SAMARA UNIVERSITY

«Cyclic Interplanetary Motion of a Cargo Solar Sail»

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New-York, 2023

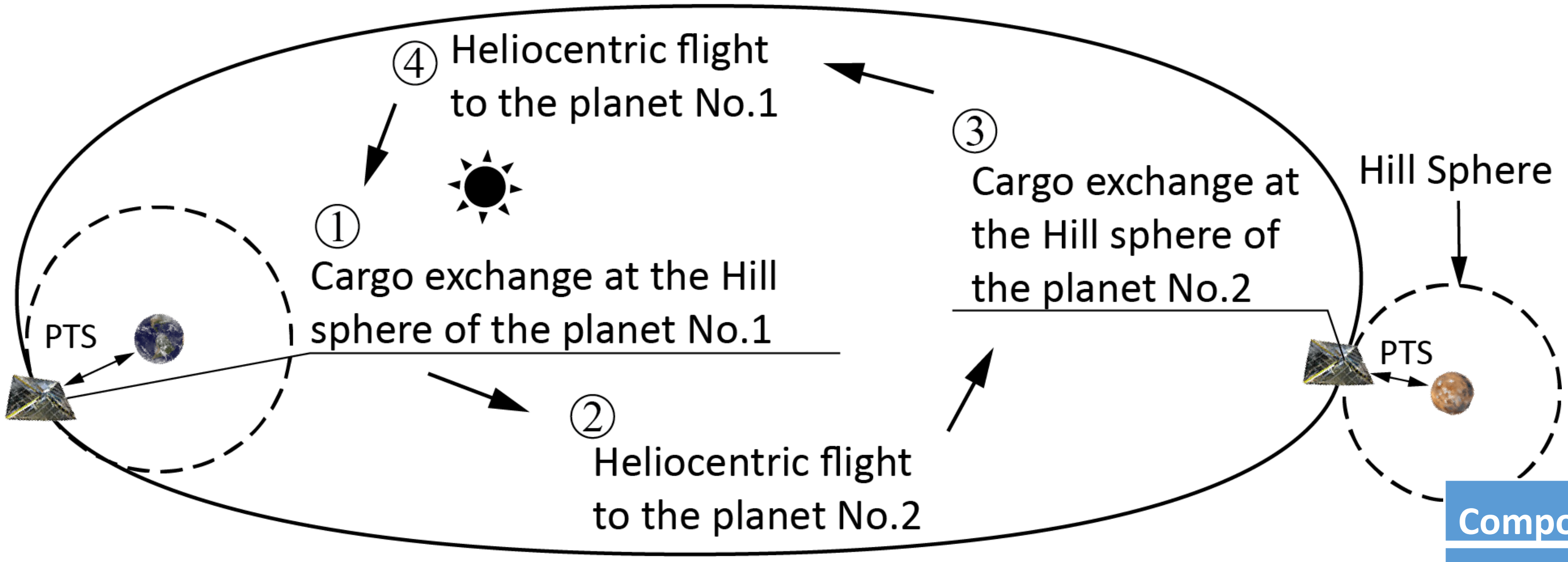


Fig. 1 Schematic illustration of cyclic solar sail mission for continuous cargo delivery.

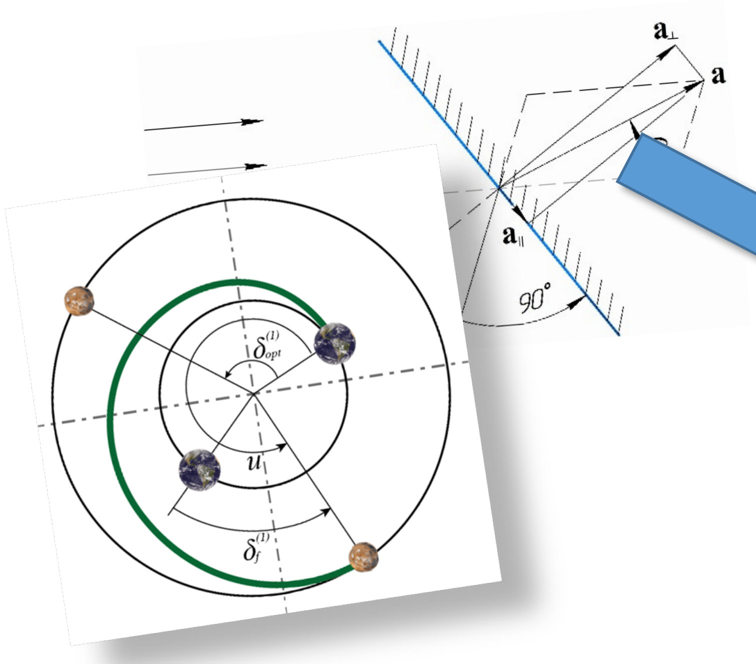
Table 1 Solar sail that capable to carry onboard up to 1905 kg of cargo payload with characteristic acceleration 0.25 mm/s^2 (sail area 72564 m^2).

Component	Mass, *kg*
Total Sail payload mass	1905
2-μm CP1 film, 2.86 g/m^2	216
0.1-μm Al coating, 0.54 g/m^2	41
Bonding, 10% coated mass	26
Sail booms, ABLE 0.94-m booms at 70 g/m	54
Mechanical systems, 40% contingency	111
Total sail assembly mass	448
Total mission launch mass	2353

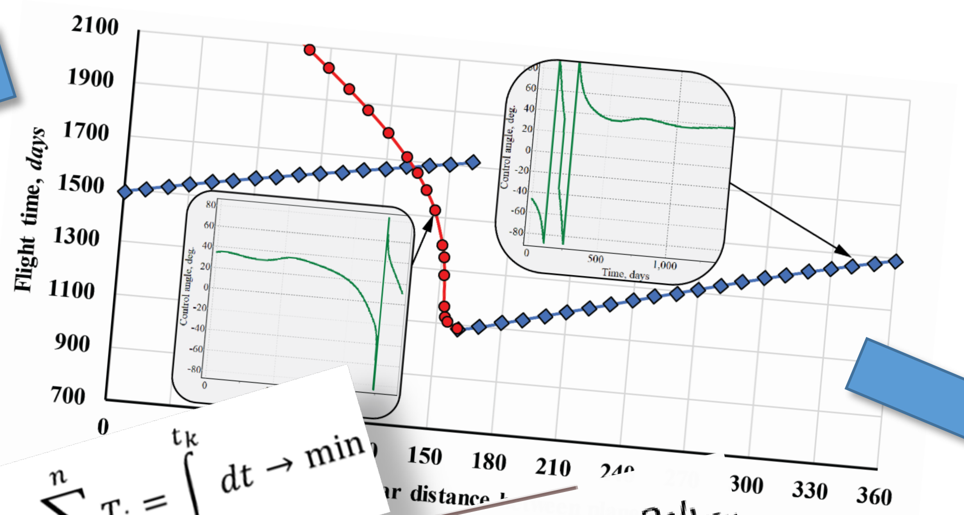
G.W. Hughes, M. Macdonald, C.R. McInnes et al., Sample Return from Mercury and Other Terrestrial Planets Using Solar Sail Propulsion, *Journal of Spacecraft and Rockets*, 43 (2006).

Year	Paper	Authors	Key Points
2001	Optimization of Interplanetary Flights of Spacecraft with Low-Thrust Engines Taking into Account the Ellipticity and Noncoplanarity of Planetary Orbits, *Cosm. Res.*	V. V. Salmin, O. L. Starinova	Fundamental approach of interplanetary flight optimization for low-thrust motion.
2005	Preliminary Design of Earth–Mars Cyclers Using Solar Sails, *J. Spacecraft & Rockets*	R. Stevens, M. I. Ross	One of the first approach to calculate cycler based on solar sails.
2007	Solar Sails for Mars Cargo Missions, *AIP Conf, Proc.*	R. H. Frisbee	Concept of a huge sail (20 and 25 km^2) without Mars orbit insertion.
2007	Solar-Sail-Based Stopover Cyclers for Cargo Transportation Mission, *J. Spacecr. Rockets*	G. Mengali, A. A. Quarta	Thorough research of a solar sail cycling trajectory with waiting for appropriate launch opportunity.
2018	Time-optimal solar sail heteroclinic-like connections for an Earth-Mars cycler, *Acta Astronaut.*	M. Vergaaij, J. Heiligers	Optimization of solar sail cycler between Earth-Moon L2 point and the Sun-Mars L1 point.

Part 1 Materials and Methods



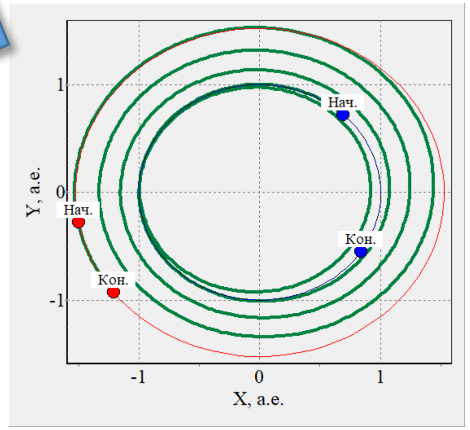
Part 2 Trajectory Optimization



$$t_k = \sum_{i=1}^{n} T_i = \int_0^{t_k} dt \to \min$$

$$\operatorname{tg}\theta = \frac{\sqrt{9\psi_{Vr}^2 + 8\psi_{Vu}^2} - 3\psi_{Vr}}{4\psi_{Vu}}$$

