CONTROLLABILITY OF SOLAR SAILS

Alesia Herasimenka
Université Côte d'Azur, CNRS, Inria, LJAD, France
ESA contract no 4000134950/21/NL/GLC/my

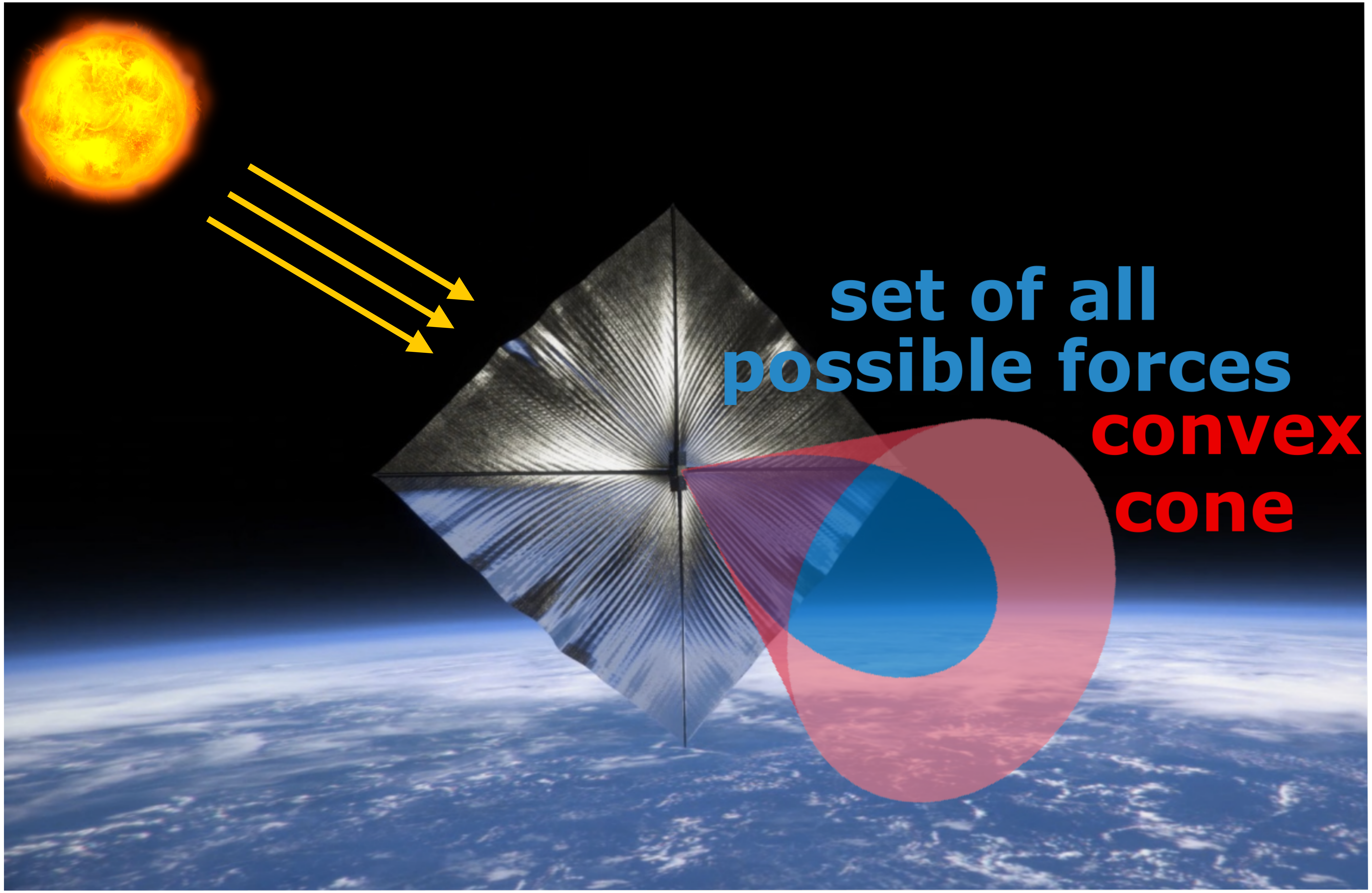
CONTROLLABILITY OF SOLAR SAILS

Alesia Herasimenka

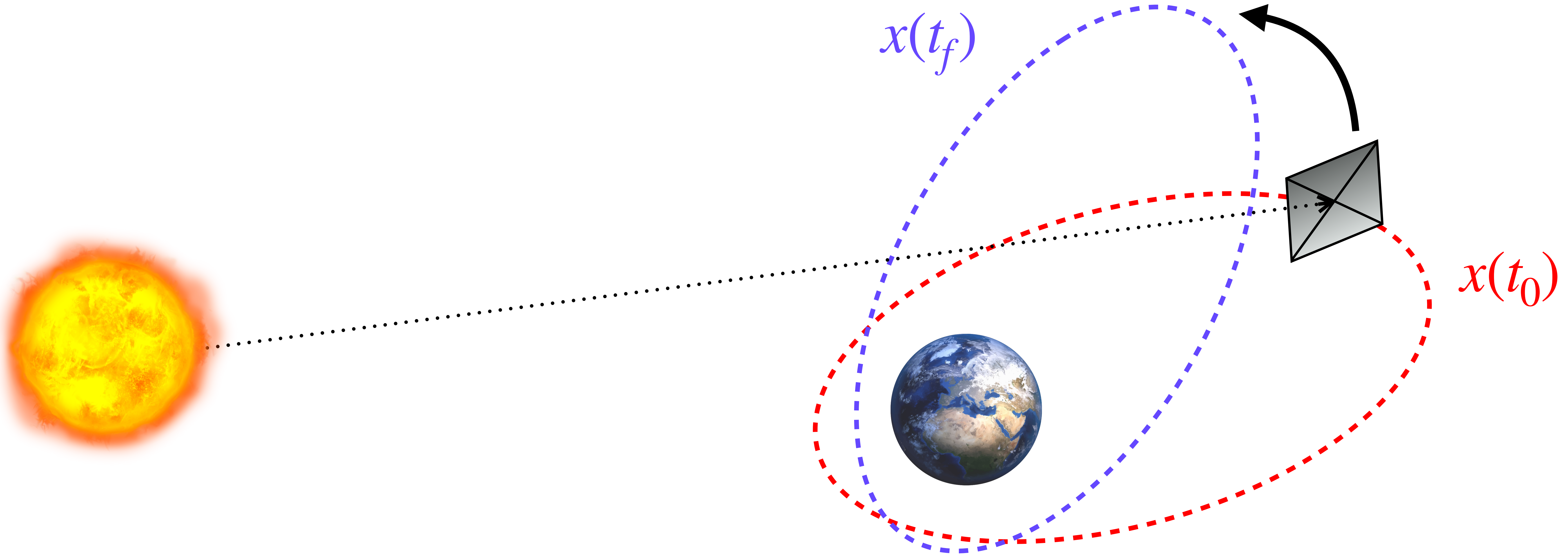
Université Côte d’Azu [illegible], Inria, LJAD, France

ESA contract no 40[illegible]/21/NL/GLC/my

Non-ideal sail: a cone-constrained control problem

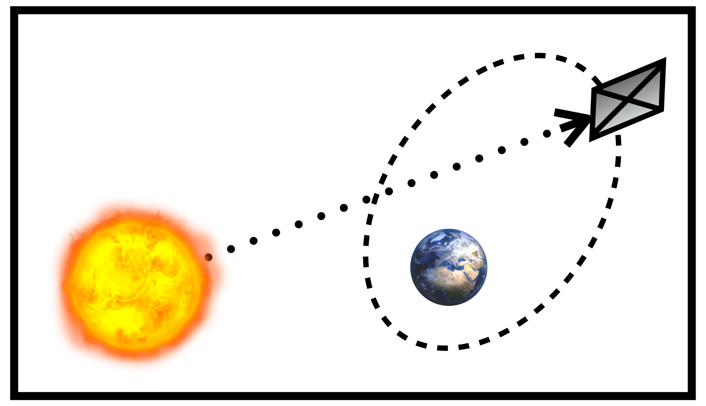


Can non-ideal sails arbitrarily change their orbit?



Is it possible to generate any $x(t_0) \longrightarrow x(t_f)$?

