Blended Locally-Optimal Control Laws for Space Debris Removal in LEO using a Solar Sail

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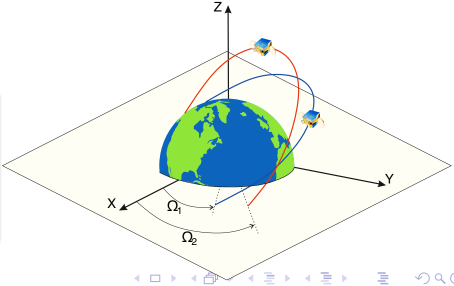
Introduction - Space debris

Space Debris Problem

- Increasing **number of objects**
- **Active removal** strategies
- **Constellations**

Space Debris Model

- **Circular** orbits
- **Same inclination**
- **Different RAAN**



Introduction - Mission Scenario

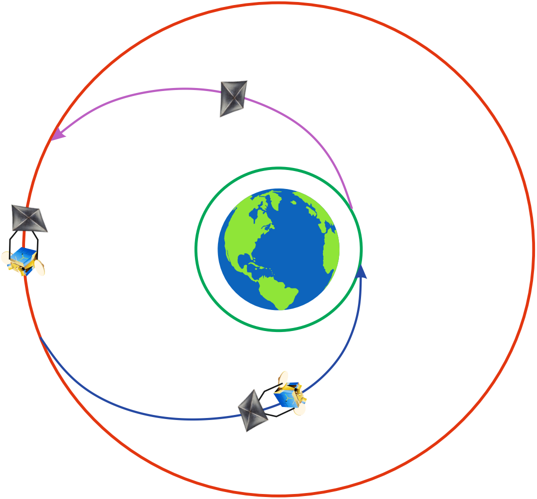
Description

Solar Sail to:

- **reach** debris orbit
- **collect** it
- **bring** it **down**

Assumption

No phasing (ν is free)



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Equations of Motion

Modified Equinoctial Orbital Elements (MEOEs)

$$p = a\,(1 - e^2)$$
$$f = e\,\cos(\Omega + \omega)$$
$$g = e\,\sin(\Omega + \omega)$$
$$h = \tan(i/2)\,\cos(\Omega)$$
$$k = \tan(i/2)\,\sin(\Omega)$$
$$L = \Omega + \omega + \nu$$

$\boldsymbol{x} = [p,\ f,\ g,\ h,\ k,\ L]^{\mathrm{T}}$ sail **state vector**

$\dot{\boldsymbol{x}} = \mathbb{A}(\boldsymbol{x})\,\boldsymbol{a} + \boldsymbol{b}(\boldsymbol{x})$ **equation of motion**

$\boldsymbol{a} = \boldsymbol{a}_{SRP} + \boldsymbol{a}_D + \boldsymbol{a}_L + \boldsymbol{a}_{J_2}$ **perturbing acceleration** (in RTN frame)

Equations of Motion

$$\mathbb{A}(\boldsymbol{x}) = \begin{bmatrix} 0 & \frac{2p}{q}\sqrt{\frac{p}{\mu}} & 0 \\ \sqrt{\frac{p}{\mu}}\sin L & \sqrt{\frac{p}{\mu}}\frac{1}{q}\left[(q+1)\cos L + f\right] & -\sqrt{\frac{p}{\mu}}\frac{g}{q}\left[h\sin L - k\cos L\right] \\ -\sqrt{\frac{p}{\mu}}\cos L & \sqrt{\frac{p}{\mu}}\frac{1}{q}\left[(q+1)\sin L + g\right] & \sqrt{\frac{p}{\mu}}\frac{f}{q}\left[h\sin L - k\cos L\right] \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{s^2\cos L}{2q} \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{s^2\sin L}{2q} \\ 0 & 0 & \sqrt{\frac{p}{\mu}}\frac{1}{q}\left[h\sin L - k\cos L\right] \end{bmatrix}$$