Part 3 Flight Simulation Results



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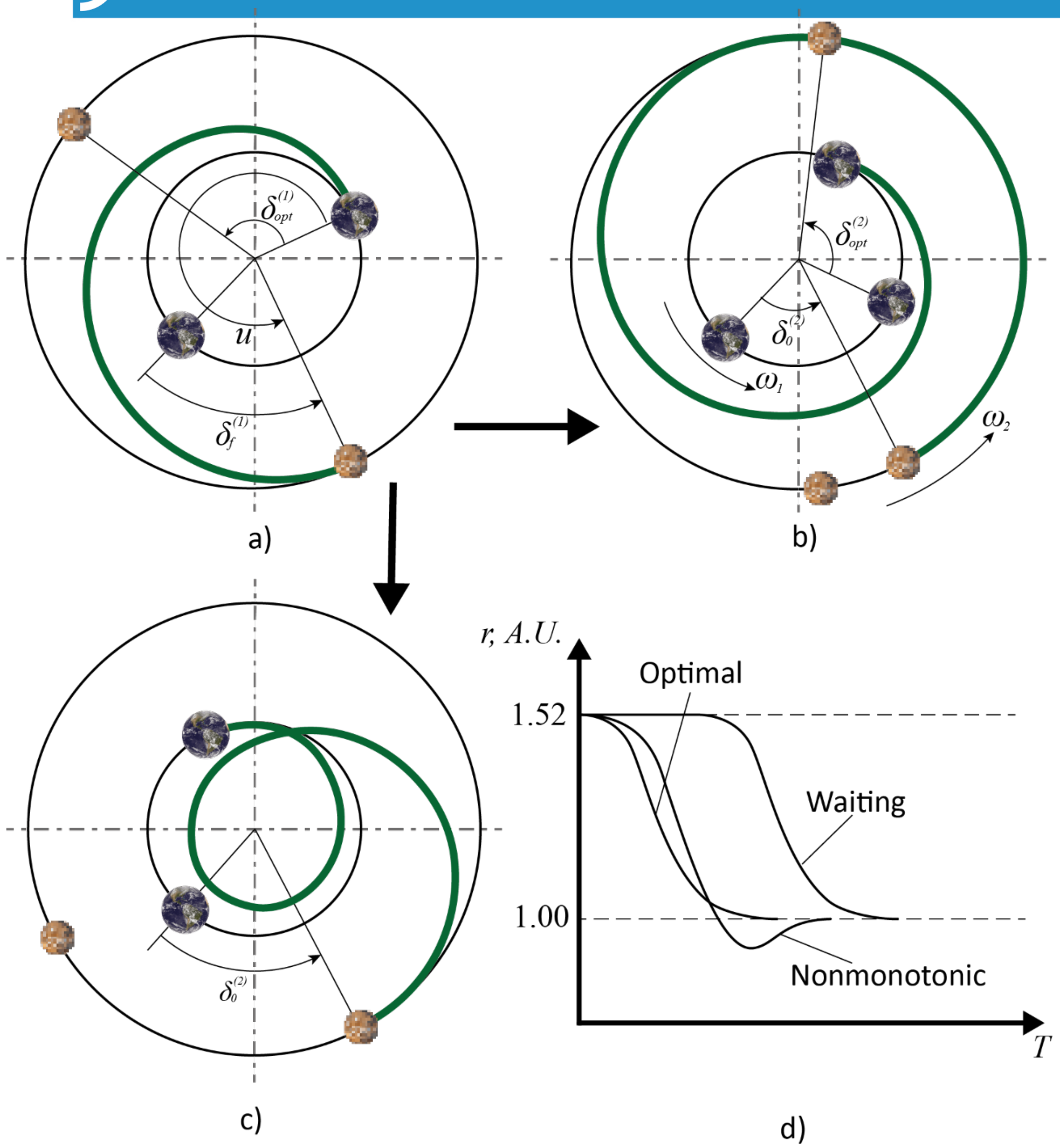

Fig. 2 Example of one cycle of motion:

a – Earth-Mars flight with the optimum planets position δ_{opt};

b – return flight after waiting for optimum planets position;

c – return flight without waiting;

d – change of a heliocentric distance with time.

- δ_0 – initial angular distance between planets.
- δ_{opt} – initial angular distance that ensures time-optimal rendezvous mission.
- ω_1 and ω_2 – angular velocities of planets.

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Ideal reflection

$$\mathbf{T} = 2P_r A\cos^2\theta = 2\frac{S_e}{c}\left(\frac{1}{r}\right)^2 A\cos^2\theta\,,$$

P_r – solar radiation pressure at heliocentric distance r; A – sail area;
c – speed of the light; S_e - solar irradiance on Earth's orbit;
θ – installation angle (incident angle).

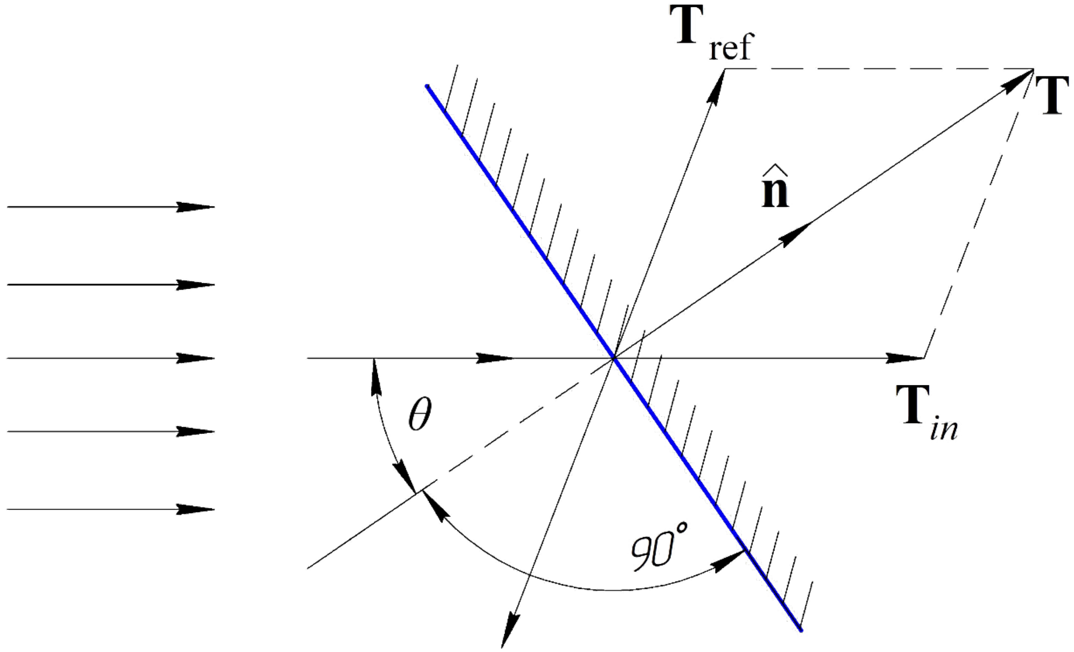

Fig. 3 Direction of the thrust for ideally reflecting sail is always along the back surface normal.

R. L. Forward, Grey Solar Sails, *Journal of the Astronautical Sciences* (1989)

$$a_\perp = 2\frac{P_r}{m}A\cdot\cos\theta\cdot(a_1\cos\theta + a_2),$$

$$a_{||} = -2\frac{P_r}{m}A\cdot\cos\theta\cdot a_3\sin\theta,$$

$$a_1 = \frac{1}{2}(1+\varsigma\rho),\; a_3 = \frac{1}{2}(1-\varsigma\rho),$$

$$a_2 = \frac{1}{2}\left(B_f(1-\varsigma)\rho + (1-\rho)\frac{\varepsilon_f B_f - \varepsilon_b B_b}{\varepsilon_f + \varepsilon_b}\right).$$

ρ – reflection coefficient;
ς – specular reflection factor;
ε_f, ε_b – emission coefficients;
B_f, B_b – non-Lambertian coefficients.

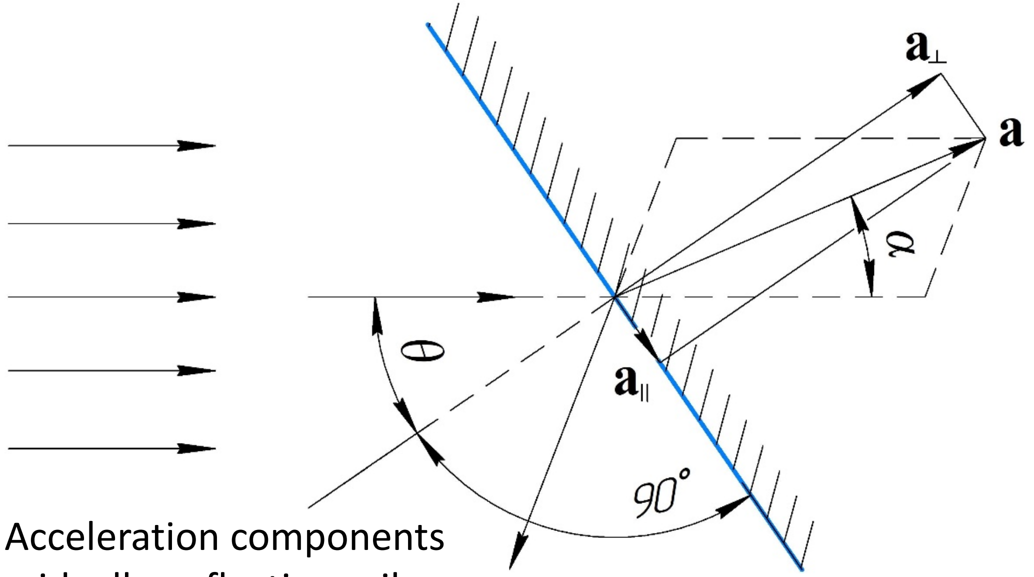

Fig. 4 Acceleration components for nonideally reflecting sail.

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B. Dachwald, G. Mengali, A.A. Quarta, M. Macdonald Parametric model and optimal control of solar sails with optical degradation, *Journal of Guidance, Control, and Dynamics* (2006)

Optical parameters exponential dependence on an accumulated solar radiation doze:

$$\frac{p(t)}{p_0} = \begin{cases} \dfrac{1+de^{-\lambda\Sigma(t)}}{1+d} & if \;\; p \in \{\rho, \varsigma\}, \\ 1+d\left(1-e^{-\lambda\Sigma(t)}\right) & if \;\; p = \varepsilon_f, \\ 1 & if \;\; p \in \{\varepsilon_b, B_f, B_b\}. \end{cases}$$

Dimensionless solar radiation dose:

$$\Sigma(t) = \frac{\tilde{\Sigma}(t)}{\tilde{\Sigma}_0} = \frac{1}{T_e}\int_{t_0}^{t} \frac{\cos\theta\,(t)}{r(t)^2}\,dt$$

Degradation factor d defines optical parameters values at which their change with time becomes infinitely low $\lim_{t\to\infty} p(t) = p_\infty$:

$$\rho_\infty = \frac{\rho_0}{1+d}, \; \varsigma_\infty = \frac{\varsigma_0}{1+d}, \; \varepsilon_{f\infty} = \varepsilon_{f0}(1+d)$$

Degradation coefficient λ corresponds to an amount of solar radiation doze that leads to change of the optical parameters by half:

$$\lambda = \frac{\ln 2}{\hat{\Sigma}} \qquad \hat{p} = \frac{p_0 + p_\infty}{2}$$

J. A. Dever, S. K. Miller, E. A. Sechkar, T.N. Wittberg Space Environment Exposure of Polymer Films on the Materials International Space Station Experiment: Results from MISSE 1 and MISSE 2, *High Performance Polymers* (2008)

Table 3 MISSE 2 experiment results for solar sail polymer films materials.

