University of Glasgow | James Watt School of Engineering

Università di Pisa

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Space and Exploration Technology Group

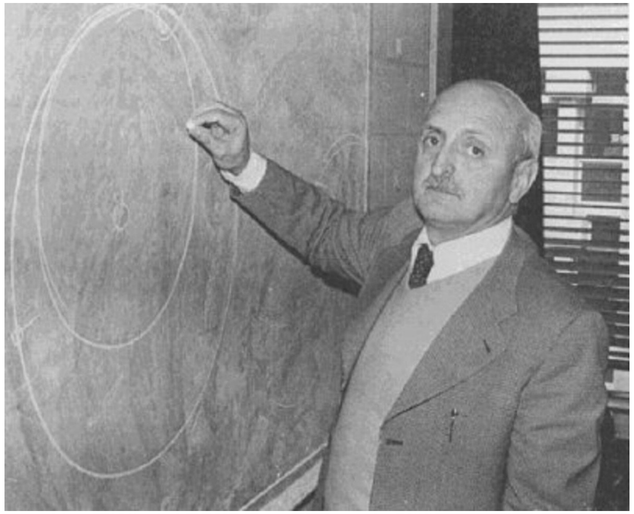
Advances in preliminary solar-sail trajectory design

Shape-Based Methods and Artificial Neural Networks

Matteo Ceriotti, Giulia Viavattene, Andrea Caruso, Giovanni Mengali, Alessandro Quarta

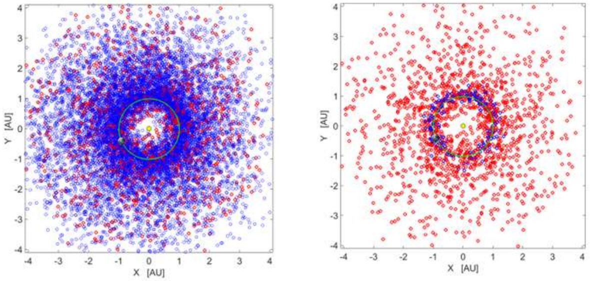
Why preliminary trajectory design?

- Early mission design phase
  - Concept study, pre-phase A
- Fast preliminary trajectory design
- Interested in trajectory key figures of merit:
  - Δv, TOF, maximum thrust, propellant mass, feasibility
- Not interested in full control history (as long as feasible)
- Approximate solution
  - Design margins are high
- Fast to calculate
  - Can calculate hundreds to millions of options



Giuseppe “Bepi” Colombo. Credit: ESA

Multi-target missions



- Multi-target missions, wide launch window
  - Appealing for solar sailing: “infinite” Δv
- Multi-asteroid rendezvous mission
  - Complete database: *n* = ~13,000 NEAs
  - Near-Earth Object Human Space Flight Accessible Target Study (NHATS) (for a low-thrust return mission)
    Reduced database: *n* = ~1,800 objects

$$\text{NHATS criteria:}\begin{cases}\text{total } \Delta v \text{ required} \leq 8 \text{ km/s} \\ \text{total mission duration} \leq 450 \text{ days} \\ \text{stay time at the object} \geq 8 \text{ days} \\ \text{launch: } 2015\text{–}2040 \\ H \leq 26 \text{ magnitude} \\ \text{OCC} \leq 7 \end{cases}$$

- Number of unique sequences of *q* objects out of *n*:

$$P_q^n = \frac{n!}{(n-q)!}$$

Database	n	$q = 3$	$q = 4$	$q = 5$
Complete database	12,840	2.1×10^{12}	2.7×10^{16}	3.5×10^{20}
Reduced database	1,801	5.8×10^{9}	1.0×10^{13}	1.9×10^{16}

Peloni, A., Ceriotti, M. and Dachwald, B. (2016) Solar sail trajectory design for a multiple near-Earth asteroid rendezvous mission. Journal of Guidance, Control, and Dynamics, 39(12), pp. 2712-2724. (doi: 10.2514/1.G000470)