$$\boldsymbol{b}(\boldsymbol{x}) = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \sqrt{\mu p}\left(\frac{q}{p}\right)^2 \end{bmatrix}^{\mathrm{T}}$$

$$q = 1 + f\cos L + g\sin L \qquad s^2 = 1 + h^2 + k^2$$

Solar Radiation Pressure Acceleration

Ideal Force Model

- **Flat** plate
- Only **specular reflection**

$$\boldsymbol{a}_{SRP} = \eta \, a_c \, \cos^2 \alpha \, \hat{\boldsymbol{n}}$$

η : **shadow factor** ($\eta = 0$ eclipse, $\eta = 1$ sunlight)

a_c : **characteristic acceleration**

$\alpha \in [0, \pi/2]$: **cone angle**

Assumption

The dependence of $\boldsymbol{a}_{SRP}$ on the distance from the Sun is neglected

Solar Radiation Pressure Acceleration

Sunlight reference frame $\mathcal{T}_S$:

$\hat{\boldsymbol{x}}_S$: Sun-sail direction

$$\hat{\boldsymbol{y}}_S = \hat{\boldsymbol{z}}_I \times \hat{\boldsymbol{x}}_S$$

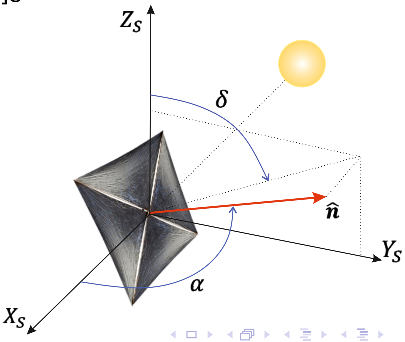
$$[\hat{\boldsymbol{n}}]_S = \cos\alpha\,\hat{\boldsymbol{x}}_S + \sin\alpha\sin\delta\,\hat{\boldsymbol{y}}_S + \sin\alpha\cos\delta\,\hat{\boldsymbol{z}}_S$$

$$[\hat{\boldsymbol{n}}]_{RTN} = \mathbb{R}_{S\to RTN}\,[\hat{\boldsymbol{n}}]_S$$

Control Variables

Cone angle $\alpha \in [0, \pi/2]$

Clock angle $\delta \in [0, 2\pi)$



Atmospheric Drag and Lift

Atmospheric Model

- **Hyperthermal** free-molecular flow ($v_{S/C} \gg v_{\text{thermal}}$)
- **NRLMSISE-00** model (MATLAB function *atmosnrlmsise00*)
- **Flat plate**

$$\boldsymbol{a}_{\text{D}} = -\frac{1}{2}\frac{\rho\, v^2}{\sigma} C_D\, \hat{\boldsymbol{v}}$$

$$\boldsymbol{a}_{\text{L}} = \frac{1}{2}\frac{\rho\, v^2}{\sigma} C_L\, \hat{\boldsymbol{L}}$$

ρ : **atmospheric density** (*atmosnrlmsise00*)

σ : **sail loading** ($\sigma = m/A$)

v : sail inertial **velocity**

Atmospheric Drag and Lift

Drag and **Lift coefficients**:

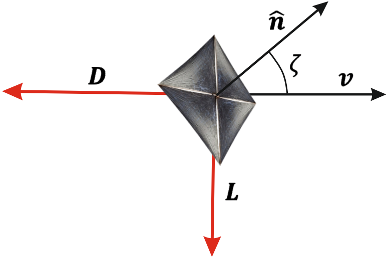
$$C_D = 2\left[\sigma_T + \sigma_N V_R \left|\cos\zeta\right| + (2 - \sigma_N - \sigma_t)\cos^2\zeta\right]\,\left|\cos\zeta\right|$$

$$C_L = 2\left[\sigma_N V_R + (2 - \sigma_N - \sigma_T)\cos^2\zeta\right]\,\left|\cos\zeta\right|\,\sin\zeta$$