Samples with AL for Gossamer sail	Absorption coefficient			Reflection coefficient		
	Flight	Control	$\Delta\alpha$	Flight	Control	$\Delta\rho$
50.8 μm FEP	0.128	0.120	0.008	0.872	0.880	-0.008
25.4 μm Kapton HN	0.400	0.346	0.054	0.600	0.654	-0.054
25.4 μm Upilex S	0.487	0.437	0.050	0.513	0.563	-0.050
25.4 μm CP1	0.255	0.223	0.032	0.745	0.777	-0.032

Plane polar coordinate system.
Initial and target orbits are considered co-planar and circular.

$$\begin{cases} \dfrac{dr}{dt} = V_r, \\ \dfrac{du}{dt} = V_u, \\ \dfrac{dV_r}{dt} = a_r(r,\theta) - \dfrac{1}{r^2} + \dfrac{V_u^2}{r}, \\ \dfrac{dV_u}{dt} = a_u(r,\theta) - \dfrac{V_u V_r}{r}. \end{cases}$$

If degradation is considered:

$$\frac{d\Sigma}{dt} = \frac{1}{T_e}\frac{\cos\theta}{r^2} \Rightarrow a(\Sigma, r, \theta)$$

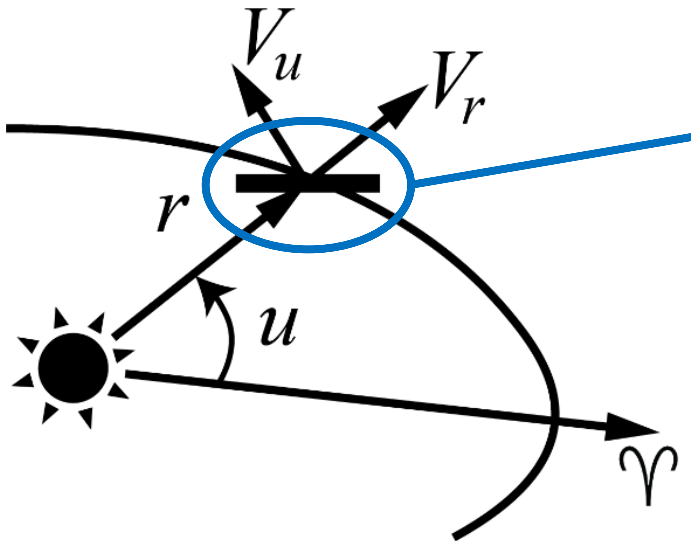

Fig. 5 Schematic illustration of polar coordinate system.

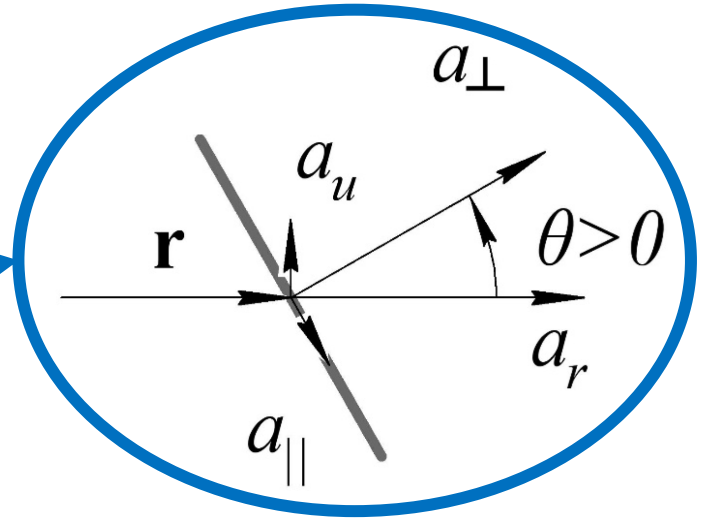

Fig. 6 Acceleration components for non-ideally reflecting solar sail and their projections on polar coordinate system.

$$a_r = a_\perp \cos\theta - a_{||} \sin\theta,$$
$$a_u = a_\perp \sin\theta + a_{||} \cos\theta.$$

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- Possible control: $\theta(t) \in U \in [-\pi/2, \pi/2]$ for $t \in [t_0, t_f]$;
- Ballistic parameter: δ_0 corresponds to a launch date of a solar sail;
- Spacecraft design parameters: $\mathbf{D} = \{m, A, \rho_0, \varsigma_0, \varepsilon_{f_0}, \varepsilon_b, B_f, B_b\}^{\mathrm{T}}$;
- Boundary conditions:

$$t = t_{i-1}, \qquad \boldsymbol{X}_{0,i} = \{r_{f,i-1}, u_{f,i-1}, V_{r_{f,i-1}}, V_{u_{f,i-1}}, \Sigma_{f,i-1}\}^{\mathrm{T}},$$

$$t = t_{i-1} + T_i, \qquad \boldsymbol{X}_{f,i} = \{r_{f,i}, u_{f,i}, V_{r_{f,i}}, V_{u_{f,i}}, \Sigma_{f,i} - \text{unfixed}\}^{\mathrm{T}}$$

To find minimum overall flight time for n cycles:

$$t_k^* = \min_{\theta(t),\, \delta_0} t_k \left(\theta(t),\, \delta_0 \mid \mathbf{D}, n = \text{fixed}, \theta(t) \in U, \mathbf{X}_0 = \mathbf{X}(t_0), \mathbf{X}_f = \mathbf{X}(t_f)\right).$$

For chosen δ_0 problem can be decomposed to a set of consequent $2n$ optimization problems:

$$T_i^* = \min_{\theta(t)} T_i \left(\theta(t) \mid \mathbf{D} = \text{fixed}, \theta(t) \in U, \delta_{0,i} = \delta_{f,i-1}, \mathbf{X}_{0,i} = \mathbf{X}(t_{i-1}), \mathbf{X}_{f,i} = \mathbf{X}(t_{i-1} + T_i)\right).$$

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Hamiltonian of the system with sail's degradation:

$$H = V_r\psi_r + V_u\psi_u + \left(a_r(\Sigma, r, \theta) - \frac{1}{r^2} + \frac{V_u^2}{r}\right)\psi_{Vr} + \left(a_u(\Sigma, r, \theta) - \frac{V_u V_r}{r}\right)\psi_{Vu} + \frac{1}{T_0}\frac{\cos\theta}{r^2}\psi_\Sigma$$

For any optimal control along with the optimal state trajectory it is necessary to Hamiltonian has maximum value:

$$\frac{\partial H}{\partial \theta} = \frac{a_c}{r^2}(a_3 \sin^3\theta + 2(a_1 + a_2)\sin\theta\cos\theta - (3a_1 + 2a_3)\cos^2\theta\sin\theta) + \frac{a_c}{r^2}(\cos^3\theta - 2\sin^2\theta\,(a_2 + \cos\theta) + a_2) - \frac{1}{T_0}\frac{\sin\theta}{r^2}\psi_\Sigma = 0,$$

where $a_c = 2\frac{S_e}{mc}A$ – solar sail's characteristic acceleration.

Flight time minimum optimal control for ideally reflecting solar sail

$$\operatorname{tg}\theta = \frac{\sqrt{9\psi_{Vr}^2 + 8\psi_{Vu}^2} - 3\psi_{Vr}}{4\psi_{Vu}}.$$

A.N. Zhukov, V.N. Lebedev Variational Problem of Transfer between Heliocentric Orbits by Means of a Solar Sail, translated from *Kosmicheskie Issledovaniya* (Cosmic Research), 1964.

$$\begin{cases} \dfrac{d\psi_r}{dt} = -\dfrac{\partial H}{\partial r} = \dfrac{1}{r^2}\left[V_u\psi_u + \left(\dfrac{2}{r}(S-1) + V_u^2\right)\psi_{Vr} + \left(\dfrac{2}{r}T - V_uV_r\right)\psi_{Vu} + \dfrac{2}{r}\dfrac{\cos\theta}{T_e}\psi_\Sigma\right], \\ \dfrac{d\psi_u}{dt} = -\dfrac{\partial H}{\partial u} = 0 \quad \rightarrow \quad \psi_u(t) \equiv const, \\ \dfrac{d\psi_{Vr}}{dt} = -\dfrac{\partial H}{\partial V_r} = -\psi_r + \dfrac{V_r}{r}\psi_u, \\ \dfrac{d\psi_{Vu}}{dt} = -\dfrac{\partial H}{\partial V_u} = \dfrac{1}{r}(V_r\psi_{Vu} - 2V_u\psi_{Vr} - \psi_u), \\ \dfrac{d\psi_\Sigma}{dt} = -\dfrac{\partial H}{\partial \Sigma} = -\dfrac{a_c}{r^2}\left(\psi_{Vr}\left(\dfrac{\partial a_2}{\partial\Sigma}\cos^2\theta + \dfrac{\partial a_3}{\partial\Sigma}\cos\theta\,(\sin^2\theta - \cos^2\theta)\right) + \psi_{Vu}\left(\sin\theta\cos\theta\left(\dfrac{\partial a_2}{\partial\Sigma} - 2\dfrac{\partial a_3}{\partial\Sigma}\cos\theta\right)\right)\right), \end{cases}$$

$$S = a_c\left(\cos^2\theta\,(a_1(\Sigma)\cos\theta + a_2(\Sigma)) + a_3(\Sigma)\cos\theta\sin^2\theta\right),$$
$$T = a_c\left(\cos\theta\sin\theta\,(a_1(\Sigma)\cos\theta + a_2(\Sigma)) - a_3(\Sigma)\cos^2\theta\sin\theta\right).$$

$$\frac{\partial a_3}{\partial\Sigma} = \frac{\lambda d e^{-\lambda\Sigma}}{(1+d)^2}(1 + d e^{-\lambda\Sigma})\rho_0\sigma_0 = -\frac{\partial a_1}{\partial\Sigma},$$

$$\frac{\partial a_2}{\partial\Sigma} = 0.5\left(B_f\left(\frac{2(1+de^{-\lambda\Sigma})}{1+d}\sigma_0 - 1\right)\frac{\lambda d e^{-\lambda\Sigma}}{1+d}\rho_0 + \left(1 + \frac{\lambda d e^{-\lambda\Sigma}}{1+d}\rho_0\right)\frac{\lambda d e^{-\lambda\Sigma}\,\varepsilon_f B_f - \varepsilon_b B_b}{\lambda d e^{-\lambda\Sigma}\,\varepsilon_f + \varepsilon_b}\right).$$