Solar sailing vs. low thrust (EP)

- Continuous, low acceleration

Solar sailing

- Thrust direction and magnitude related
- Dependent on sun direction

$$\mathbf{a} = a_c \left(\frac{r_\oplus}{r} \right)^2 \cos^2 \alpha \, \hat{\mathbf{N}}$$

$$a_c = 2P \frac{A}{M}$$

$$\hat{\mathbf{N}} = \begin{bmatrix} N_r \\ N_\vartheta \\ N_h \end{bmatrix} = \begin{bmatrix} \cos\alpha \\ \sin\alpha\cos\delta \\ \sin\alpha\sin\delta \end{bmatrix}$$

$$\alpha = \angle\left(\mathbf{r}_{sun}, \hat{\mathbf{N}}\right)$$

$$\alpha < 90^\circ$$

- Objective:
  - Time of flight

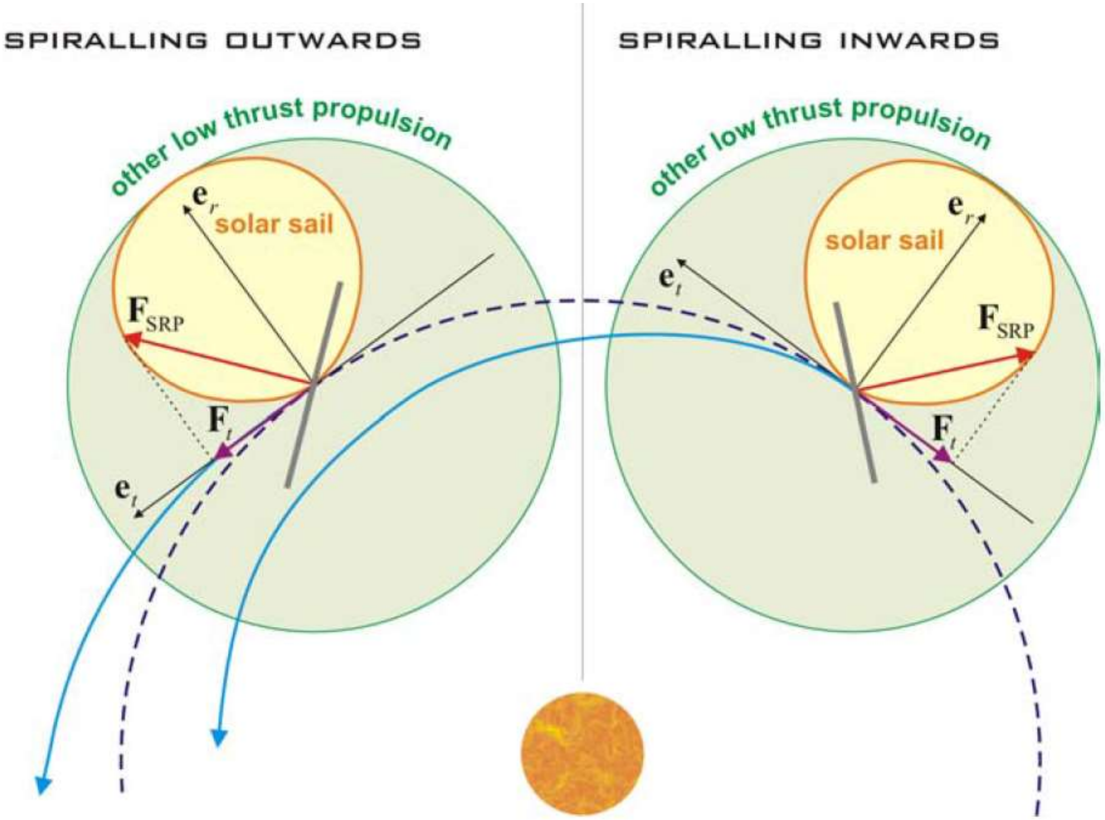
Low Thrust (EP)

