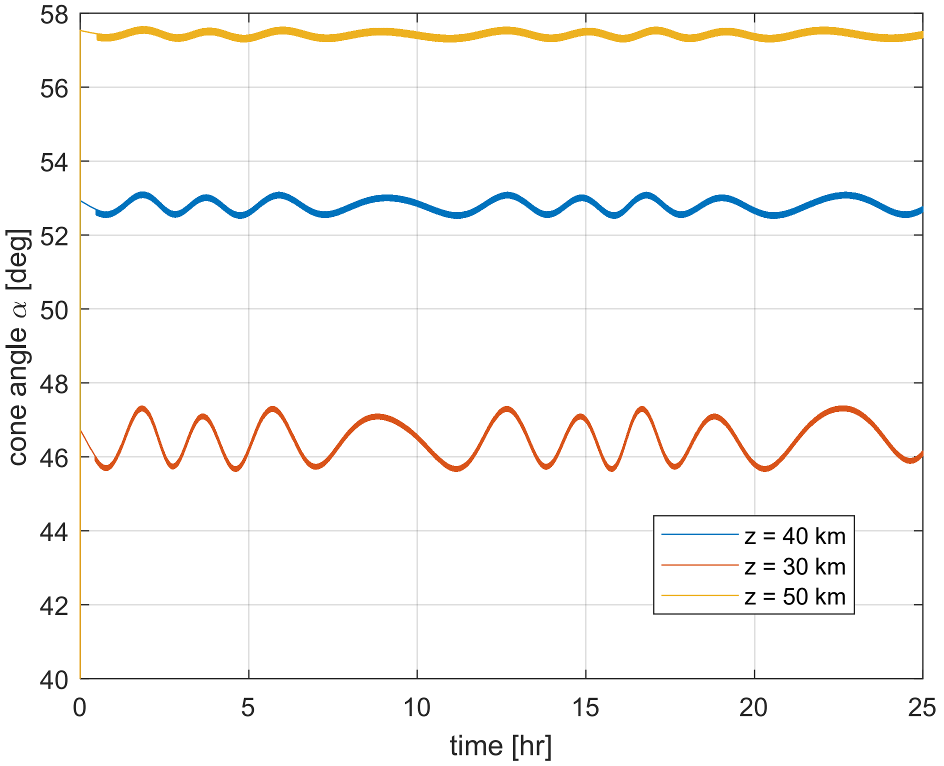
$\rho\nearrow$, $\alpha\nearrow$, oscillation of $\alpha\nearrow$

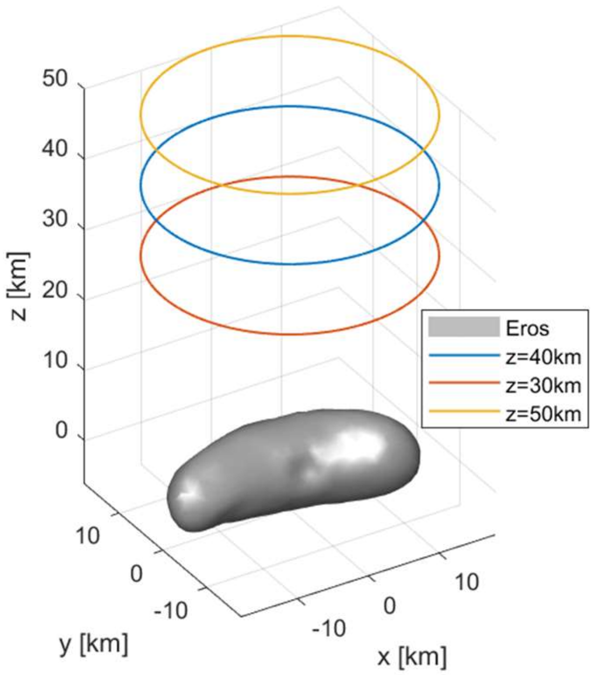
Results

Effect of Hovering Height

Response to cone angle with different height

ρ = 18 km, z = 40 km (blue), z = 30 km (red), z = 50 km (yellow)





$z\nearrow$, $\alpha\nearrow$, oscillation of $\alpha\searrow$

Results

Effect of Sunlight Direction

Sunlight incidence angle
is a variable, affected by:

- Eros orbital inclination
- Eros obliquity to the ecliptic

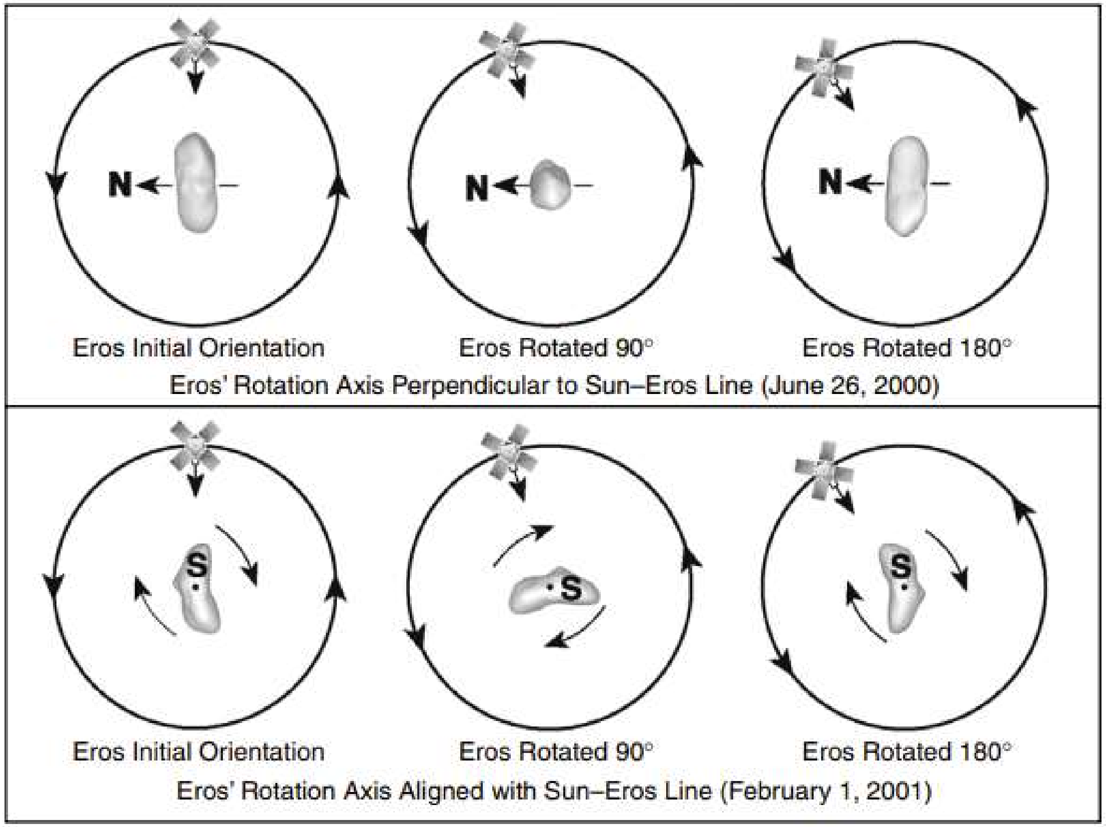
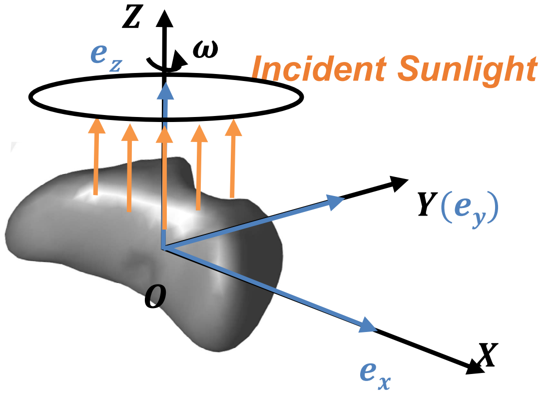
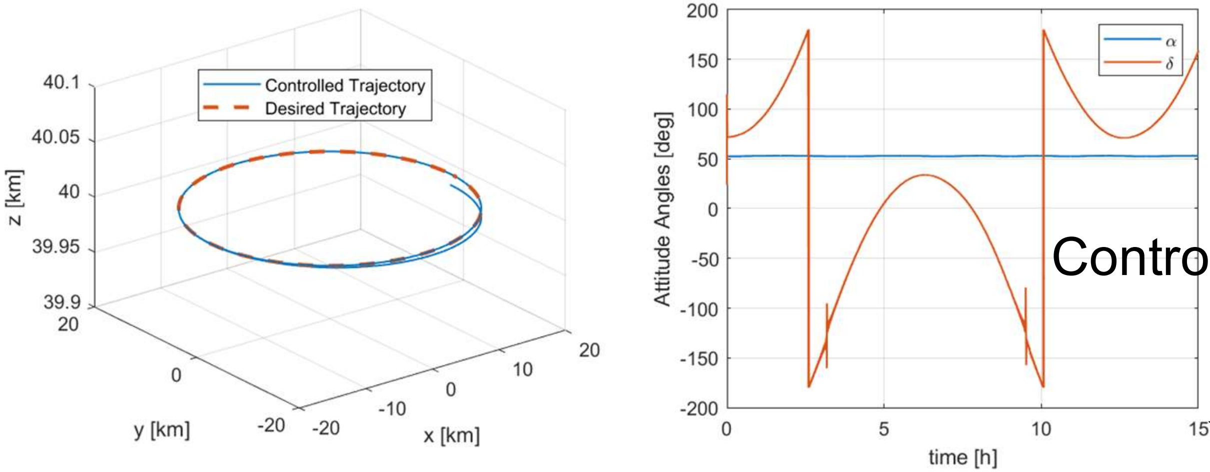