Vectors of state and costate variables

$$\mathbf{X} = \{r, u, V_r, V_u, \Sigma\}^{\mathrm{T}},$$
$$\boldsymbol{\psi} = \{\psi_r, \psi_u, \psi_{Vr}, \psi_{Vu}, \psi_\Sigma\}^T.$$

Boundary conditions

$$t = t_0, \qquad \mathbf{X}_0 = \{r_0, u_0, V_{r_0}, V_{u_0}, \Sigma_0\}^{\mathrm{T}},$$
$$t = T, \qquad \mathbf{X}_f = \{r_f, u_f, V_{r_f}, V_{u_f}\}^{\mathrm{T}}, \qquad \boldsymbol{\psi}_f = \psi_{\Sigma_f} = 0.$$

Ideally reflecting solar sail

Vectors of state and costate variables

$$\mathbf{X} = \{r, u, V_r, V_u\}^{\mathrm{T}},$$
$$\boldsymbol{\psi} = \{\psi_r, \psi_u, \psi_{Vr}, \psi_{Vu}\}^T.$$

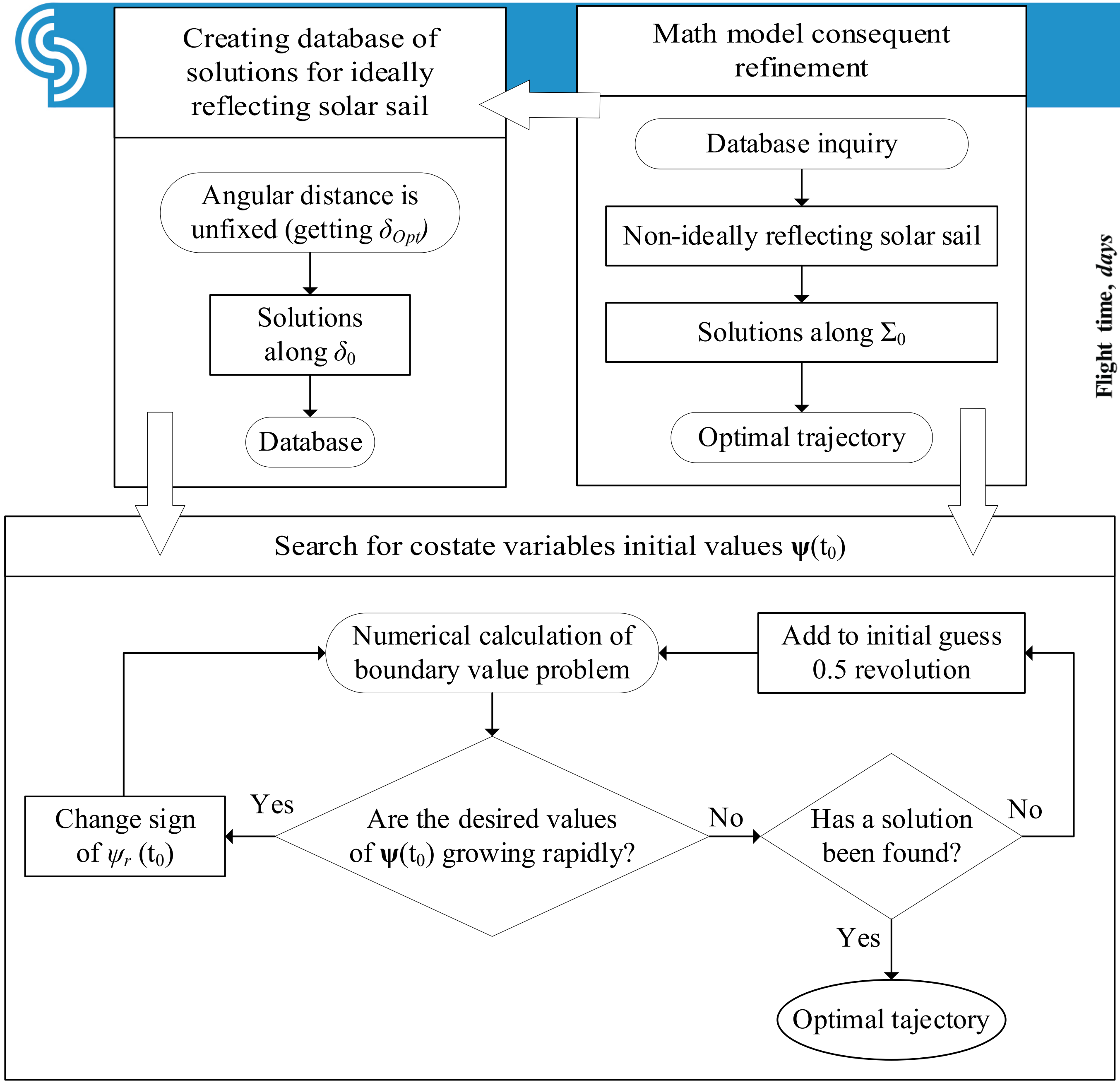
Boundary conditions

$$t = t_0, \qquad \mathbf{X}_0 = \{r_0, u_0, V_{r_0}, V_{u_0}\}^{\mathrm{T}},$$
$$t = T, \qquad \mathbf{X}_f = \{r_f, u_f, V_{r_f}, V_{u_f}\}^{\mathrm{T}}.$$

Normalization

$$\psi_r(t_0) = \pm 1,$$

TRAJECTORY OPTIMIZATION ALGORITHM



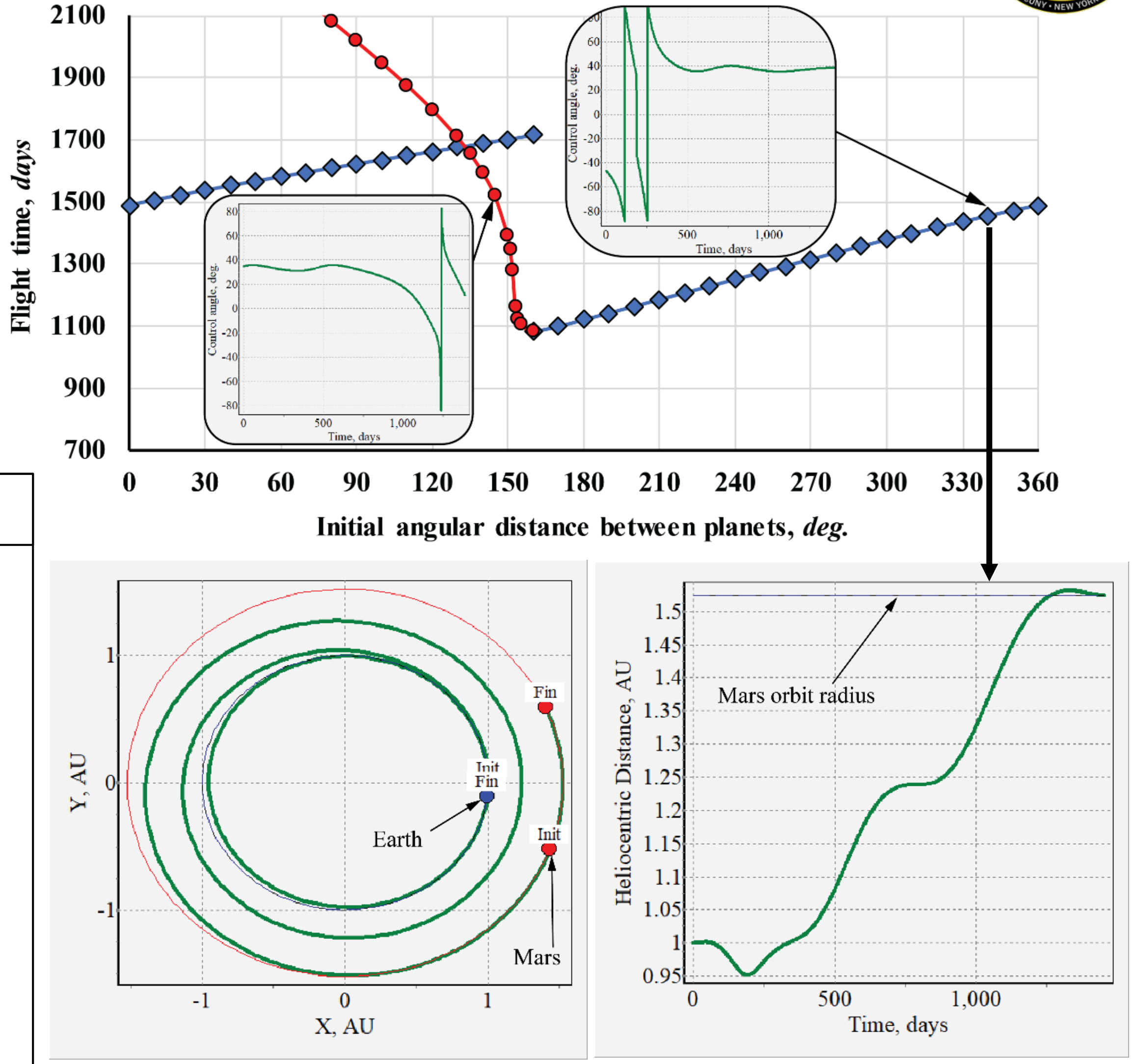


Fig. 7. Example of database for Earth-Mars trajectories.

Table 4. Spacecraft design parameters.