σ_N, σ_T : normal and tangential **accommodation coefficients** (≈ 0.8)

V_R : **ratio** of the average **thermal speed** to the **sail velocity** (≈ 0.05)

$\zeta \in [0, \pi]$: angle between $\hat{\boldsymbol{n}}$ and $\hat{\boldsymbol{v}}$



Earth's oblateness

$$[\boldsymbol{a}_{J_2}]_\mathrm{R} = -\frac{3\,\mu\,J_2\,R_\oplus^2}{2r^4}\left[1 - \frac{12\,(h\sin L - k\cos L)^2}{(1+h^2+k^2)^2}\right]$$

$$[\boldsymbol{a}_{J_2}]_\mathrm{T} = -\frac{12\,\mu\,J_2\,R_\oplus^2}{r^4}\left[\frac{(h\sin L - k\cos L)\,(h\cos L + k\sin L)}{(1+h^2+k^2)^2}\right]$$

$$[\boldsymbol{a}_{J_2}]_\mathrm{N} = -\frac{6\,\mu\,J_2\,R_\oplus^2}{r^4}\left[\frac{(1-h^2-k^2)\,(h\sin L - k\cos L)}{(1+h^2+k^2)^2}\right]$$

$\mu = 3.986 \times 10^5\,\mathrm{km}^3/\mathrm{s}^2$ Earth's **gravitational parameter**

$R_\oplus = 6378.14\,\mathrm{km}$ Earth's mean **equatorial radius**

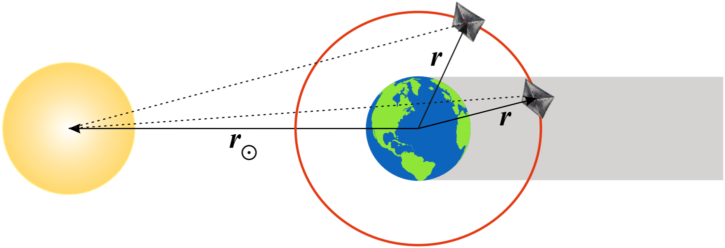
$J_2 = 1.0826 \times 10^{-3}$ Earth's **second harmonic coefficient**

r = **orbital radius** of the sail

Eclipses

Eclipse Model

- **Cylindrical** model
- **Shadow** factor $\eta = \begin{cases} 0 & \text{if} \quad \theta_\odot + \theta_{\text{sail}} < \theta \quad \rightarrow \quad \text{Eclipse} \\ 1 & \text{if} \quad \theta_\odot + \theta_{\text{sail}} \geq \theta \quad \rightarrow \quad \text{Sunlight} \end{cases}$



$$\theta = \arccos\left(\frac{\boldsymbol{r}_\odot \cdot \boldsymbol{r}}{r_\odot r}\right), \quad \theta_\odot = \arccos\left(\frac{R_\oplus}{r_\odot}\right), \quad \theta_{\text{sail}} = \arccos\left(\frac{R_\oplus}{r}\right)$$

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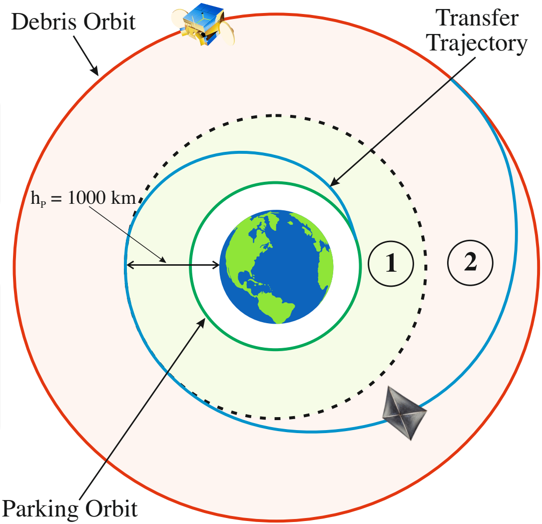
Transfer Strategy