- Free thrust direction
- Free magnitude (up to the max)

$$\mathbf{a} = \begin{bmatrix} a_x \\ a_y \\ a_z \end{bmatrix}, \qquad |\mathbf{a}| < a_{max}$$

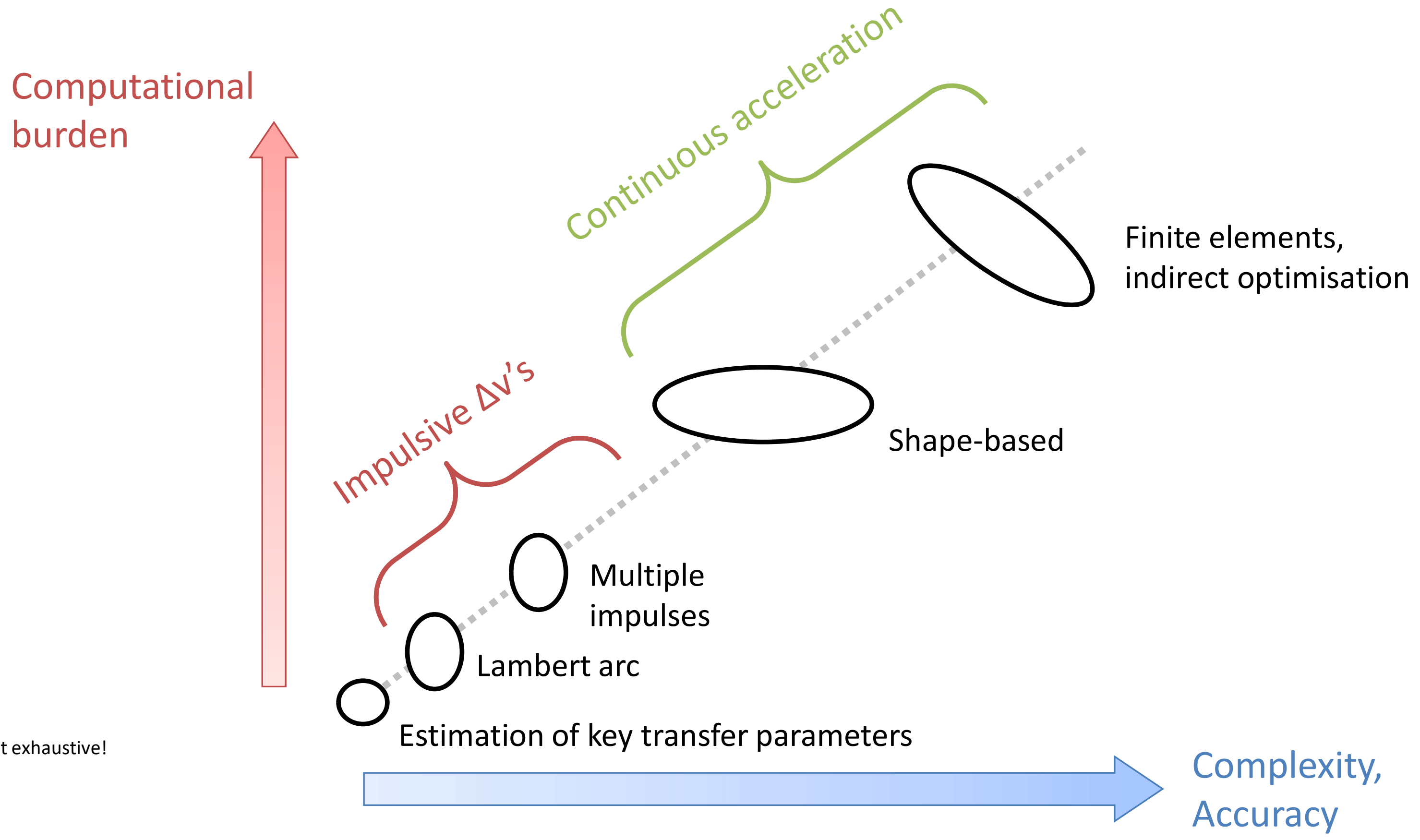
- (Multi-) Objective:
  - Propellant mass
  - Time of flight

Solar sailing vs. low thrust (EP)



Dachwald, B., “Low-Thrust Trajectory Optimization and Interplanetary Mission Analysis Using Evolutionary Neurocontrol”, Institut fur Raumfahrttechnik, Universitaet der Bundeswehr Muenchen, 2004.