Sail area, m^2	Sail mass, *kg*	*Sail Construction mass, kg*	Total mass, *kg*	Characteristic acceleration, *mm/s²*
75625	257	191	2353	0.25

Optical Parameters:

$$\hat{\rho}_{Al} = 0.911;\ \varsigma = 0.94;$$
$$\varepsilon_f = 0.05; \varepsilon_b = 0.55;$$
$$B_f = 0.79; B_b = 0.55.$$

Degradation parameters:

$$\rho_\infty = 0.32$$
$$d = 1.75;$$
$$\lambda = 0.02;$$

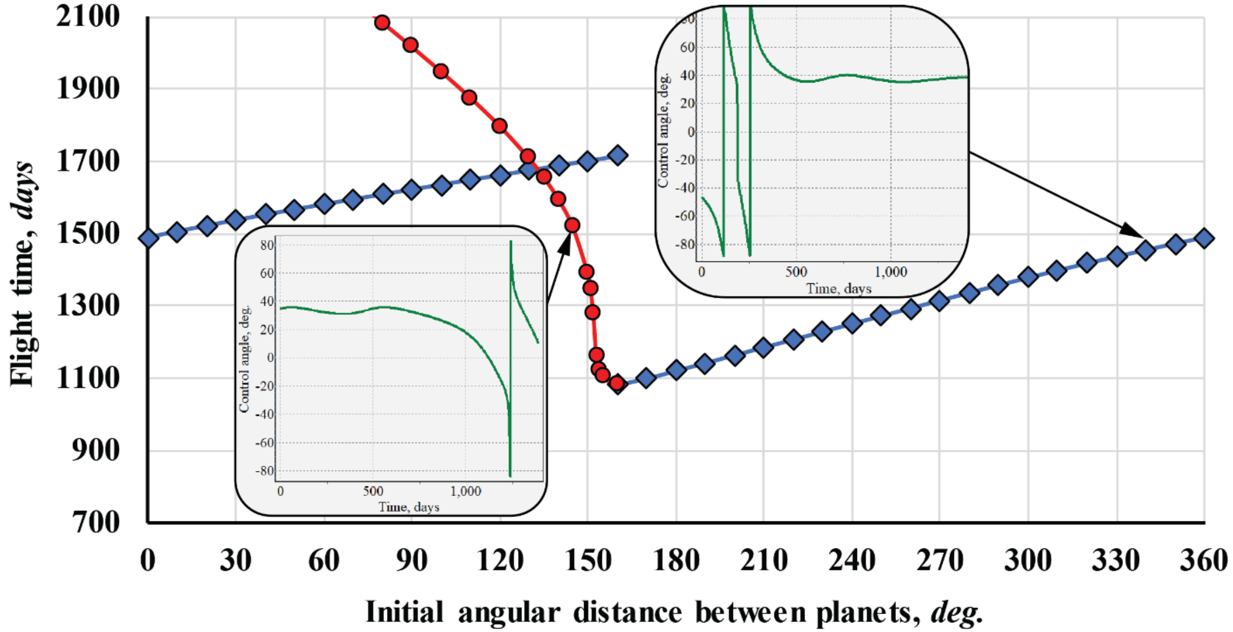


Fig. 8. Variation of T_i^* with δ_0 for Earth-Mars flights.

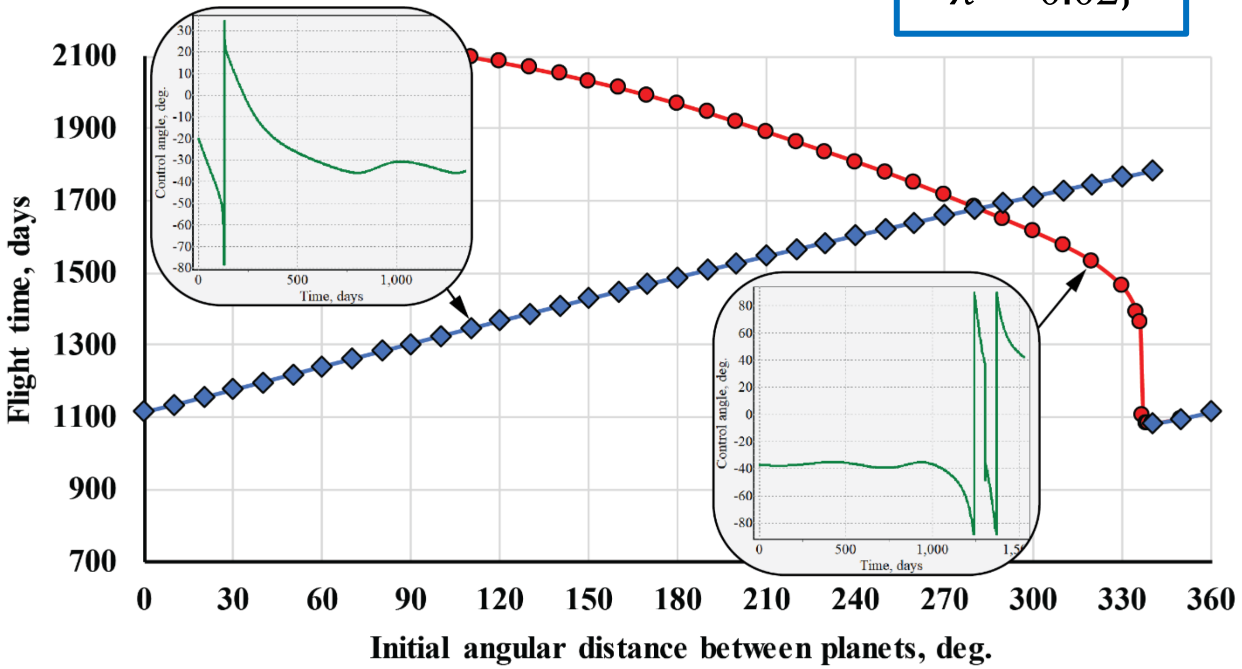


Fig. 9. Variation of T_i^* with δ_0 for Mars-Earth flights.

Two types of optimal control are shown:
● – synchronization is performed at the end of a trajectory, ♦ – at the beginning.

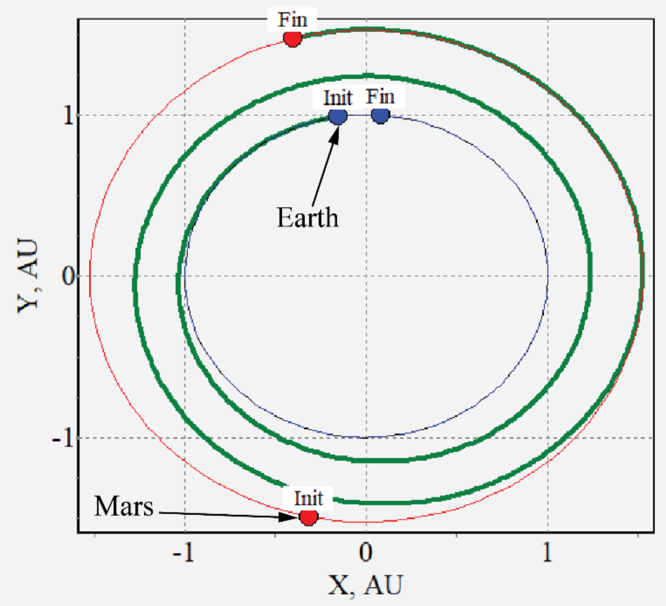


Table 5. Simulation results for 4 Earth-Mars-Earth cycles motion of ideally reflecting solar sail.

	First cycle		Second cycle		Third cycle		Fourth cycle	
	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.
Earth-Mars	2.96	159	3.23	207	3.23	207	3.23	207
Mars-Earth	3.16	20	3.18	23	3.18	23	3.18	23
Total time	6.12		12.53		18.94		25.35	

Launch date: 2023/12/31

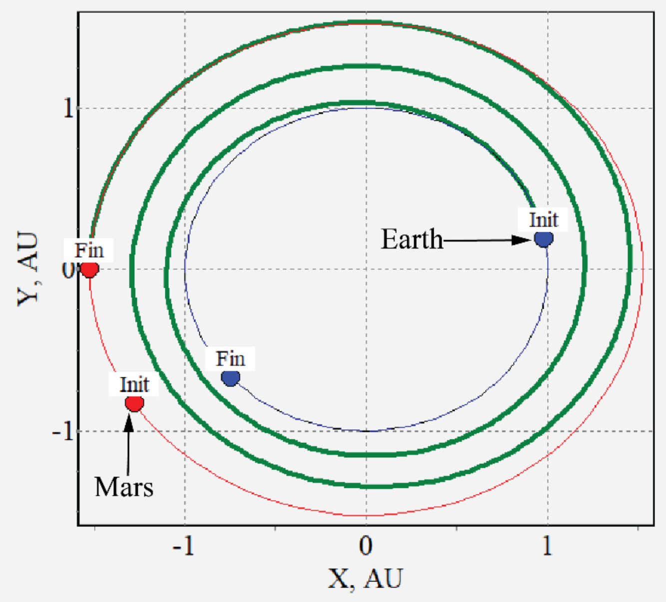


Table 6. Simulation results for 4 Earth-Mars-Earth cycles motion of non-ideally reflecting solar sail.

	First cycle		Second cycle		Third cycle		Fourth cycle	
	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.	T_f, year	δ_0, deg.
Earth-Mars	3.58	202	4.11	196	6.29	166	5.78	144
Mars-Earth	5.00	318	4.62	135	4.52	174	5.03	251
Total time	8.58		17.30		28.11		38.91	