Optimization Technique

Locally-optimal control laws (orbital perturbations)

Phases

- 1st : SMA increase
- 2nd : debris targeting
- Descent



1st Phase - Description

Objective

Find **optimal sail attitude** $\{\alpha_{opt}, \delta_{opt}\}$ that **minimizes** the **cost index** J at each time (MATLAB *fmincon*)

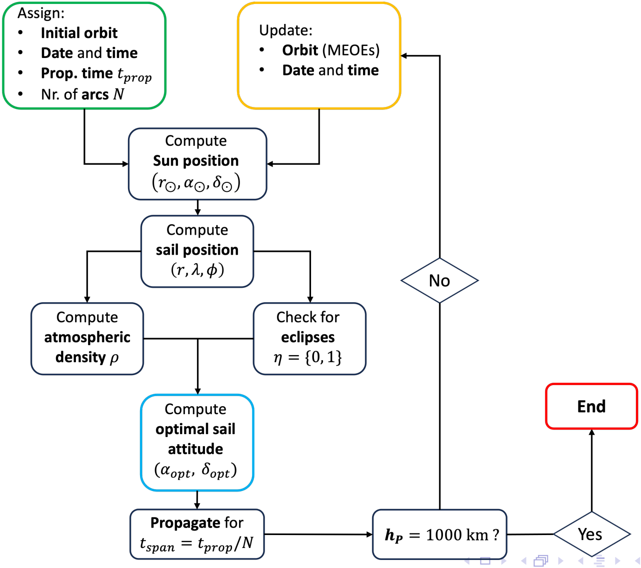
Cost Index

$$J = -\frac{da}{dt} = -2\sqrt{\frac{a^3}{\mu\,(1-e^2)}}\left[e \sin\nu\, a_R + (1 + e \cos\nu)\, a_T\right]$$

Stopping Criterion

The 1st phase ends when the **perigee altitude** of the osculating orbit reaches 1000 km

1st Phase - Optimization Procedure



2^{nd} Phase - Blended Control Law

Objective

Find **optimal sail attitude** $\{\alpha_{opt},\, \delta_{opt}\}$ that **minimizes** the **cost index** J at each time (MATLAB *fmincon*)

Cost Index

$$J = W_a\, R_a\, \frac{d(a/a_0)}{dt} + W_e\, R_e\, \frac{de}{dt} + W_i\, R_i\, \frac{di}{dt}$$

$W_e,\ W_e,\ W_i \in [0,\, 1]$: **constant weights**

$R_a,\ R_e,\ R_i$: **variable weights**

$$\frac{de}{dt} = \sqrt{\frac{a\,(1-e^2)}{\mu}}\left[\sin\nu\, a_R + \left(\cos\nu + \frac{e+\cos\nu}{1+e\cos\nu}\right) a_T\right]$$

$$\frac{di}{dt} = \sqrt{\frac{a\,(1-e^2)}{\mu}}\,\frac{\cos(\omega+\nu)}{1+e\cos\nu}\, a_N$$

2nd Phase - Blended Control Law

Variable Weights

- Adjust the **relative importance** of each orbital element according to the "distance" from the target
- The **sign** indicates if the o.e. time derivative has to be **maximized** ($R < 0$) or **minimized** ($R > 0$)

$$R_a = \frac{a - a_t}{|a_0 - a_t|}, \quad R_e = \frac{e - e_t}{|e_0 - e_t|}, \quad R_i = \frac{i - i_t}{|i_0 - i_t|}$$

where

$\{a_t,\ e_t,\ i_t\}$ are the **target semimajor axis**, **eccentricity** and **inclination**

$\{a,\ e,\ i\}$ are the **sail instantaneous** orbital elements

$\{a_0,\ e_0,\ i_0\}$ are the **sail** orbital elements at the **end of the 1st phase**