Trajectory models



Computational burden

Continuous acceleration

Impulsive Δv's

Finite elements, indirect optimisation

Shape-based

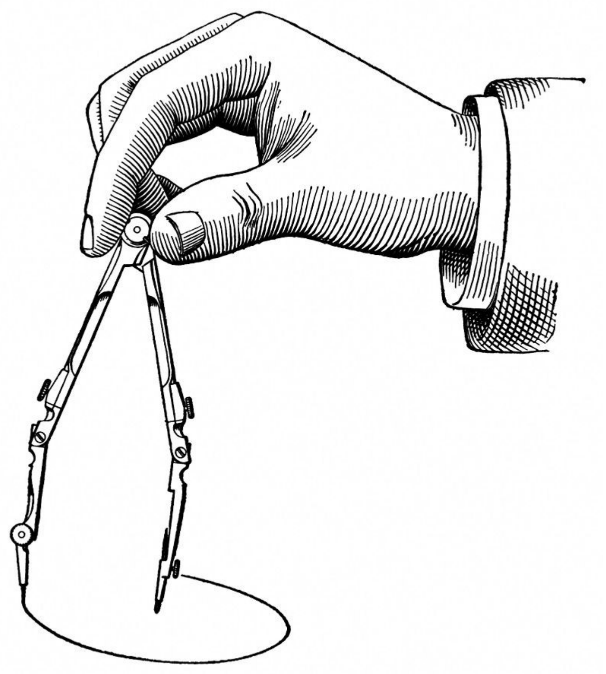
Multiple impulses

Lambert arc

Estimation of key transfer parameters

Complexity, Accuracy

Not exhaustive!



SHAPE-BASED TRAJECTORIES FOR SOLAR SAILING

Shape-based methods

- Take the Equations of Motion: $\ddot{\mathbf{x}} = f\left(\mathbf{x}, t\right) + \mathbf{a}\left(t\right)$
- Assign a “shape”, i.e. trajectory function of time: $\mathbf{x}\left(t\right) = \mathbf{x}^*\left(t, \mathbf{p}\right),\ \dot{\mathbf{x}},\ \ddot{\mathbf{x}}$
- Set shape parameters **p** to meet the boundary conditions (e.g. rendezvous): $\mathbf{x}^*\left(t_0, \mathbf{p}\right) = \mathbf{x}_0,\ \mathbf{x}^*\left(t_f, \mathbf{p}\right) = \mathbf{x}_f$
- “Invert” the EoM to find the control (acceleration) profile: $\mathbf{a}\left(t\right) = \ddot{\mathbf{x}}^* - f\left(\mathbf{x}^*, t\right)$

➕

- Usually fast to calculate
- Provides a thrust profile
  - Can be used as initial guess for OPC

➖

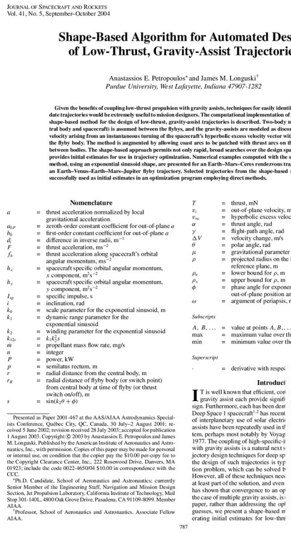
- Inversion of EoM only possible for specific shapes
- Does not always satisfy the constraints!