Launch date: 2023/10/05

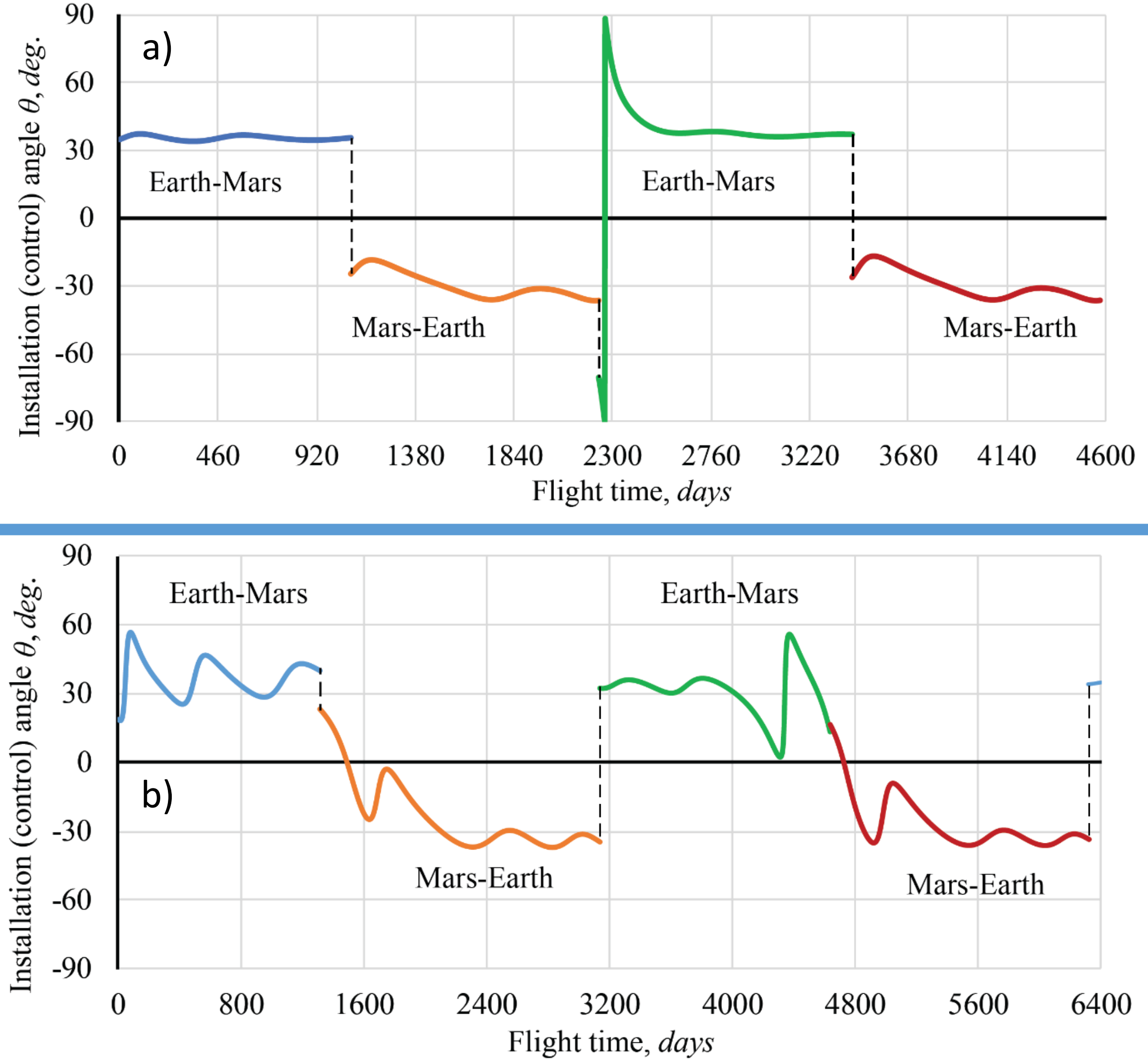


Fig. 9. Control program of the 1st and 2nd cycles for a) ideally and b) non-ideally reflecting solar sail.

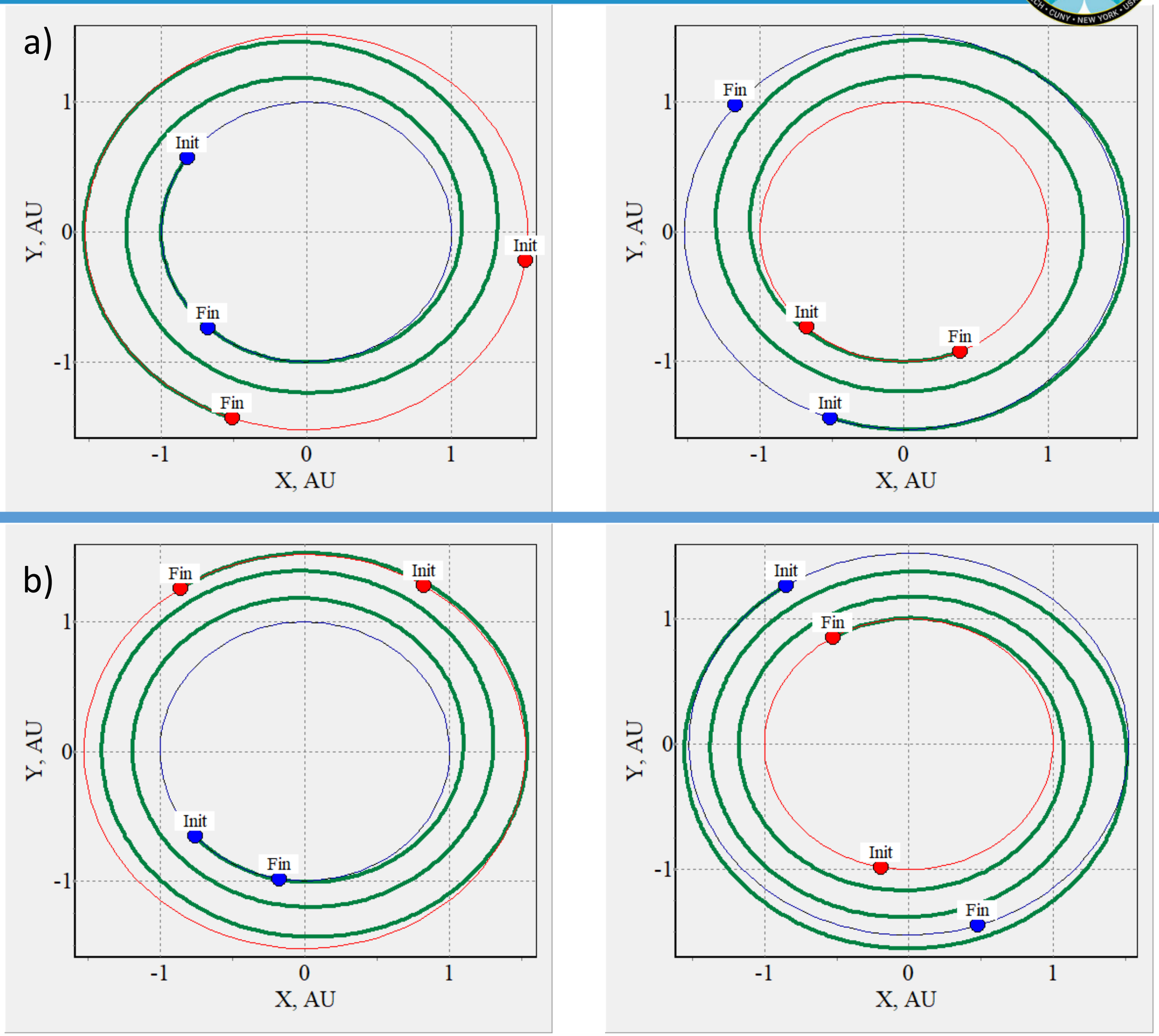


Fig. 10. Heliocentric trajectories of the 2nd cycle for a) ideally and b) non-ideally reflecting solar sail.

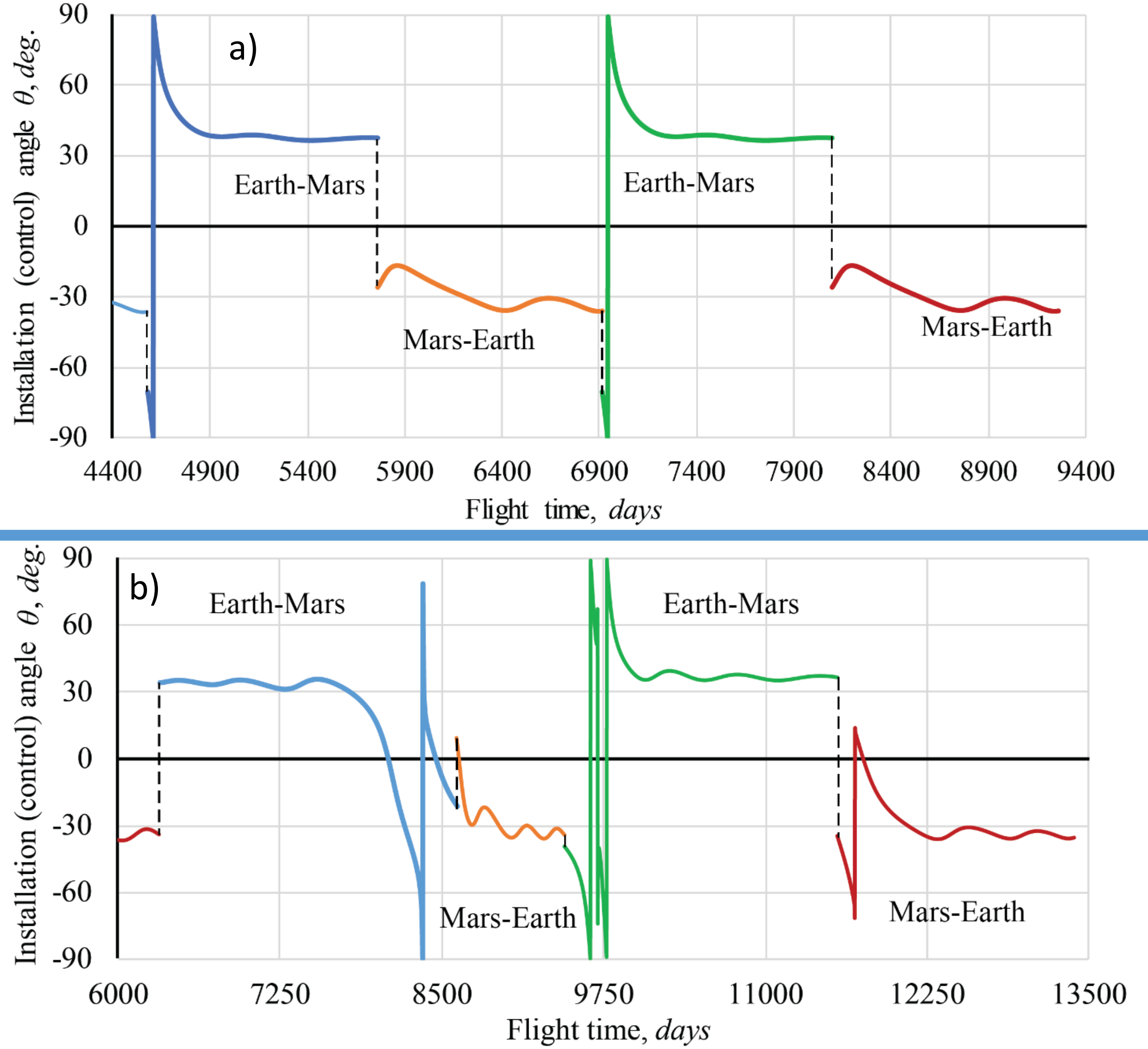


Fig. 11. Control program of the 3rd and 4th cycles for a) ideally and b) non-ideally reflecting solar sail.