2^{nd} Phase - RAAN Matching

J2 Perturbation

- a, e, i undergo short-term oscillations
- Ω has a **secular drift** ($\dot{\Omega} < 0$ for prograde orbits)

Assumption

RAAN behaviour depends only on J_2 (contribution of SRP is negligible)

RAAN of target **debris**: $\Omega_t(t) = \Omega_{t,0} + \dot{\Omega}_t\, t$

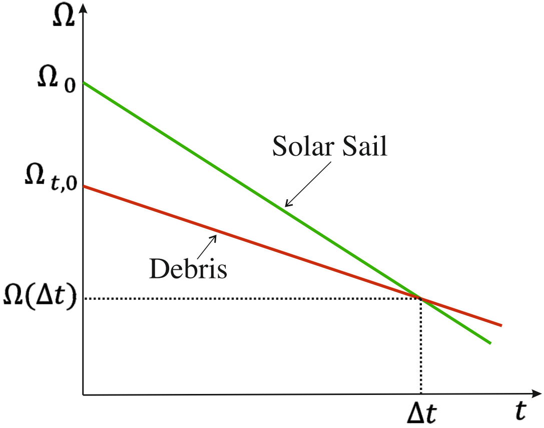
RAAN of **solar sail**: $\Omega(t) = \Omega_0 + \dot{\Omega}_{avg}\, t$

$$\dot{\Omega}_{avg} = -\frac{3}{2} \frac{J_2 \sqrt{\mu}\, R_\oplus^2}{a_{avg}^{7/2} (1 - e_{avg}^2)^2} \cos i_{avg}$$

$$a_{avg} = (a_0 + a_t)/2 \qquad e_{avg} = (e_0 + e_t)/2 \qquad i_{avg} = (i_0 + i_t)/2$$

2^{nd} Phase - RAAN Matching

Estimated flight time Δt s.t. $\Omega_t(\Delta t) = \Omega(\Delta t)$



2nd Phase - Genetic Algorithm

Objective

Find the 3 **optimal constant weights** $W_{a,e,i}$ that **minimize** the **objective function** F

GA Objective Function

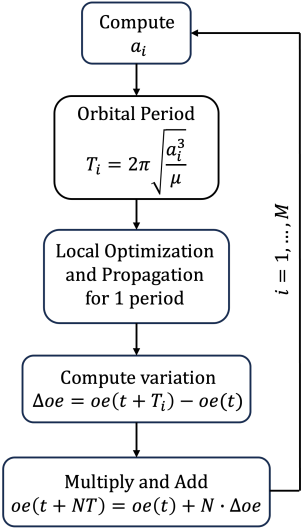
$$F = \left(\frac{a_{\text{fin}} - a_t}{a_0}\right)^2 + (e_{\text{fin}} - e_t)^2 + (i_{\text{fin}} - i_t)^2 + (\Omega_{\text{fin}} - \Omega_t)^2$$

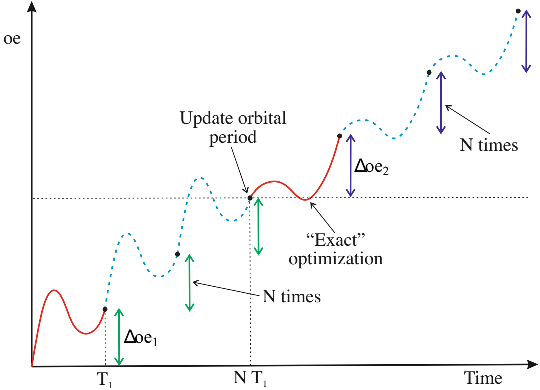
a_{fin}, e_{fin}, i_{fin}, Ω_{fin}: sail orbital elements at the end of propagation time Δt

Approximate Model

An **approximate model** for the transfer is necessary to **reduce the computational time** of the GA