Shape-based methods

- Exponential sinusoid (Petropoulos & Longusky)
- Inverse polynomial (Wall & Conway)
- Pseudo-equinoctial (De Pascale & Vasile)
- Hodographic (Gondelach & Noomen)
- Finite Fourier series (Taheri & Abdelkhalik)
- …

Usually low-thrust (EP) with tangential acceleration

- Shape-based for solar sailing (Peloni et al.)
  - 2D
  - Sail magnitude/direction constraint not considered



JOURNAL OF SPACECRAFT AND ROCKETS
Vol. 41, No. 5, September–October 2004

Shape-Based Algorithm for Automated Design of Low-Thrust, Gravity-Assist Trajectories

Anastassios E. Petropoulos* and James M. Longuski†
Purdue University, West Lafayette, Indiana 47907-1282

Given the benefits of coupling low-thrust propulsion with gravity assists, techniques for easily identifying candidate trajectories would be extremely useful to mission designers. The computational implementation of an analytic, shape-based method for the design of low-thrust, gravity-assist trajectories is described. Two-body motion (central body and spacecraft) is assumed between the flybys, and the gravity-assists are modeled as discontinuities in velocity arising from an instantaneous turning of the spacecraft's hyperbolic excess velocity vector with respect to the flyby body. The method is augmented by allowing coast arcs to be patched with thrust arcs on the transfers between bodies. The shape-based approach permits not only rapid, broad searches over the design space, but also provides initial estimates for use in trajectory optimization. Numerical examples computed with the shape-based method, using an exponential sinusoid shape, are presented for an Earth–Mars–Ceres rendezvous trajectory and an Earth–Venus–Earth–Mars–Jupiter flyby trajectory. Selected trajectories from the shape-based method are successfully used as initial estimates in an optimization program employing direct methods.

JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 39, No. 12, December 2016

Solar-Sail Trajectory Design for a Multiple Near-Earth-Asteroid Rendezvous Mission

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DOI: 10.2514/1.G000470

The scientific interest for near-Earth asteroids as well as the interest in potentially hazardous asteroids from the perspective of planetary defense led the space community to focus on near-Earth asteroid mission studies. A multiple near-Earth asteroid rendezvous mission with close-up observations of several objects can help to improve the characterization of these asteroids. This work explores the design of a solar-sail spacecraft for such a mission, focusing on the search of possible sequences of encounters and the trajectory optimization. This is done in two sequential steps: a sequence search by means of a simplified trajectory model and a set of heuristic rules based on astrodynamics, and a subsequent optimization phase. A shape-based approach for solar sailing has been developed and is used for the first phase. The effectiveness of the proposed approach is demonstrated through a fully optimized multiple near-Earth asteroid rendezvous mission. The results show that it is possible to visit five near-Earth asteroids within 10 years with near-term solar-sail technology.