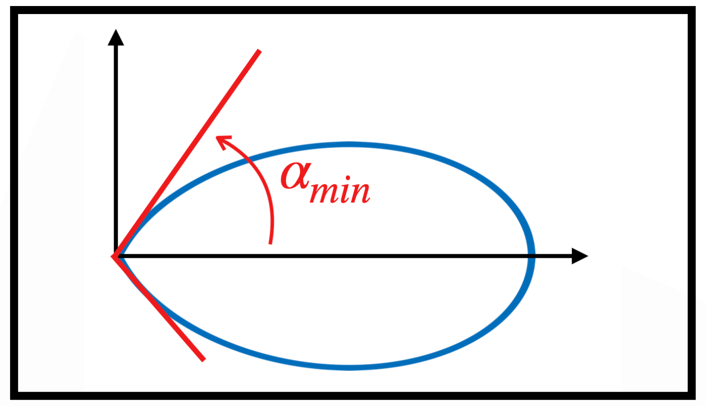
Outline



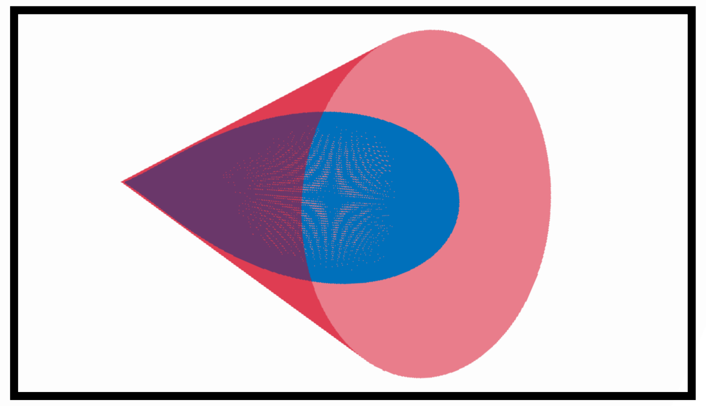
1. Dynamics of the system

$$\left\langle p_I, \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq 0$$

2. Necessary condition for local controllability

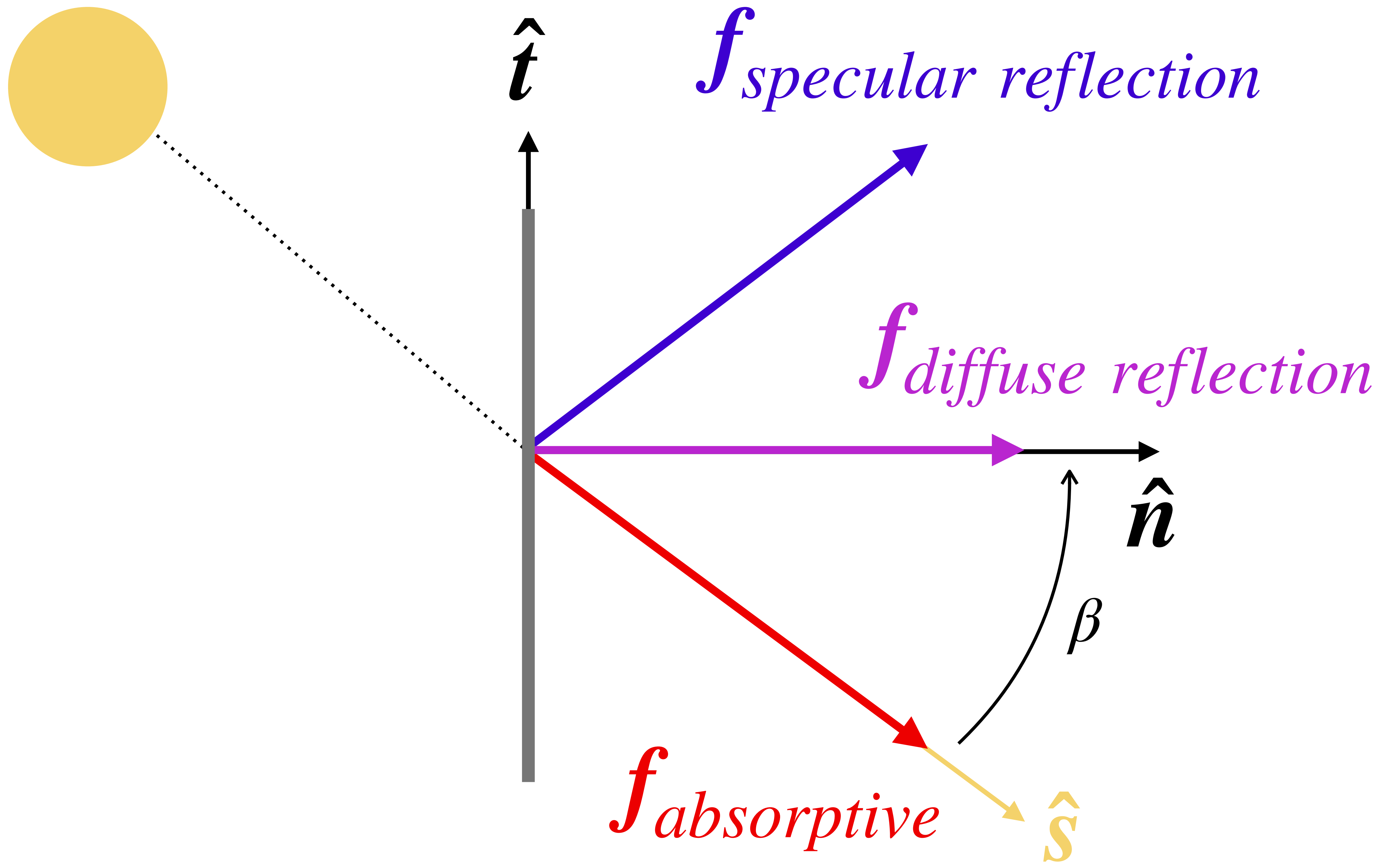


3. Minimum optical requirements



4. Way forward

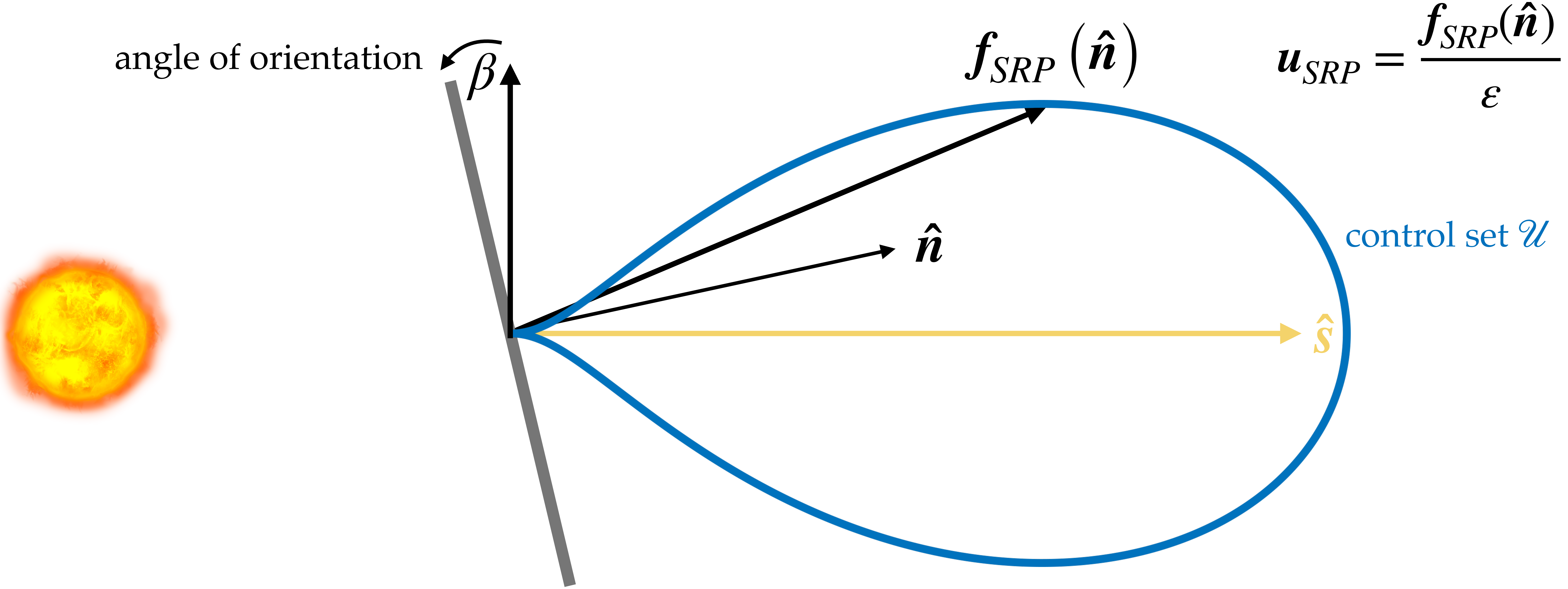
1. Force components of solar sail



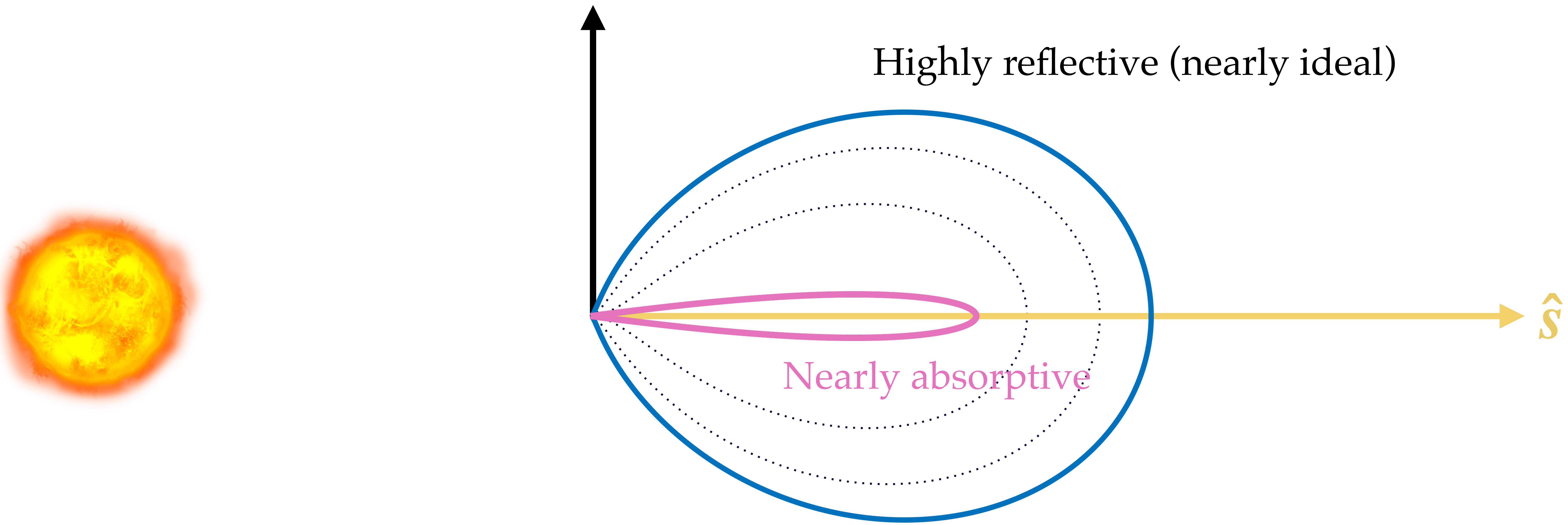
$$f_{SRP} = f_{absorptive} + f_{specular\ reflection} + f_{diffuse\ reflection}$$

1. Control set

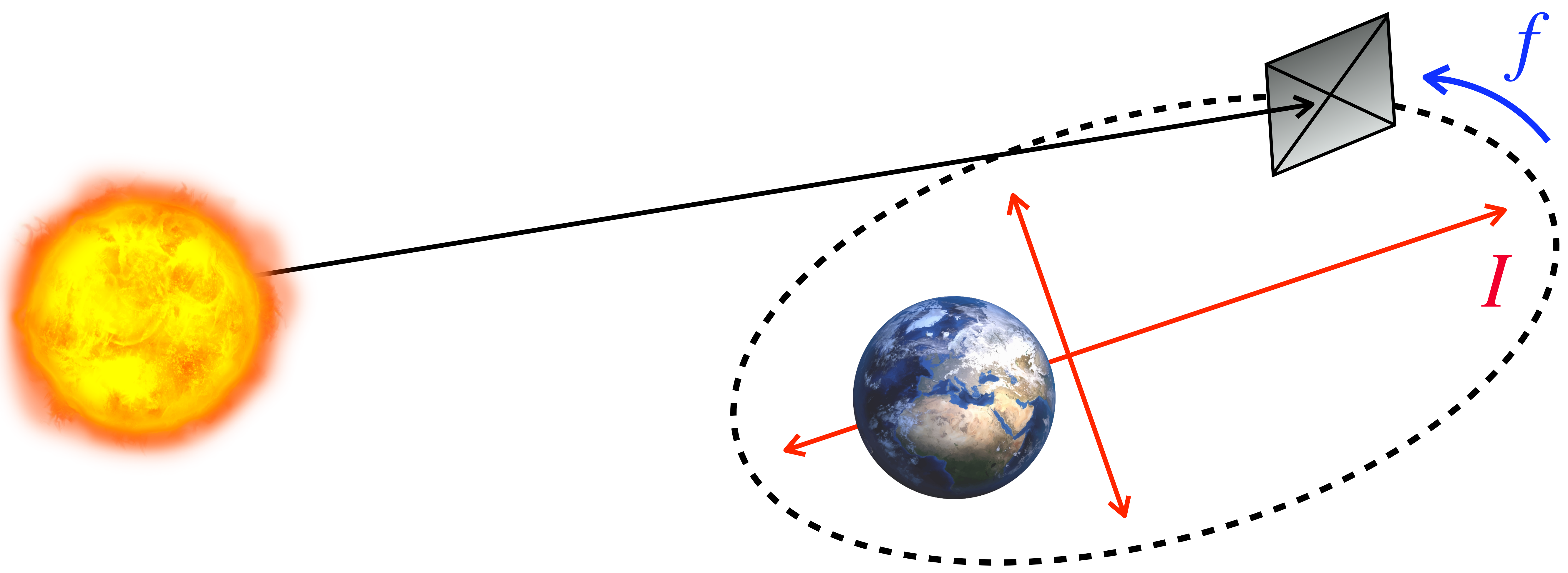
$$\dot{x} = F_0(x) + \sum_i u_i F_i(x), \quad u \in \mathscr{U}$$



1. Parametrisation of the control set



1. Dynamical system



Assumptions:

No eclipses

Sun motion neglected over one orbit

SRP is the only perturbation

$$\dot{x} = F_0(x) + \sum_i u_i F_i(x), \quad u \in \mathcal{U}, \quad i = 1, 2, 3$$

with $x = (I, f)$, $I \in M$, $f \in \mathbb{S}^1$, F_0, F_i given by Gauss variational equations