2nd Phase - Approximate Model





Descent Phase - Description

Assumptions

- Flight **time** is **not constrained**
- **RAAN** matching is **not necessary**
- **Sail loading** $\sigma_{\text{desc}} = 2\sigma$ due to **debris mass**
- $\sigma_{\text{desc}} = 2\sigma \implies a_{c,\text{desc}} = a_c/2$

Initial Orbit (= Debris Orbit)

$a_0 = a_t = R_\oplus + 1200\,\text{km}$

$e_0 = e_t = 0$

$i_0 = i_t$

Final Orbit (= Parking Orbit)

$a_{\text{fin}} = R_\oplus + 600\,\text{km}$

$e_{\text{fin}} = 0$

$i_{\text{fin}} = i_t$

Descent Phase: blended control law

Objective

Find **optimal sail attitude** $\{\alpha_{opt}, \delta_{opt}\}$ that **minimizes** the **cost index** J at each time (MATLAB *fmincon*)

Cost Index

$$J = R_a \frac{d(a/a_0)}{dt} + R_e \frac{de}{dt} + R_i \frac{di}{dt}$$

$$R_a = \frac{a - a_{\text{fin}}}{|a_0 - a_{\text{fin}}|}, \quad R_e = e - e_{\text{fin}}, \quad R_i = i - i_{\text{fin}}$$

Weights

Necessary to redefine R_e and R_i since $e_0 = e_{\text{fin}} = 0$ and $i_0 = i_{\text{fin}}$

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Numerical Simulations - 1st Phase - Data