2D shape for solar sailing

EoM
Modified Equinoctial Elements

In-plane motion

$$p = a(1 - e^2)$$

$$f = e\cos(\Omega + \omega)$$

$$g = e\sin(\Omega + \omega)$$

$$h = \tan(i/2)\cos(\Omega)$$

$$k = \tan(i/2)\sin(\Omega)$$

$$L = \Omega + \omega + \nu$$

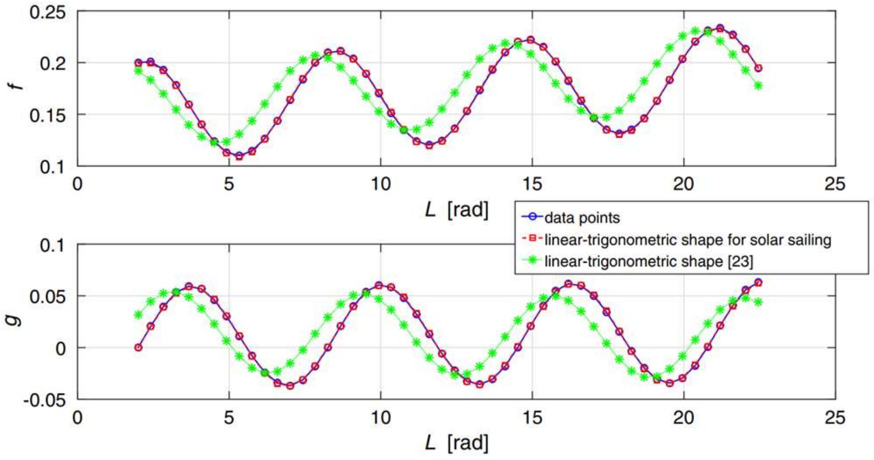
2D Shape

$$p = p_I \exp[p_F(L - L_0)] + \lambda_p \sin(L + \varphi_p)$$

$$f = f_I + f_F(L - L_0) + \lambda_{fg} \sin(L + \varphi_{fg})$$

$$g = g_I + g_F(L - L_0) - \lambda_{fg} \cos(L + \varphi_{fg})$$

Coplanar Earth-Mars



Peloni, A. , Ceriotti, M. and Dachwald, B. (2016) Solar sail trajectory design for a multiple near-Earth asteroid rendezvous mission. Journal of Guidance, Control, and Dynamics, 39(12), pp. 2712-2724. (doi: 10.2514/1.G000470)

3D shape for solar sailing

EoM
Modified Equinoctial Elements

3D Shape

In-plane motion

$$p = a(1 - e^2)$$

$$f = e\cos(\Omega + \omega)$$

$$g = e\sin(\Omega + \omega)$$

Out-of-plane motion

$$h = \tan(i/2)\cos(\Omega)$$

$$k = \tan(i/2)\sin(\Omega)$$

$$L = \Omega + \omega + \nu$$

$$p(L) = \tilde{p}_0 + \tilde{p}_f(L - L_0) + a_p(L - L_0)^2 + \lambda_p \sin(L - L_0 + \phi_p)$$

$$f(L) = \tilde{f}_0 + \tilde{f}_f(L - L_0) + \lambda_{fg} \sin(L - L_0 + \phi_{fg})$$

$$g(L) = \tilde{g}_0 + \tilde{g}_f(L - L_0) - \lambda_{fg} \cos(L - L_0 + \phi_{fg})$$

$$h(L) = \tilde{h}_0 + \tilde{h}_f(L - L_0) + a_h \exp[b_h(L - L_0)] + \lambda_{hk} \sin[2(L - L_0) + \phi_{hk}]$$

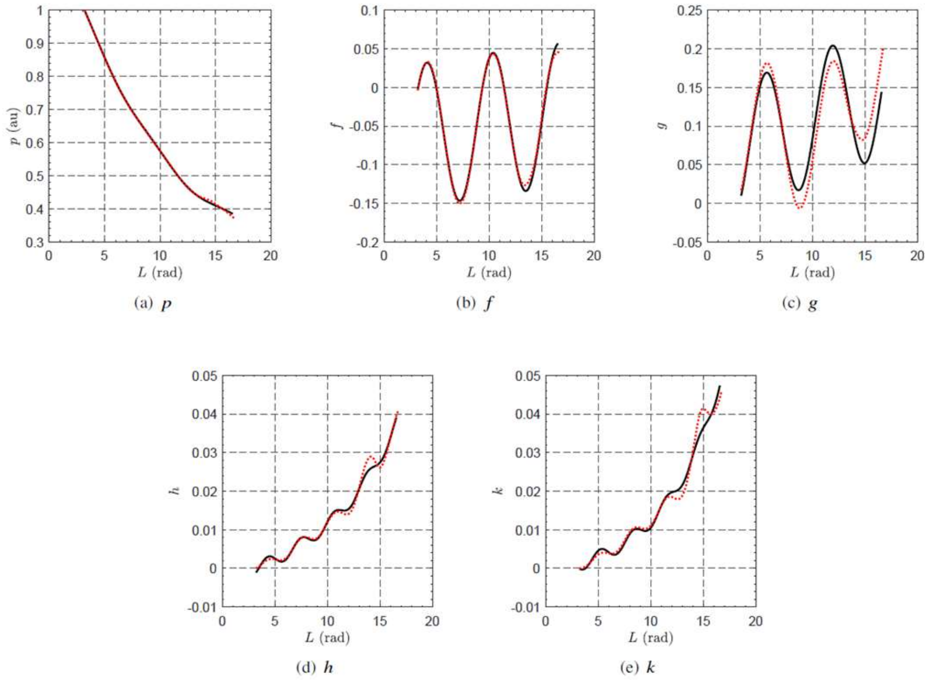
$$k(L) = \tilde{k}_0 + \tilde{k}_f(L - L_0) + a_k \exp[b_k(L - L_0)] - \lambda_{hk} \cos[2(L - L_0) + \phi_{hk}]$$