Image credit: Space Exploration of Asteroids: The 2001 Prospective (R. Farquhar)

Effect of Sunlight Direction

(a) ϕ = 0 deg



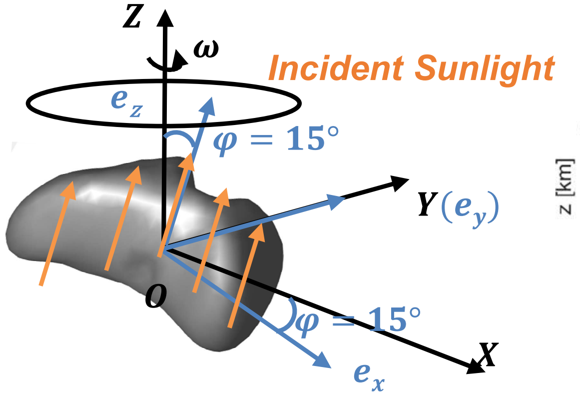


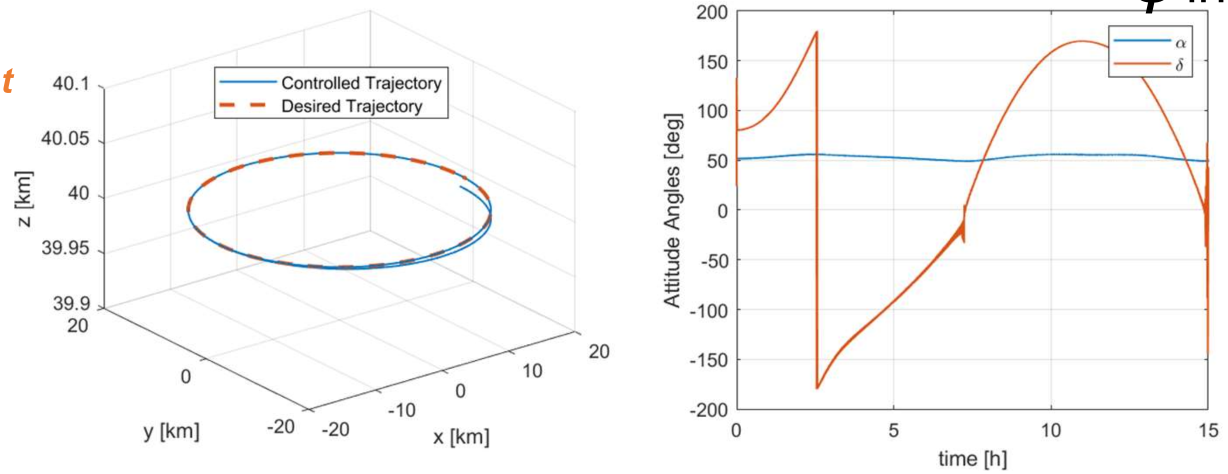
Control success!