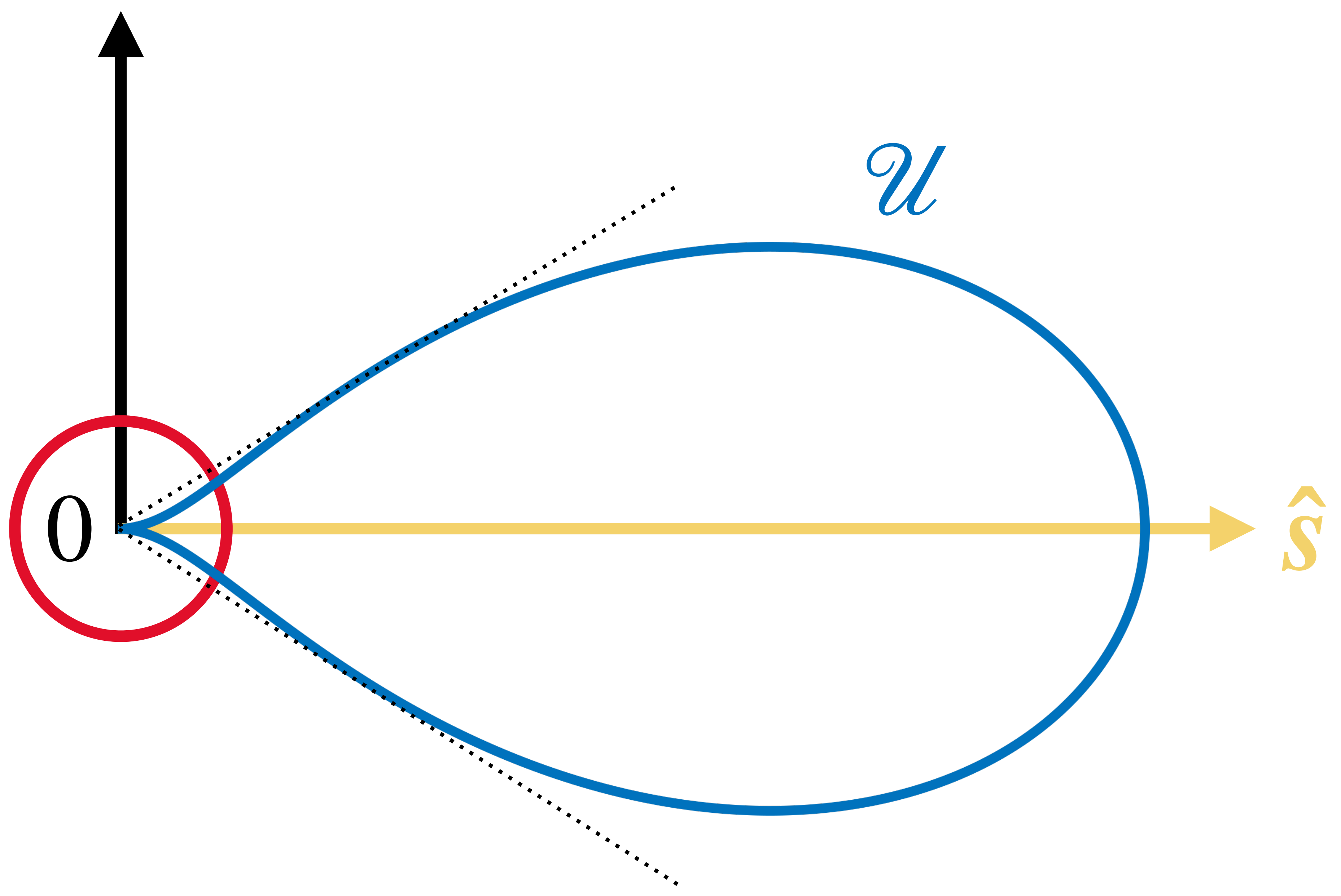
2. Classical approach using Lie brackets

Controllability if [Jurdjevic, 1996]:

i) Periodic drift ⟶ **OK**

ii) Bracket generating ⟶ **OK** if $\rho > 0$

iii) $\mathrm{Conv}\left(\mathscr{U}\right)$ is neighbourhood of the origin ⟶ **NO**



2. Proposition on controllability

Under the conditions:

(i) system is bracket generating,

(ii) control set $\mathcal{U}$ contains the origin,

(iii) $\forall I \in M, \quad \text{cone}\left\{ \sum_i u_i F_i(I, f),\ u \in \mathcal{U},\ f \in \mathbb{S}^1 \right\} = T_I M,$

the system is controllable*.

* The proof is available in [Caillau, Dell'Elce, Herasimenka Pomet, 2022]

2. Proposition on controllability

Under the conditions:

(i) system is bracket generating,

(ii) control set $\mathcal{U}$ contains the origin,

How to verify the condition?

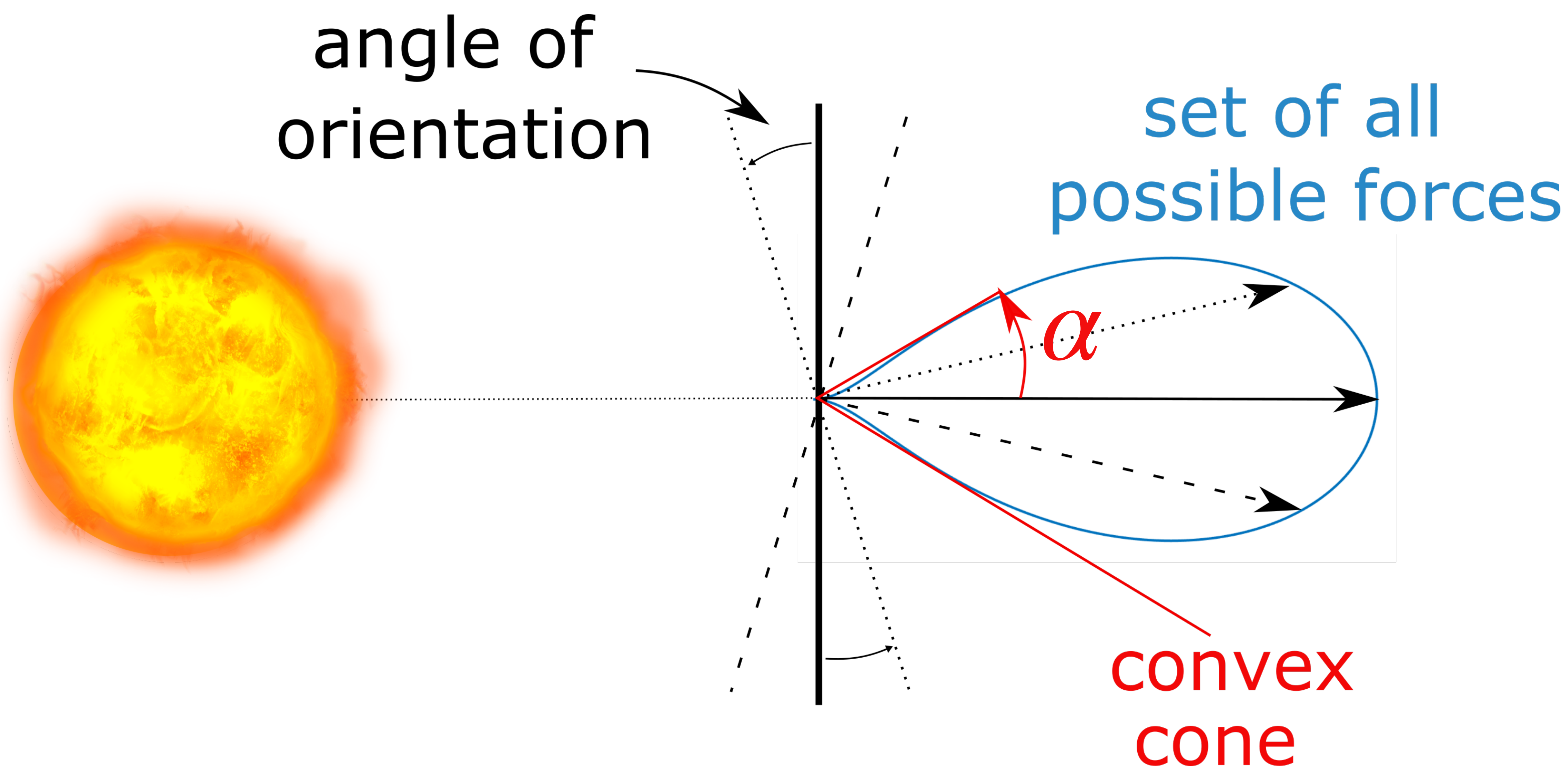
(iii) $\forall I \in M, \quad \text{cone}\left\{ \sum_i u_i F_i(I,f),\ u \in \mathcal{U},\ f \in \mathbb{S}^1 \right\} = T_I M,$

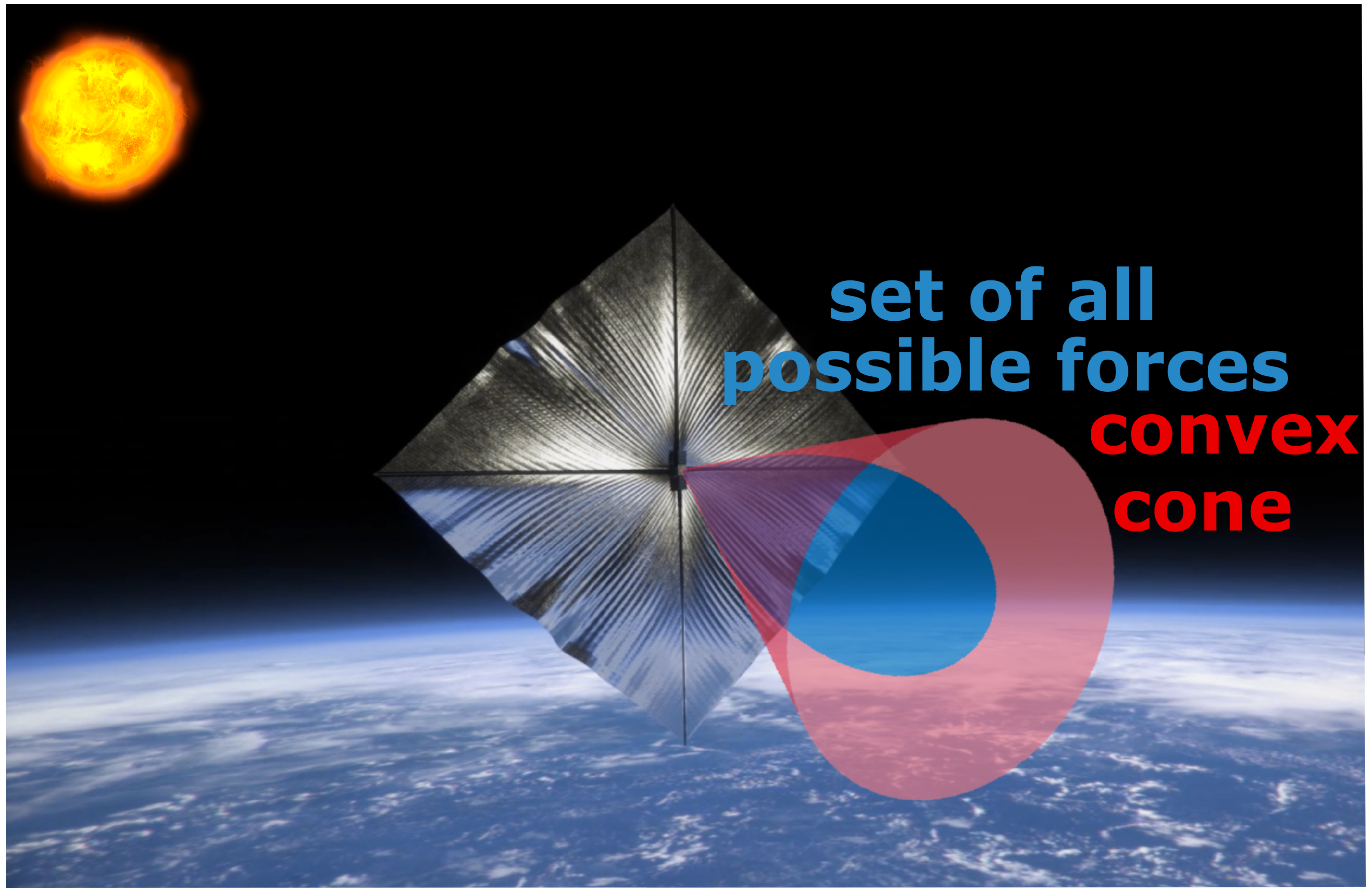
the system is controllable*.