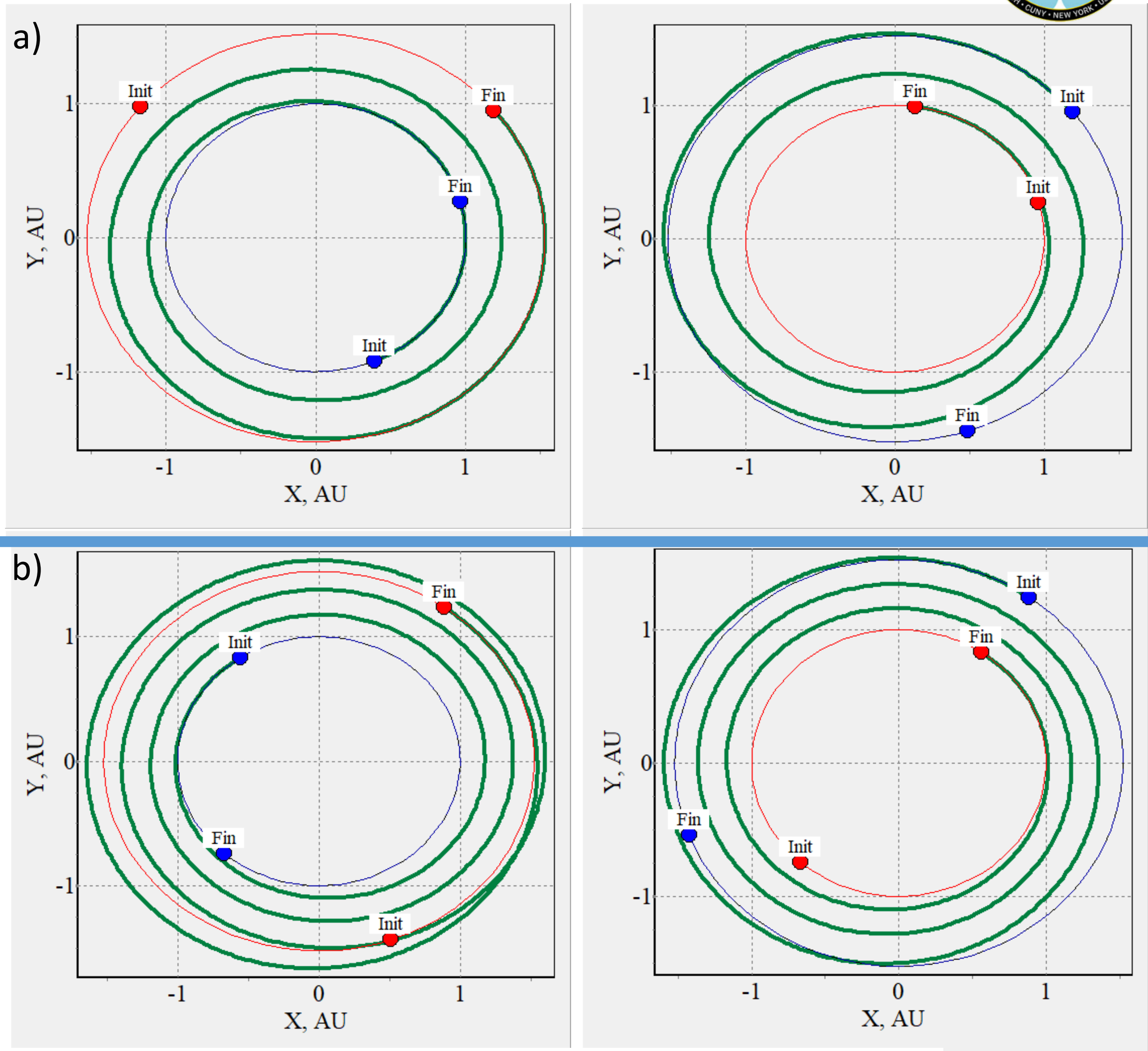
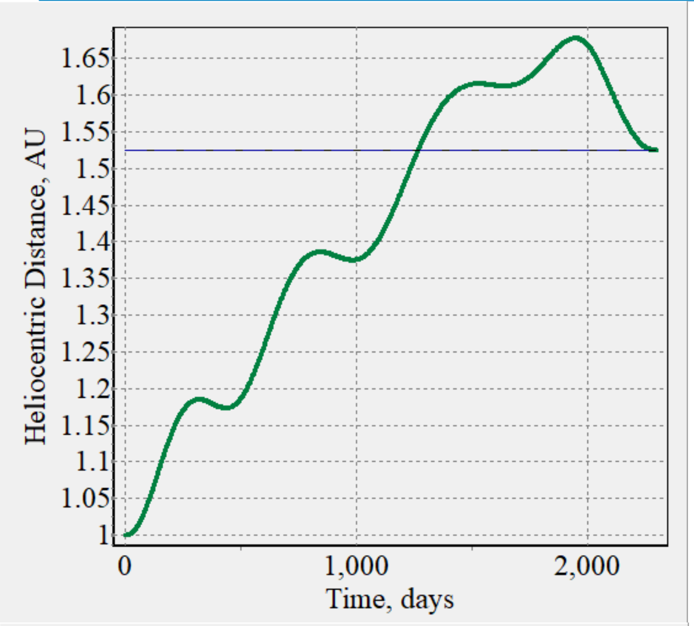
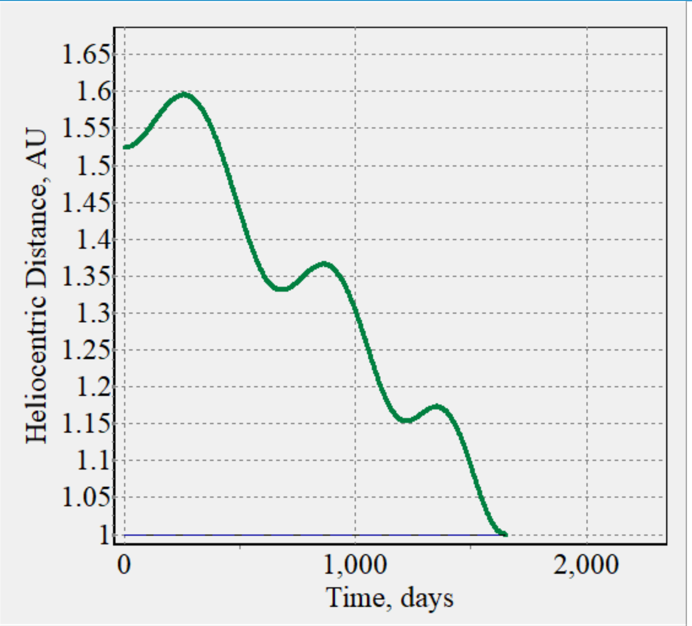
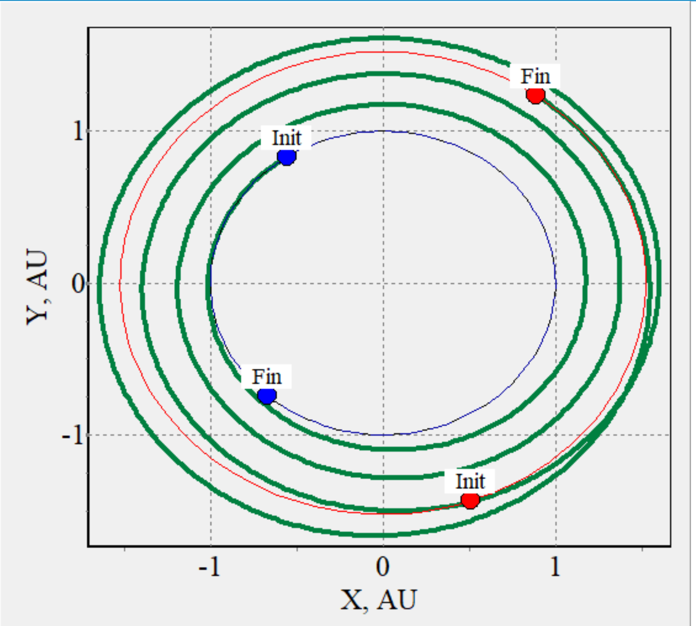


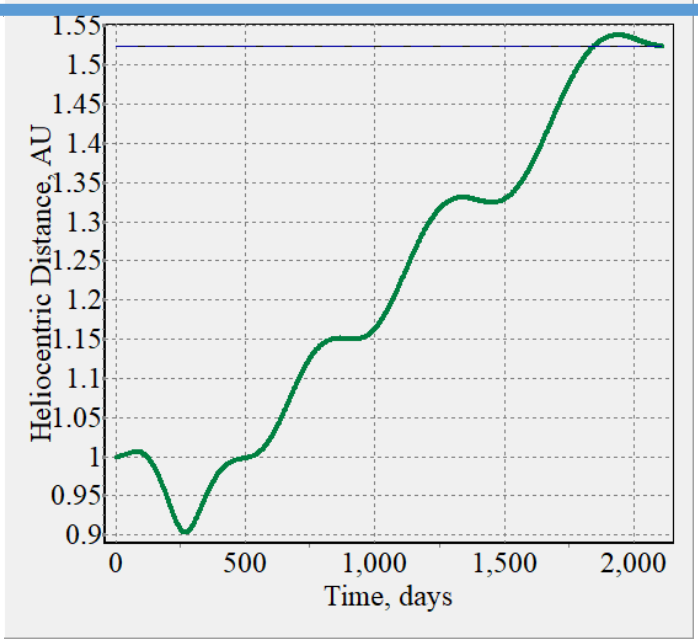
Fig. 12. Heliocentric trajectories of the 3rd cycle for a) ideally and b) non-ideally reflecting solar sail.