⇩

φ increasing…

(b) ϕ = 15 deg



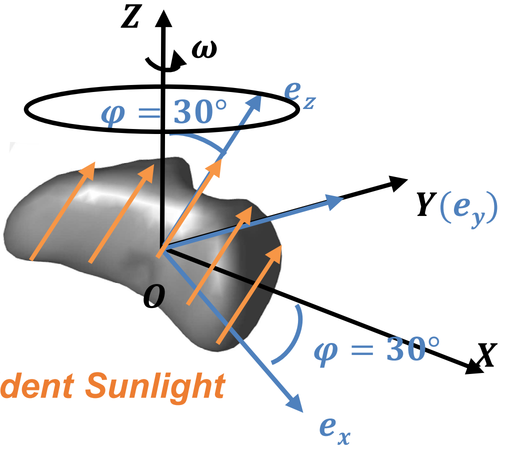


Results

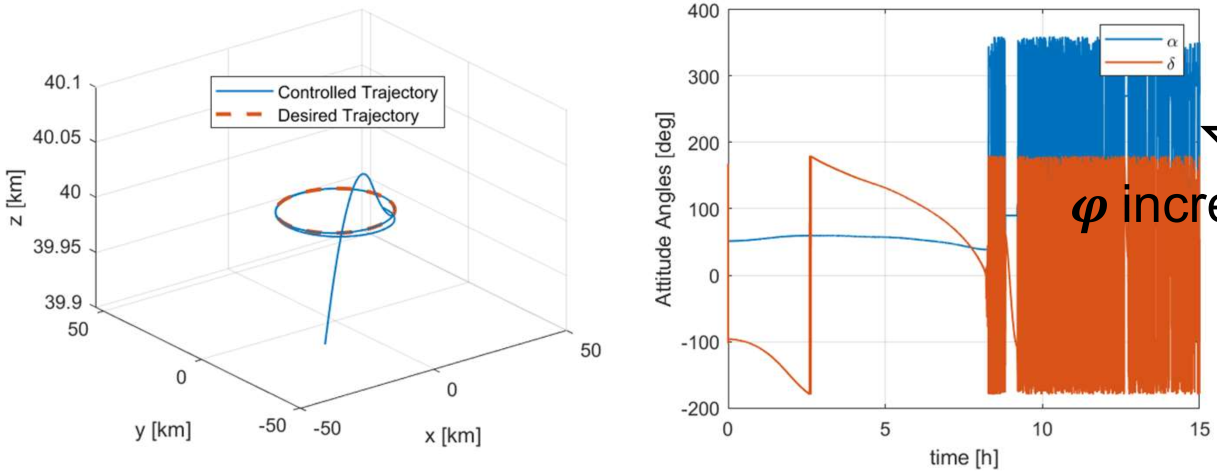
Effect of Sunlight Direction

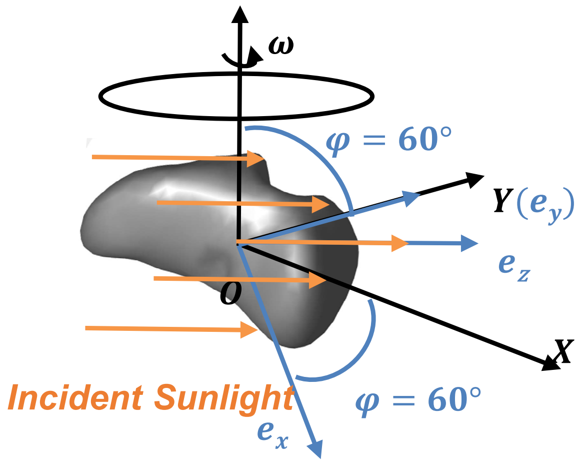
Orbit maintenance failure

φ increasing…

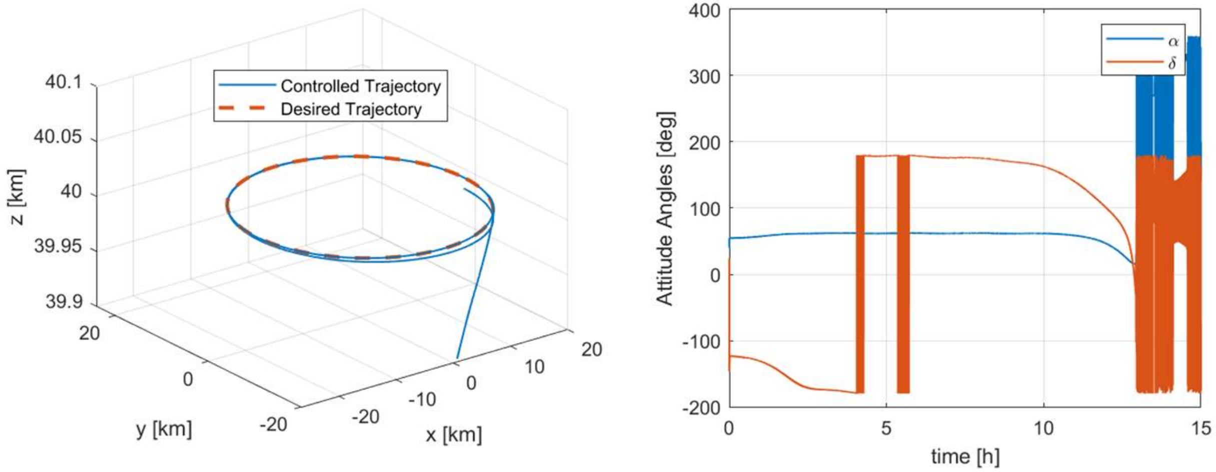


(c) ϕ = 30 deg





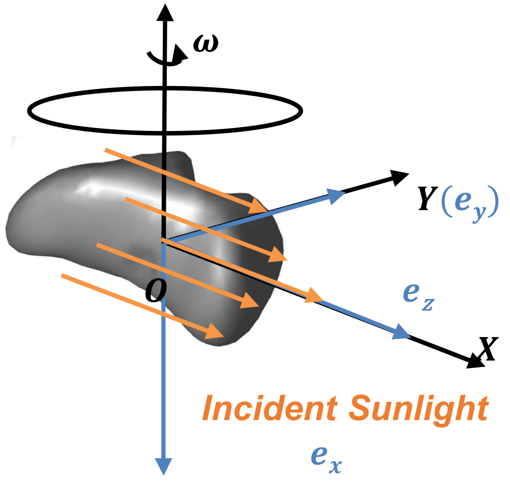
(d) ϕ = 60 deg

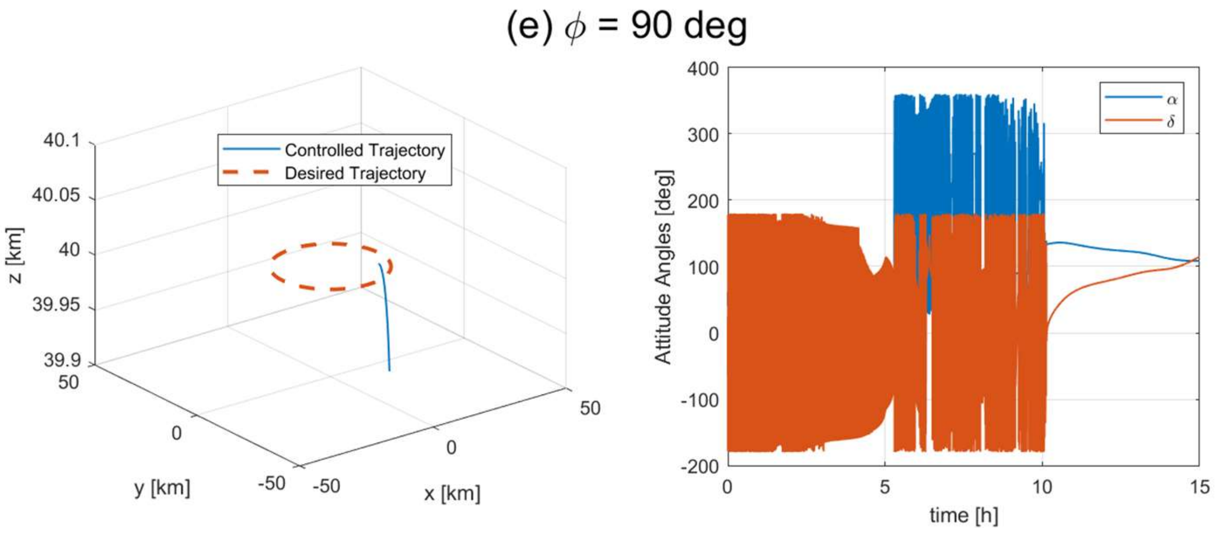


Effect of Sunlight Direction

φ increasing… ⇩

Injection failure




(e) ϕ = 90 deg

Control breaks down when required control force is sunward.

Results

Conclusions

- It finds a path to tackle underactuated and non-affine control of solar sail;
- It is robust to gravity disturbances and unmodelled sail error;
- Small hovering radius and height → small cone angle response;
- Small hovering radius and large hovering height → slight cone angle oscillation;
- Control is only effective when sunlight incidence angle is small.

Future works

- Quantitative research on feasible hovering radius, height and sunlight incidence angles;
- Direct adaptive estimation on gravity property and sail performance, instead of boundary of gravity disturbances.

Space and Exploration Technology Group

Thank you for listening!

University of Glasgow

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