* The proof is available in [Caillau, Dell'Elce, Herasimenka Pomet, 2022]

2. Convexification of the control set

$$\boldsymbol{u} \in \mathscr{U} \subset K_\alpha := \text{cone}\,(\mathscr{U})$$

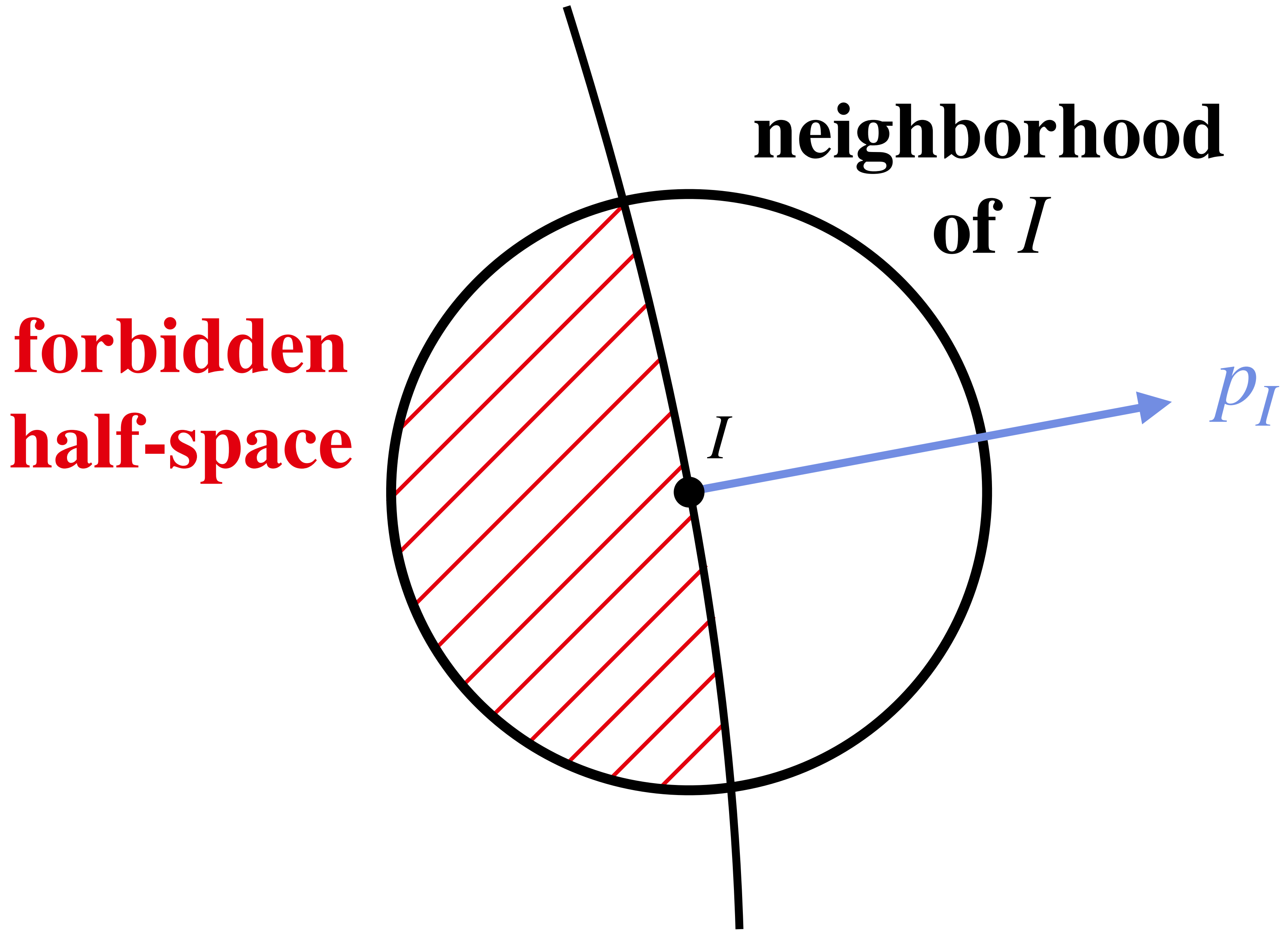




2. How to verify the condition?

Negation: If $\exists$ a one-form $p_I \in T^*M$, s.t.

$$\left\langle p_I\,,\ \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq 0, \quad \forall\, f \in \mathbb{S}^1,\, \boldsymbol{u} \in \partial K_\alpha \quad \longrightarrow \quad \text{not locally controllable}$$



2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\| \le 1} \quad J \quad \text{s.t.}$$

$$\left\langle p_I \ , \ \frac{dI(f, \boldsymbol{u})}{dt} \right\rangle \ge J, \qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

If $J^* > 0 \longrightarrow$ not locally controllable

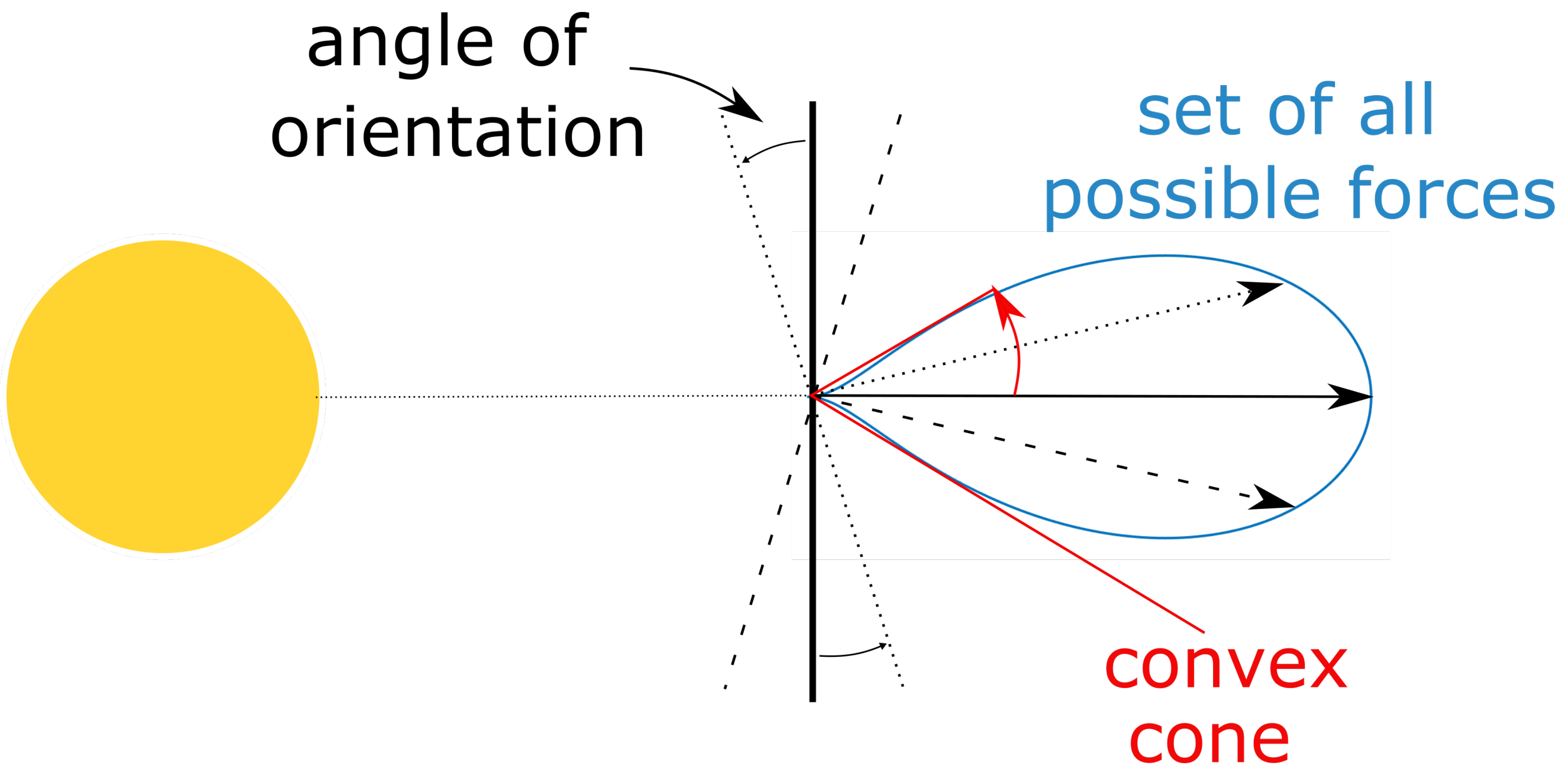
2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\|\leq 1} \quad J \quad \text{s.t.}$$

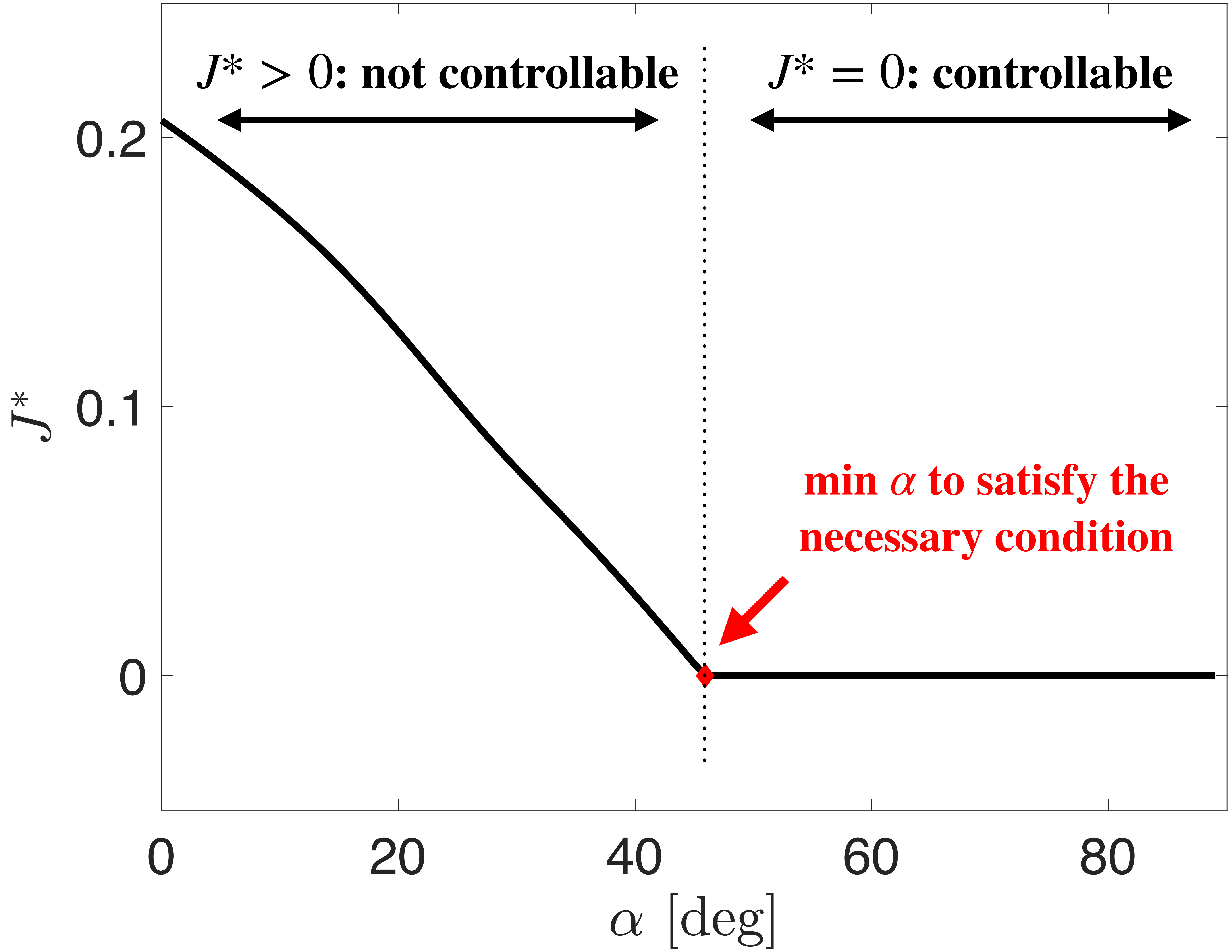
$$\left\langle p_I \, , \, \frac{dI(f, \boldsymbol{u})}{dt} \right\rangle \geq J, \qquad \forall \, f \in \mathbb{S}^1, \forall \, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

If $J^* > 0 \longrightarrow$ not locally controllable

for the fixed angle α !