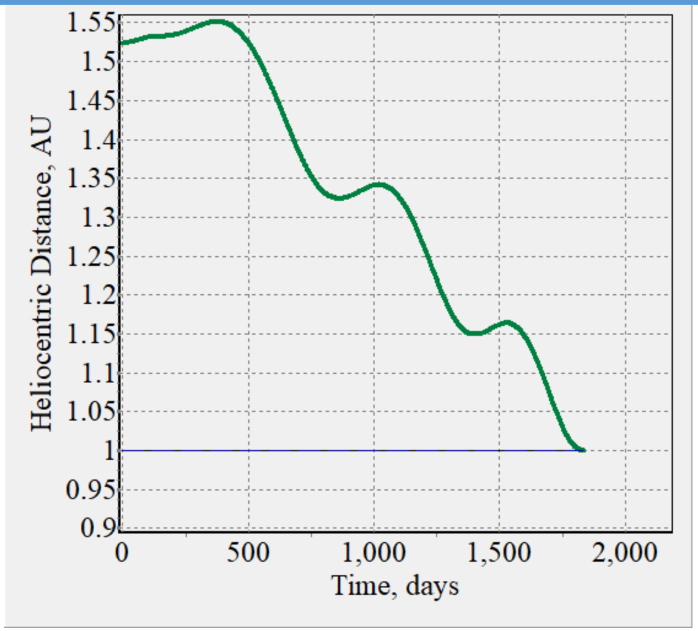
SYNCHRONIZATION AT THE END AND BEGINNING OF THE 3RD AND 4TH EARTH-MARS FLIGHTS

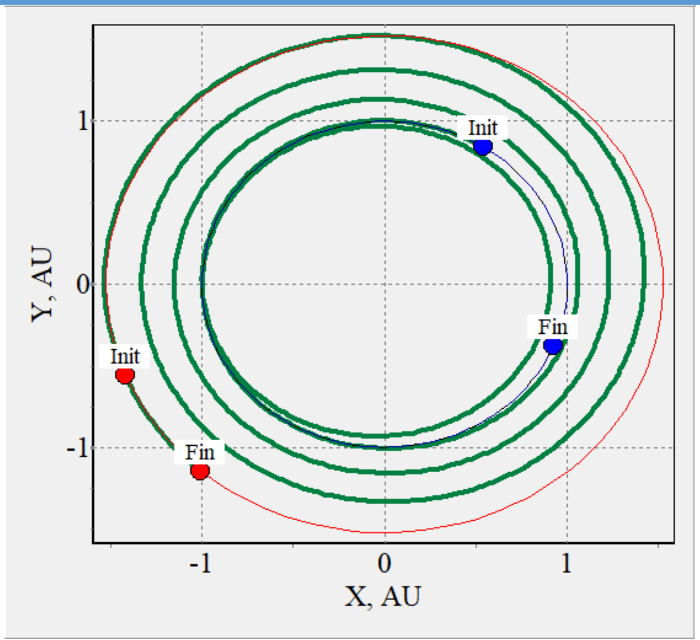


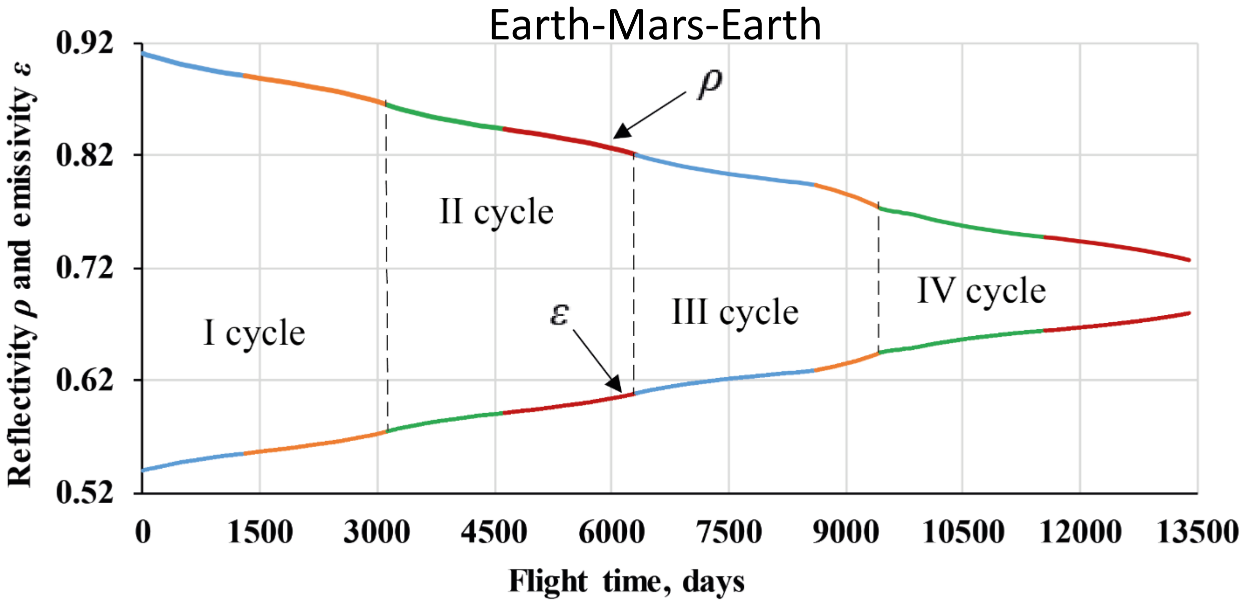


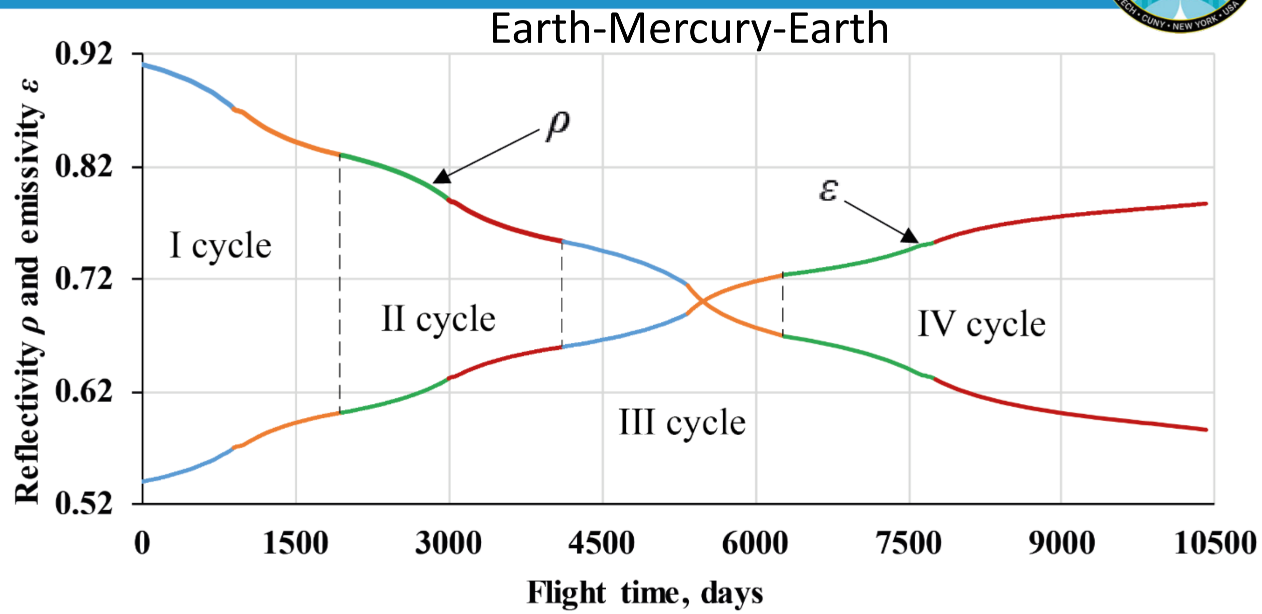


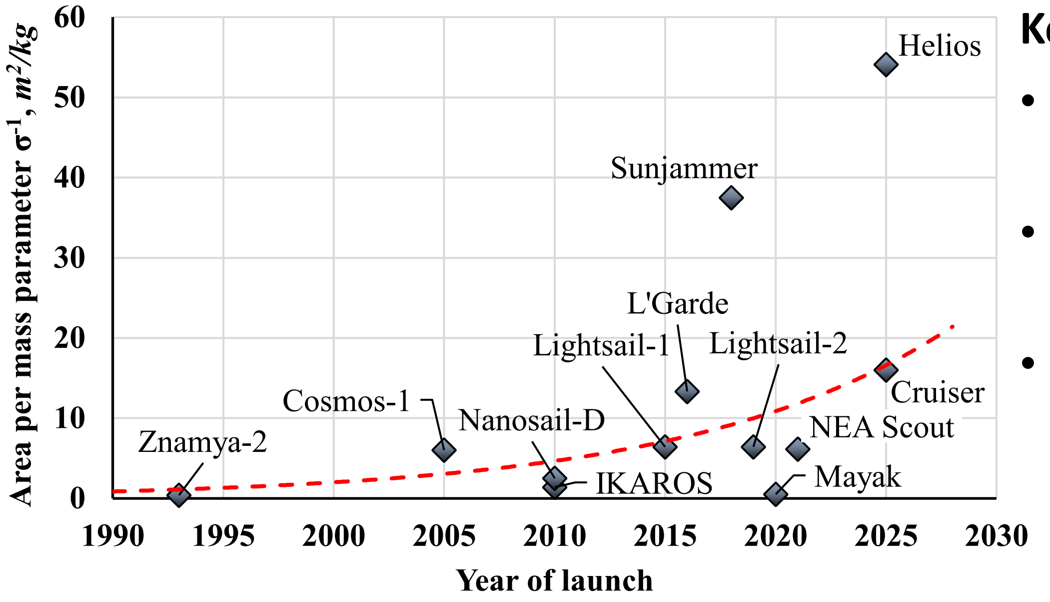












Key points:

- Degradation dramatically influences a solar sail motion, but it will be able to perform a cyclic motion even after decreasing of reflectivity by half;
- Four cycles of Earth-Mars-Earth will take almost 39 years and 29 years for Earth-Mercury-Earth for $a_c = 0.25\ mm/s^2$ solar sail;
- Degradation will not allow forming a cyclic motion with a constant cycle time, as it happens for an ideally reflecting solar sail.

Questions:

- How detailed is your mathematical interpretation of solar sail performance when planning a mission and sail design? Is it even necessary to go so deep into theory or just try it straight in space?

- How about creating a digital platform where all data relevant to the solar sails will be stored and sorted? Solar Sailing Data Center (SSDC) website, something like wikipedia for space sailing.

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THANK YOU

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