Start Date: 01/01/2030

Characteristic **Acceleration**: $a_c = 0.1\,\text{mm/s}^2$

Stopping Criterion: $h_P = 1000\,\text{km}$

Sail parking orbit

$h_0 = 600\,\text{km}$

$e_0 = 0$

$i_0 = 60\,\text{deg}$

$\Omega_0 = 0\,\text{deg}$

Target debris orbit

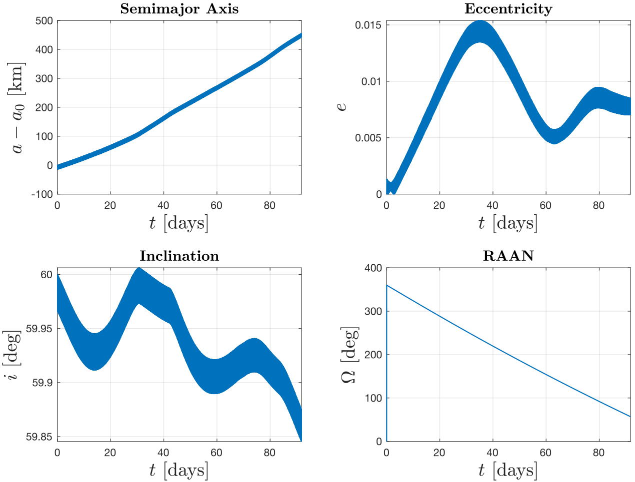
$h_t = 1200\,\text{km}$

$e_t = 0$

$i_t = 60\,\text{deg}$

$\Omega_t = \Omega_{t,0} + \dot{\Omega}_t\, t$

1st Phase - SMA increase - Plots



Results

$\Delta t = 91.8$ days

$\Delta a = 455.2$ km

$\Delta e = 0.0074$

$\Delta i = -0.1251$ deg

Eclipse Time

$$\frac{\Delta t_{ecl}}{\Delta t} \approx 23.53\,\%$$

2^{nd} Phase - Debris targeting - Results

Genetic Algorithm Settings

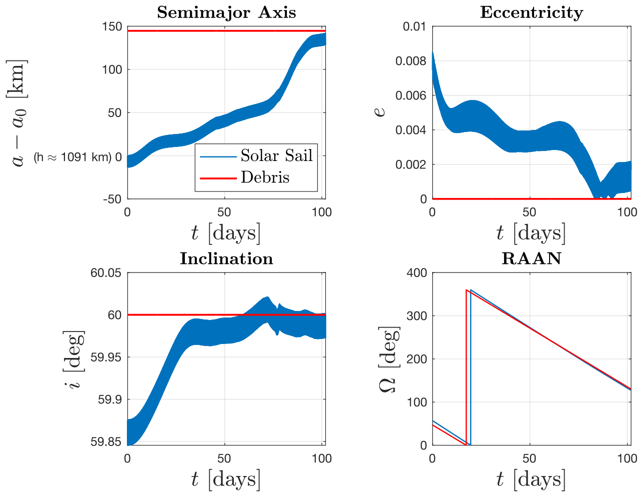
Population Size = 50
Elite Count = 2
Number of Generations = 10
Function Tolerance = 1×10^{-6}
$\Delta\Omega_0 = 10\,\text{deg}$

GA Results

$\mathbf{W_a} = 0.27849822$
$\mathbf{W_e} = 0.83082863$
$\mathbf{W_i} = 0.76324954$
$F_{opt} = 1.0907 \times 10^{-6}$
Flight Time = 101.9 days

Orbit	a [km]	e	i [deg]	Ω [deg]
Sail initial	7433.5	0.0074	59.87	56.98
Debris initial	7578.1	0	60	46.98
Debris final				130.07
Sail final **Approx**	7588.7	0.0009	59.99	130.05
Sail final **Exact**	7574.4	0.0017	60	127.23

2nd Phase - Debris targeting - Plots



Results

$\Delta t = 101.9\,\text{days}$

$\Delta a = -3.7722\,\text{km}$

$\Delta e = 0.0017$

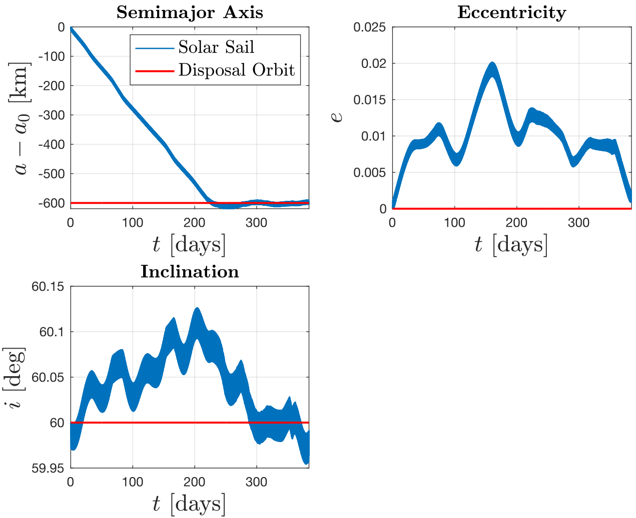
$\Delta i = -0.0002\,\text{deg}$

Right Ascension

$\Delta\Omega_0 = 10\,\text{deg}$

$\Delta\Omega_{fin} \approx 2.84\,\text{deg}$

Descent Phase - Plots



Results

$\Delta t = 384.4$ days

$\Delta a = -1.8550$ km

$\Delta e = 0.0009$

$\Delta i = -0.0361$ deg

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Conclusion

Conclusions

- **Locally-optimal laws work well** in perturbed environments
- **Blending seems effective** in controlling more orbital parameters at the same time
- **Approximate** model is pretty accurate for a **preliminary study**

Model Improvements

- Way to **estimate** the **RAAN** at the end of the 1^{st} phase
- **Increase** number of **generations** in the Genetic Algorithm

Further Developments

Further Developments

- **Switching point** between 1^{st} and 2^{nd} phase to be **optimized**
- **Higher performance sails** to reduce parking orbit's height

  $\rightarrow$ **shorter re-entry time** for debris
- **Multiple debris** removal strategy

  $\rightarrow$ best use of solar sails as **propellantless** devices

  $\rightarrow$ reduction of **launch cost**

Thank you for your attention!

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