2. Exploitation: minimum optical requirements



2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\|\leq 1} \quad J \quad \text{s.t.}$$

$$\left\langle p_I\ ,\ \frac{dI(f,\boldsymbol{u})}{dt} \right\rangle \geq J, \qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

Difficulties: $u(f) \in \partial K_\alpha, \quad f \in \mathbb{S}^1$

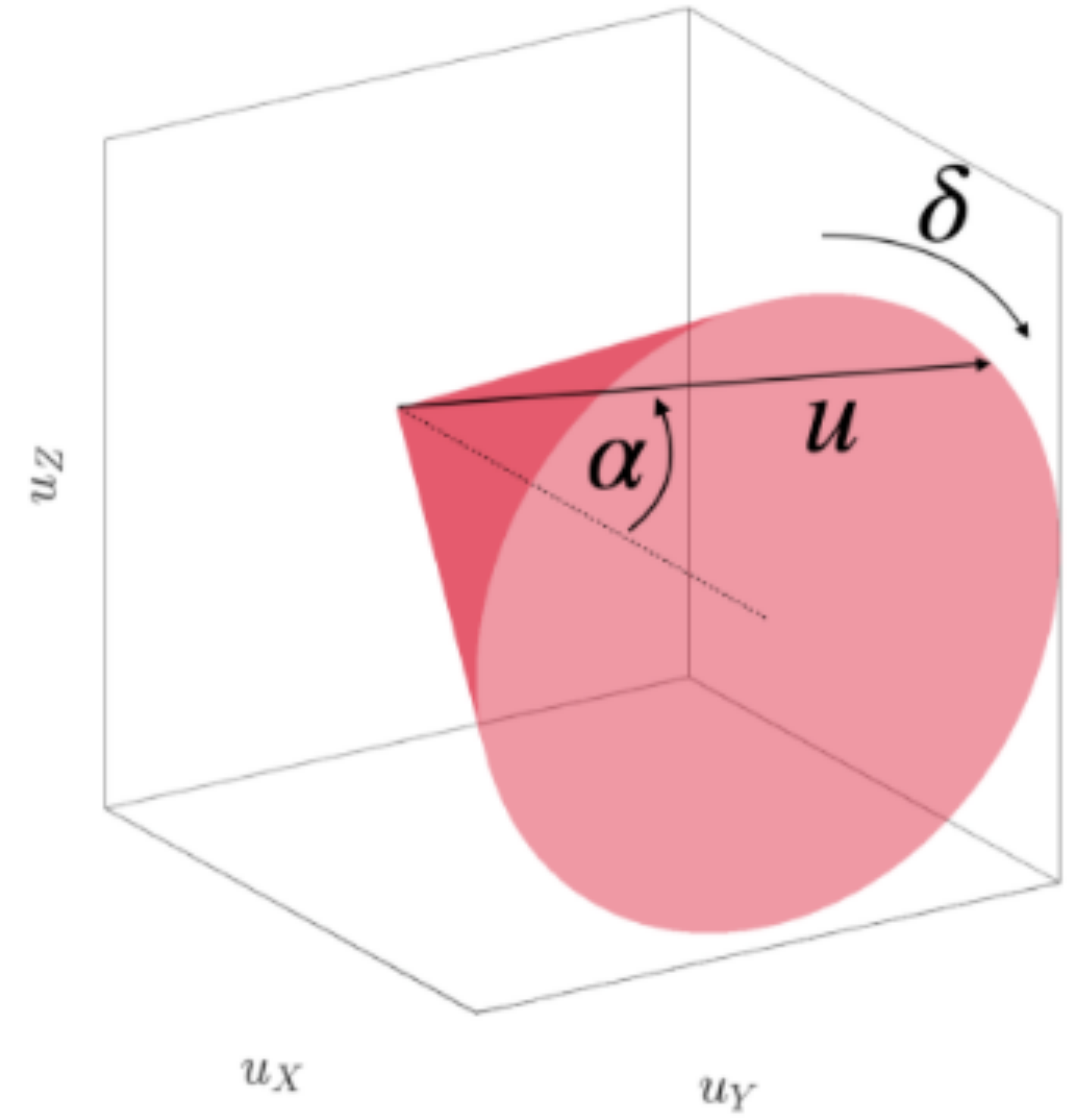
- $u(f) \in \partial K_\alpha$ → positivity constraint
- $f \in \mathbb{S}^1$ → infinite dimension

2. Numerical solution of the semi-infinite problem

Parametrization of the cone

Controls on the cone

$$\boldsymbol{u} = \begin{bmatrix} \cos\alpha \\ \cos\delta \sin\alpha \\ \sin\delta \sin\alpha \end{bmatrix}$$



2. Numerical solution of the semi-infinite problem

Fourier transform (exact) of the dynamics

Positive bivariate polynomials [Nesterov, 2000; Dumitrescu, 2007]

Leverage on formalism of squared functional systems

2. Recast into a convex optimisation problem

$$\max_{J,\ \|p_I\| \leq 1} \quad J \quad \text{s.t.}$$

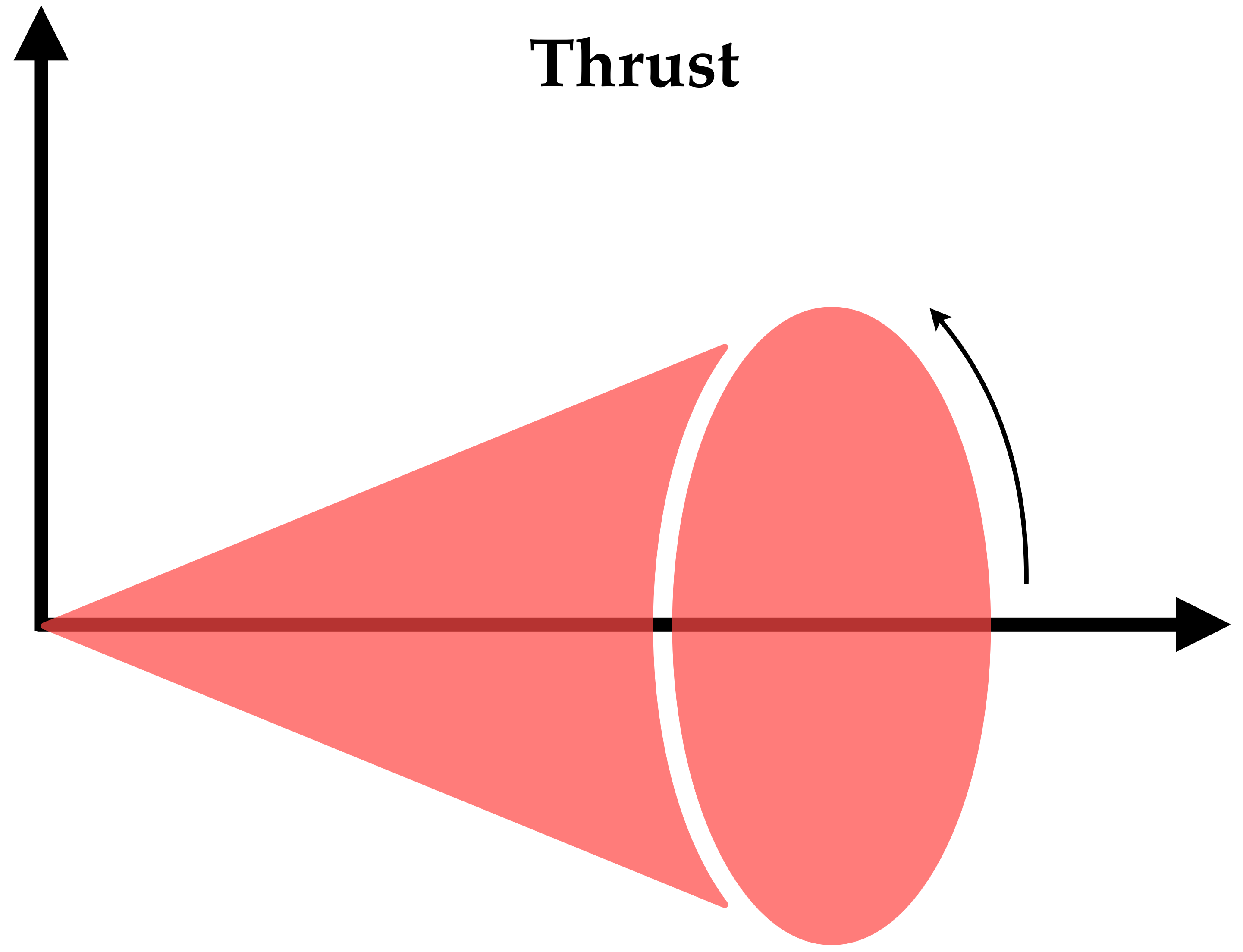
$$\left\langle p_I\ ,\ \frac{dI(f, \boldsymbol{u})}{dt} \right\rangle \geq J, \qquad \forall\, f \in \mathbb{S}^1, \forall\, \boldsymbol{u} \in \partial K_\alpha, \|\boldsymbol{u}\| = 1$$

↓

LMI

2. Effective test of the necessary condition

$$\min_{u \in K_\alpha} \alpha \quad \text{such that the system is controllable}$$



22

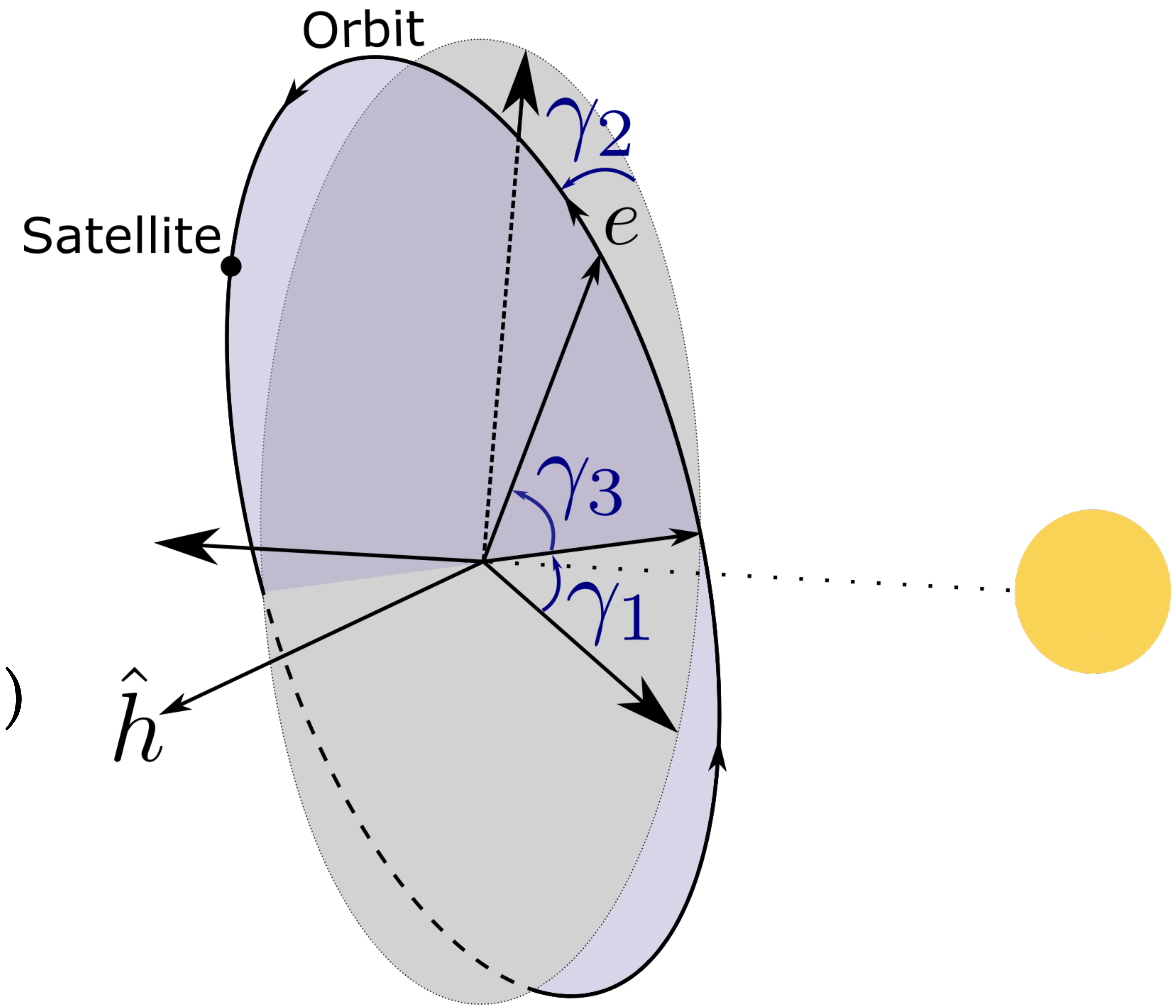
2. Convenient choice of orbital elements

$$I = (\gamma_1, \gamma_2, \gamma_3, a, e)$$

Problem independent of:

- γ_1 (axial symmetry)

- $a, \mu \rightarrow$ (planet-independent results)



2. Convenient choice of orbital elements

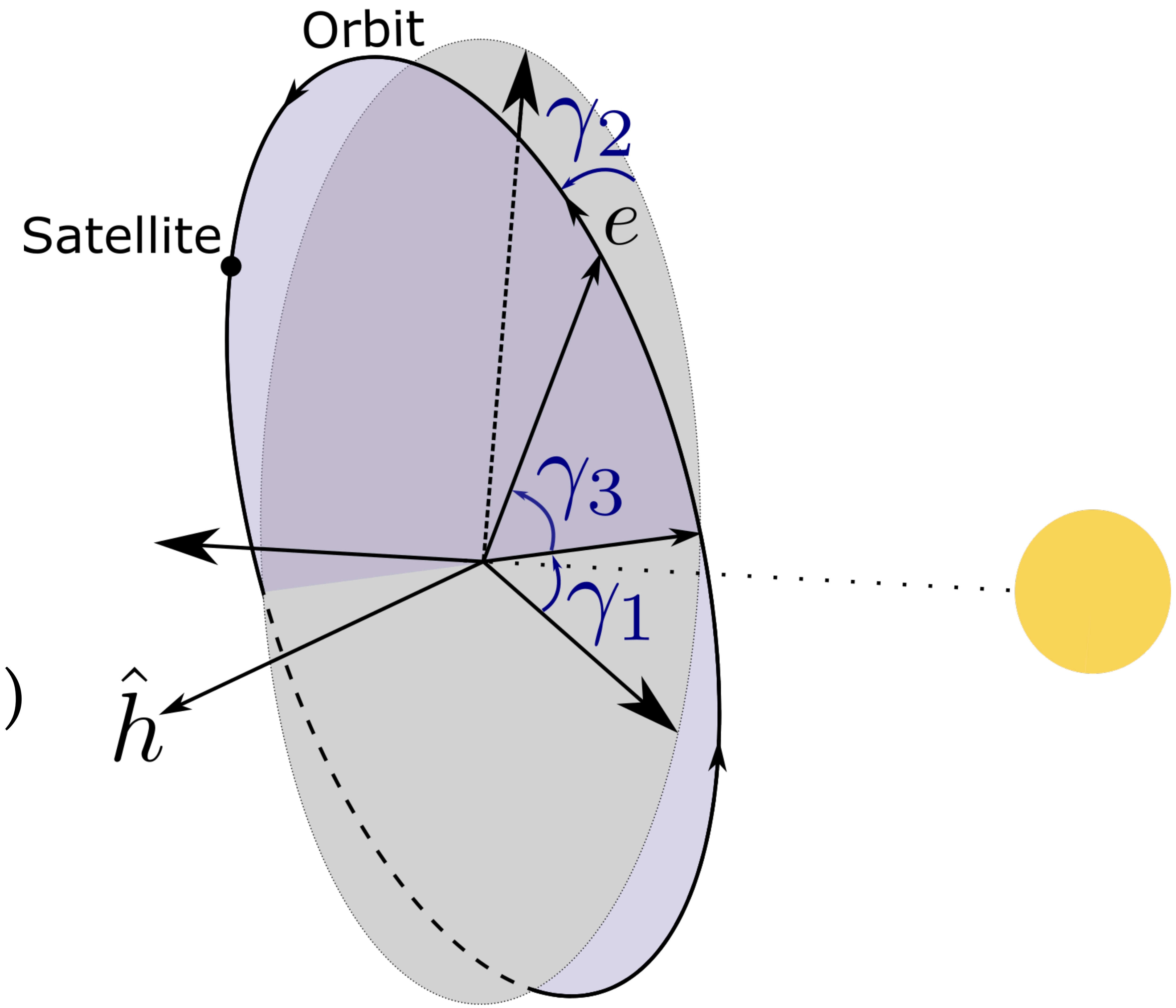
$$I = \left(\gamma_1,\, \gamma_2,\, \gamma_3,\, a,\, e\right)$$

Problem independent of:

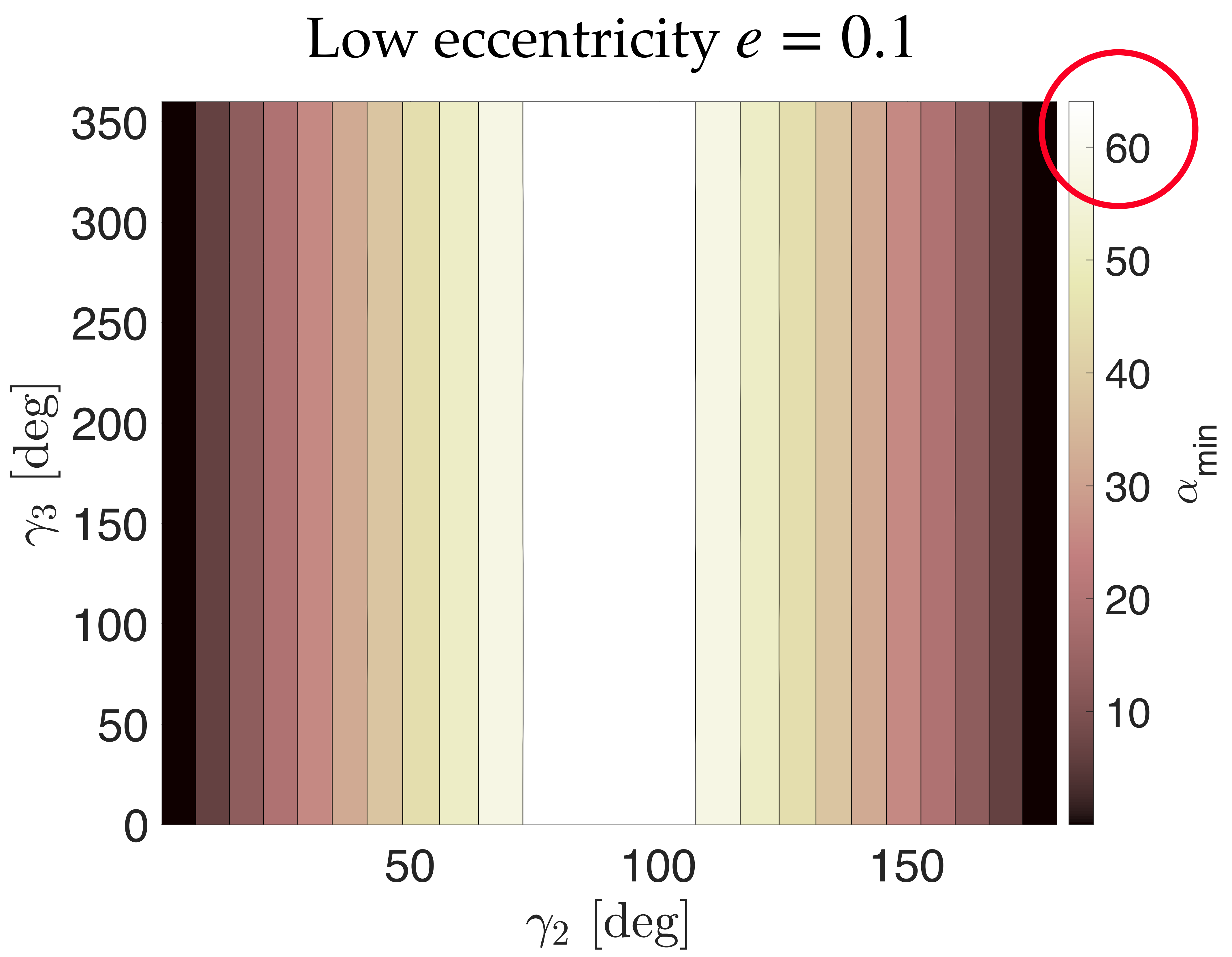
- γ_1 (axial symmetry)

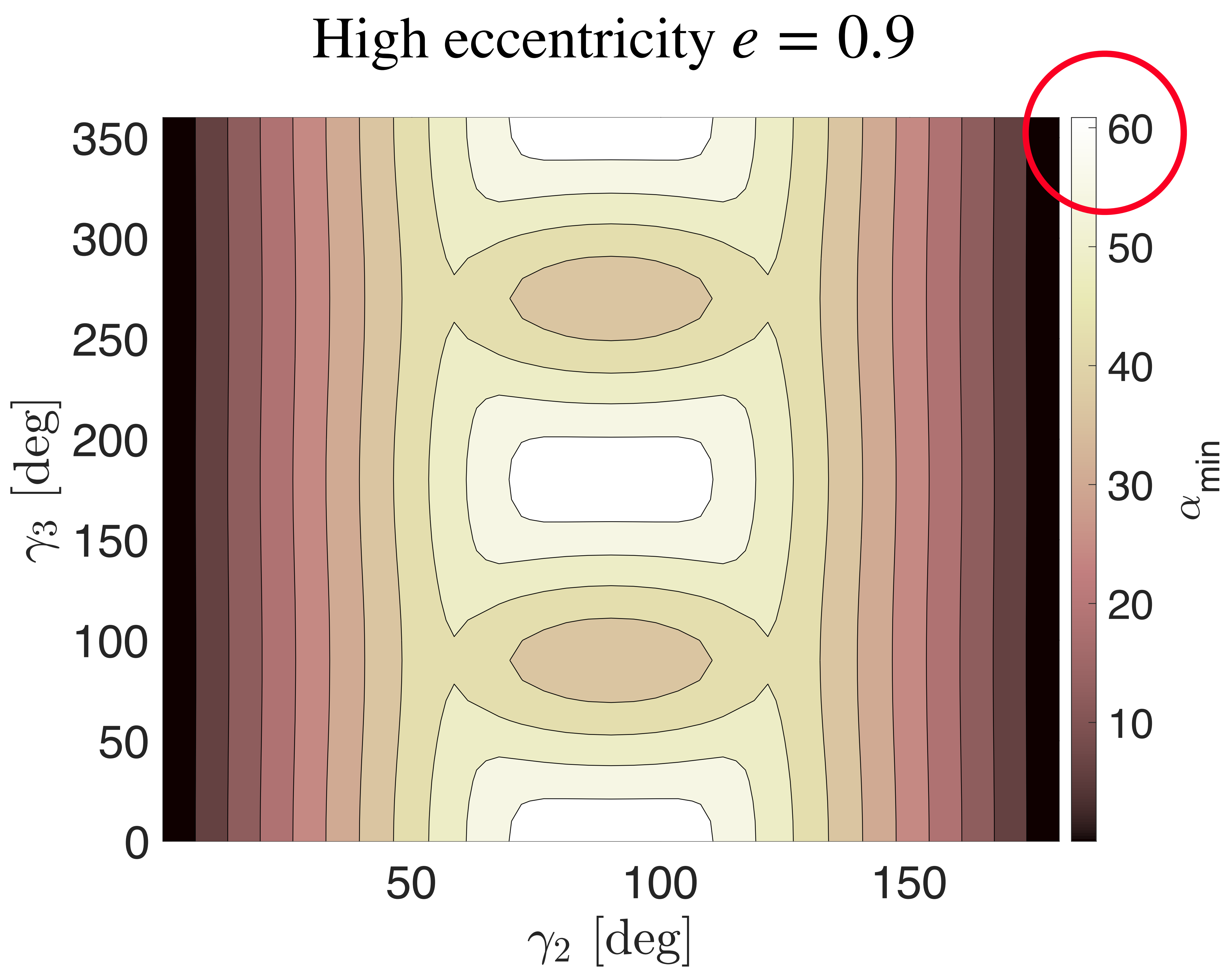
- $a, \mu \rightarrow$ (planet-independent results)

$$I = \left(\cancel{\gamma_1},\, \gamma_2,\, \gamma_3,\, \cancel{a},\, e\right)$$



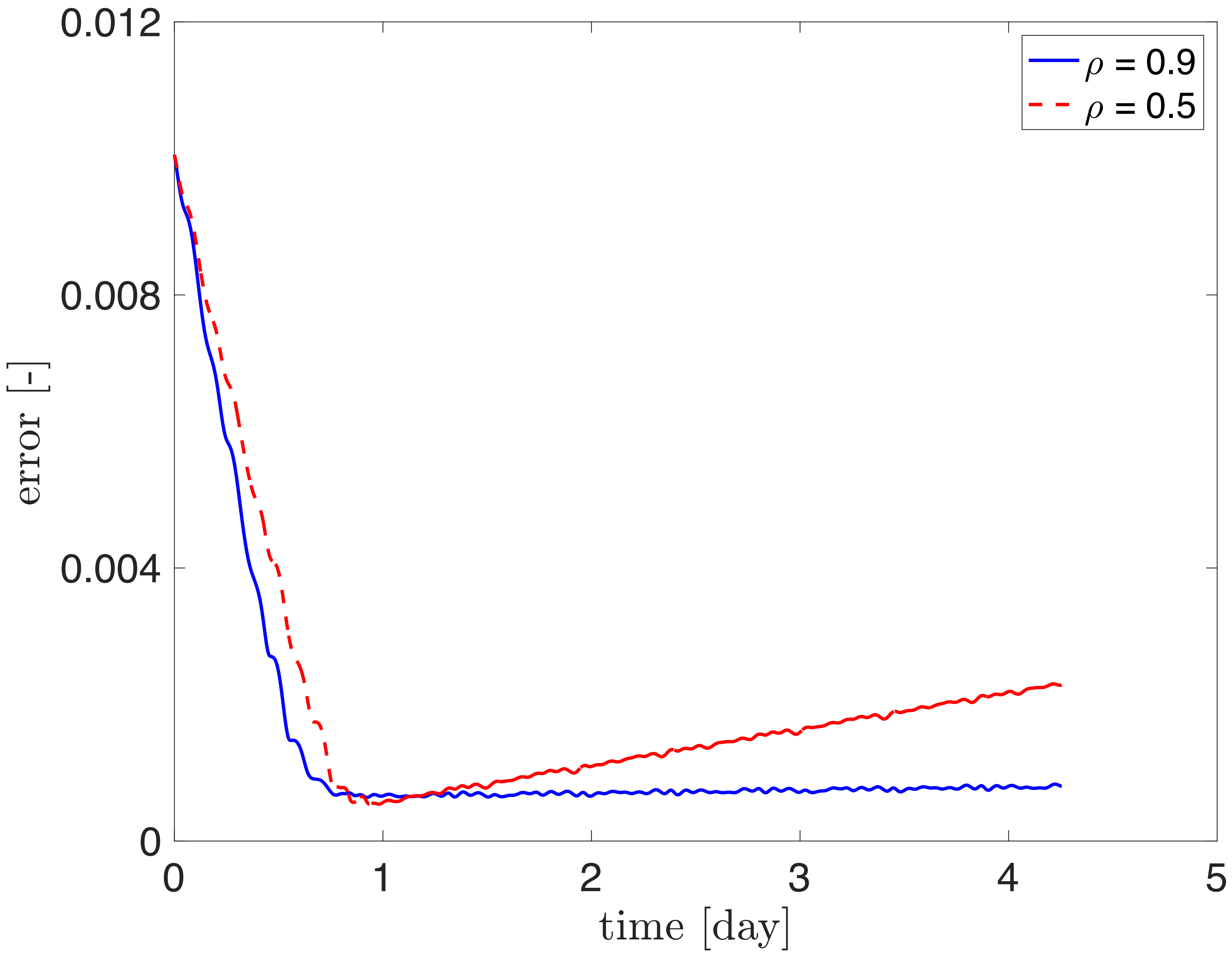
3. Results: minimal requirement





A. Herasimenka, L. Dell'Elce, J.-B. Caillau, and J.-B. Pomet, "Controllability Properties of Solar Sails," Journal of Guidance, Control, and Dynamics, vol. 46, no. 5, pp. 900–909, May 2023

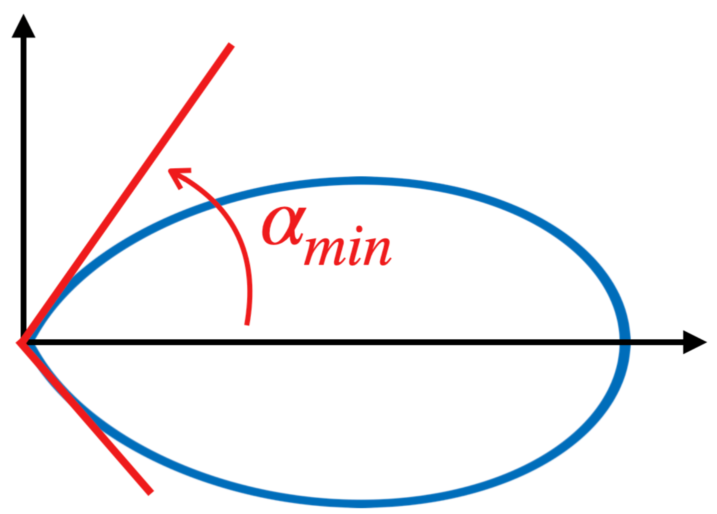
3. Validation of the minimum requirement



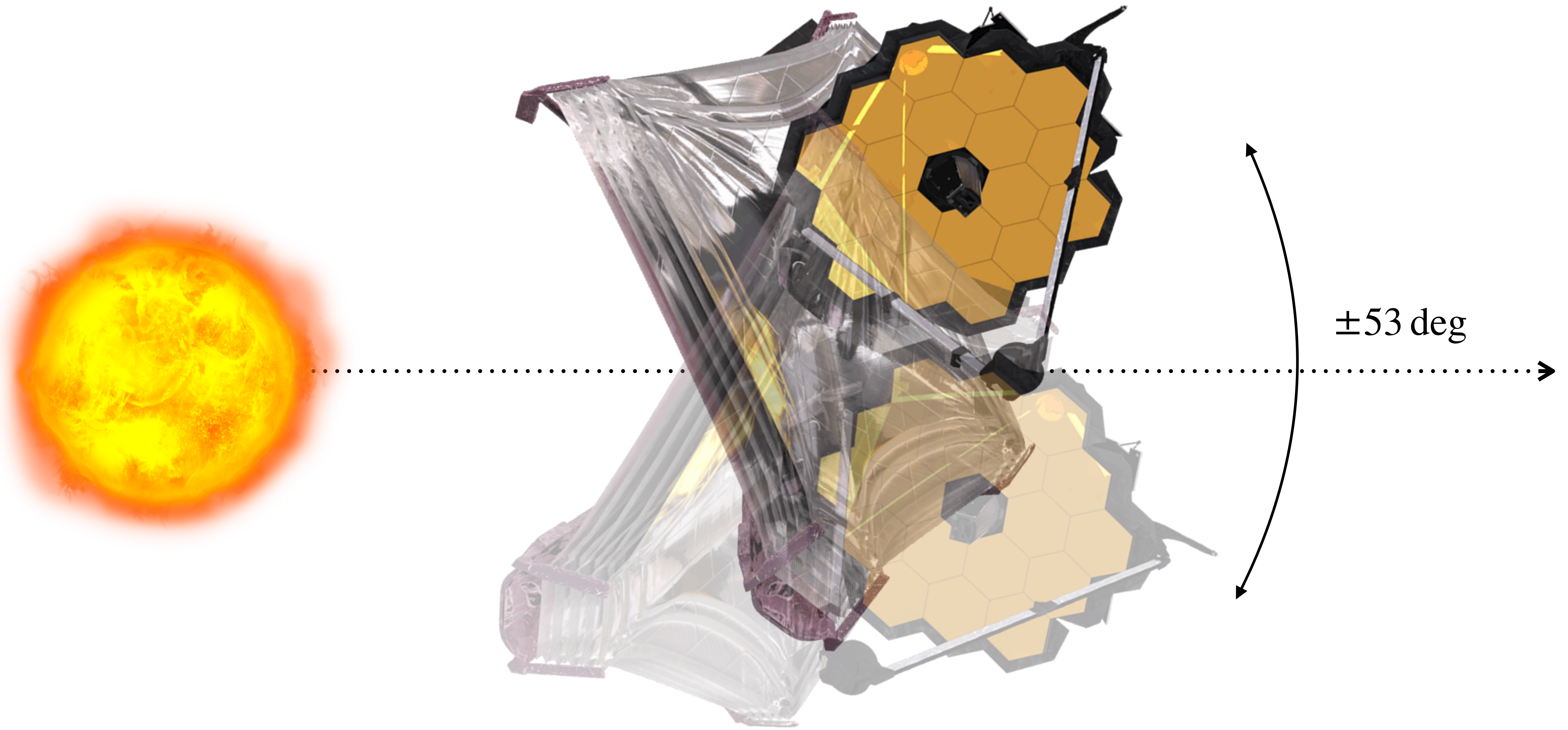
Conclusion

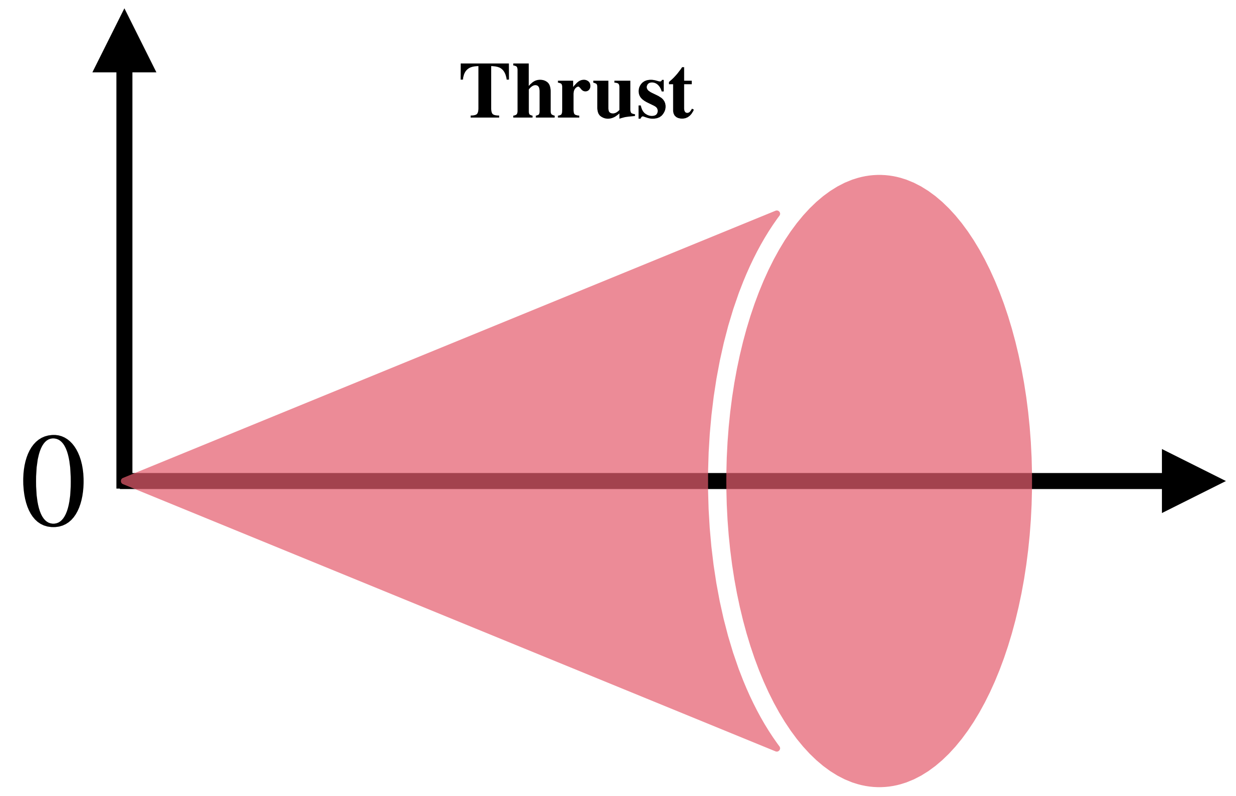
Minimum requirement in terms of optical properties:

- Mission analysis
- Lifetime of the sail



Way forward: other systems





Way forward: Algorithm for optimal control

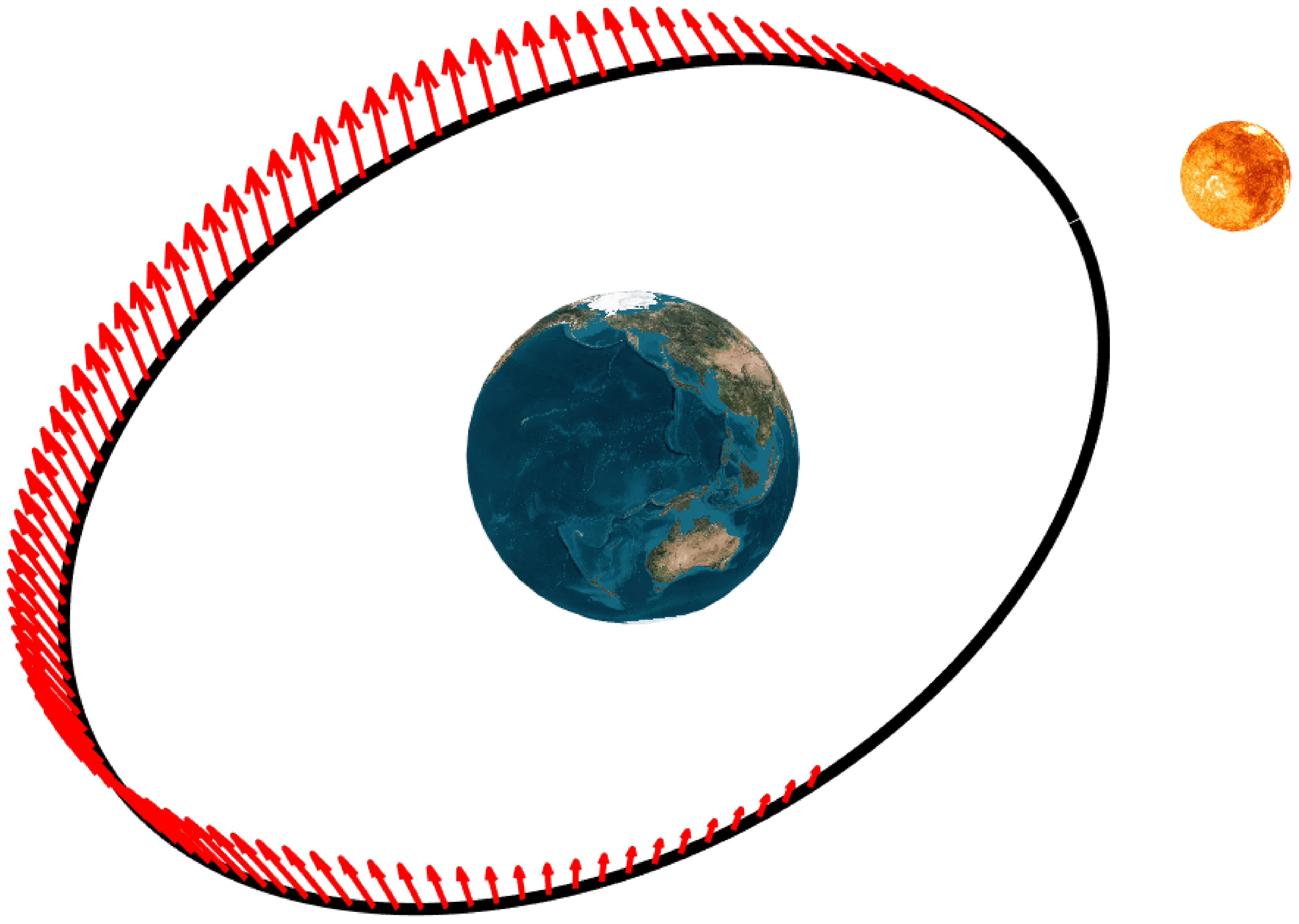
1. No initial guess

2. Optimality conditions

3. Numerical techniques:

multiple shooting, switch function,

homotopy, callback



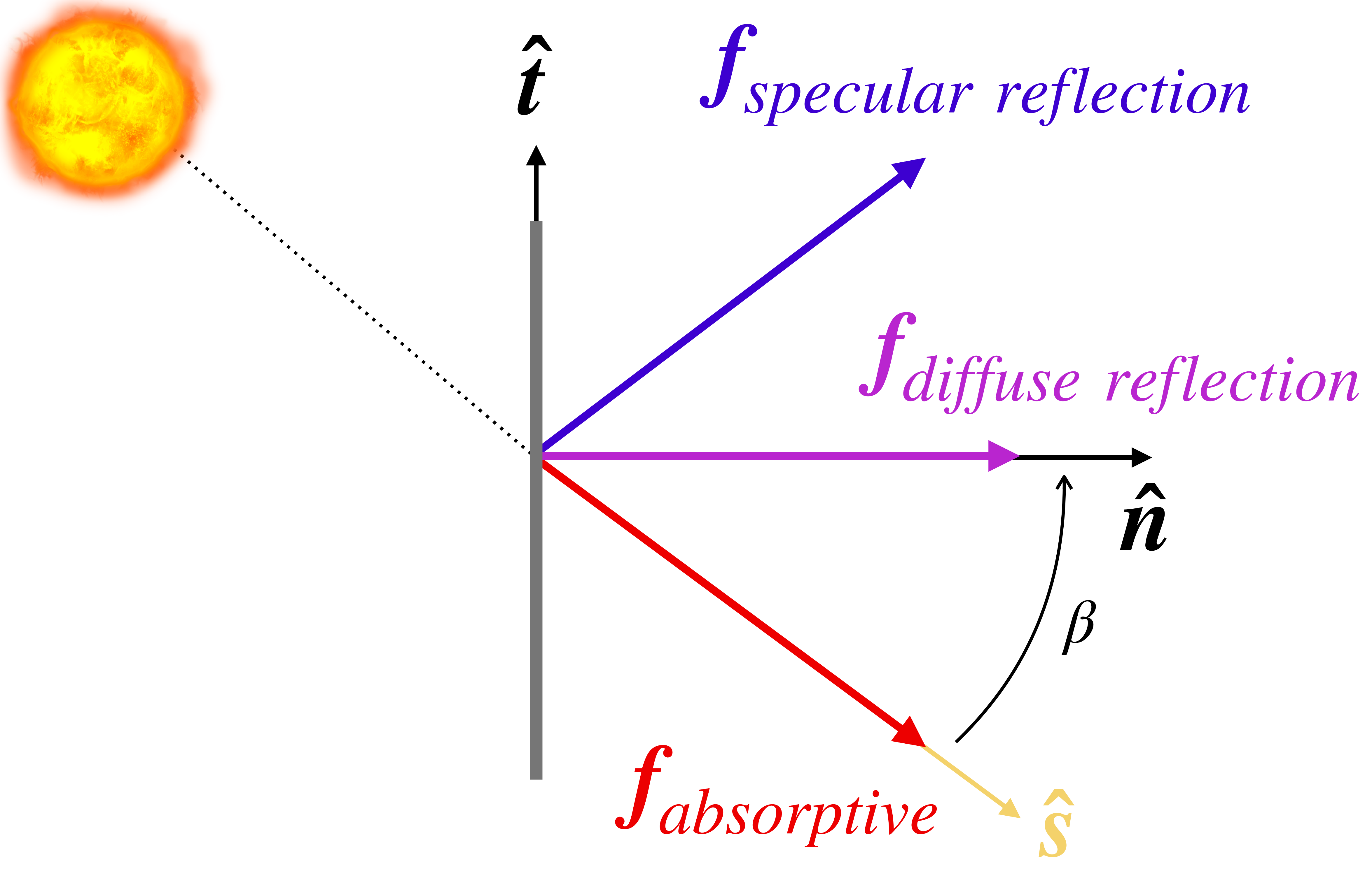
CONTROLLABILITY OF SOLAR SAILS

Alesia Herasimenka

Université Côte d’Azur, CNRS, Inria, LJAD, France

ESA contract no 4000134950/21/NL/GLC/my

1. Force components of solar sail



$$f_{SRP} = f_{absorptive} + f_{specular\ reflection} + f_{diffuse\ reflection}$$

2. Numerical solution of the semi-infinite problem

Fourier transform (exact) of the dynamics

$$\frac{\mathrm{d}I}{\mathrm{d}t} = \varepsilon \sqrt{\frac{a(1-e^2)}{\mu}} G(I,f)R(I,f)\boldsymbol{u}$$

$$G = \begin{pmatrix} 0 & 0 & \frac{\sin(\gamma_3+f)}{\sin\gamma_2(1+e\cos f)} \\ 0 & 0 & \frac{\cos(\gamma_3+f)}{1+e\cos f} \\ -\frac{\cos f}{e} & \frac{2+e\cos f}{1+e\cos f}\frac{\sin f}{e} & \frac{\cos(\gamma_3+f)}{1+e\cos f} \\ \frac{2ae}{1-e^2}\sin f & \frac{2ae}{1-e^2}(1+e\cos f) & 0 \\ \sin f & \frac{e\cos^2 f+2\cos f+e}{1+e\cos f} & 0 \end{pmatrix}$$

$(1+e\cos f)G(I,f)R(I,f)$ is a trigonometric polynomial of degree 2 in f

2. Numerical solution of the semi-infinite problem

Positive polynomials [Nesterov, 2000; Dumitrescu, 2007]

Leverage on formalism of squared functional systems:

$$\Phi(f,\delta) = \left[1,\ \mathrm{e}^{i\delta}\right]^T \otimes \left[1,\ \mathrm{e}^{if},\ \mathrm{e}^{2if}\right]^T$$
$$= \left[1,\ \mathrm{e}^{if},\ \mathrm{e}^{2if},\ \mathrm{e}^{i\delta},\ \mathrm{e}^{if}\,\mathrm{e}^{i\delta},\ \mathrm{e}^{2if}\,\mathrm{e}^{i\delta}\right]^T$$

$\Lambda_H : \mathbb{C}^N \to \mathbb{C}^{N\times N}$ a linear operator s.t. $\Lambda_H(\Phi(f,\delta)) = \Phi(f,\delta)\Phi^H(f,\delta)$

and $\Lambda_H^* : \mathbb{C}^{N\times N} \to \mathbb{C}^N$ its adjoint operator.

2. Numerical solution of the semi-infinite problem

Positive polynomials [Nesterov, 2000; Dumitrescu, 2007]

Then for any trigonometric bivariate polynomial

$$\left\langle p_I, \widetilde{G}(I,f)\boldsymbol{u} \right\rangle - J \geq 0, f \in \mathbb{S}^1, \boldsymbol{u} \in \partial K_\alpha \Leftrightarrow \exists Y \succeq 0$$

$$\text{such that } \widetilde{Gu}\, p_I - \boldsymbol{e_1} J = \Lambda_H^*(Y)$$

Fourier coefficients in the basis Φ

defined with Toeplitz matrices