Free parameters

Caruso, A., Quarta, A., Mengali, G. and Ceriotti, M. (2020) Shape-based approach for solar sail trajectory optimization. Aerospace Science and Technology, 107, 106363. (doi: 10.1016/j.ast.2020.106363)

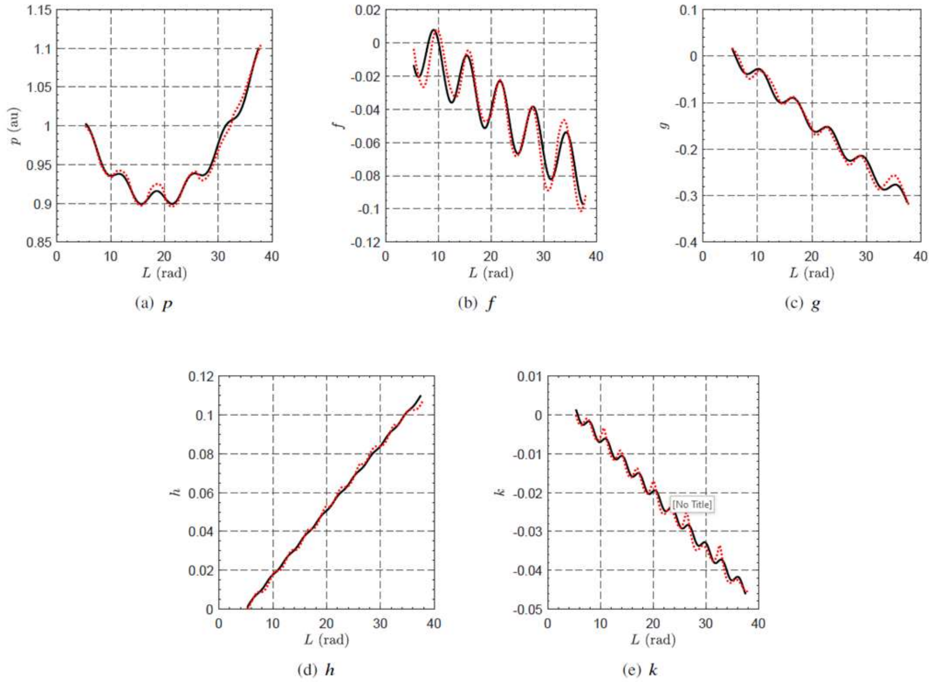
3D shape fitting

Earth-Mercury, a_c = 0.6 mm/s^2



(a) p (b) f (c) g (d) h (e) k

Earth-1620 Geographos (i = 13°), a_c = 0.2 mm/s^2



(a) p (b) f (c) g (d) h (e) k

┈┈┈ Optimal transfers computed with indirect method

——— Curve fitting to find free parameters

Shape-based trajectory design for solar sailing

$$\underbrace{\min}_{a_p,\, \lambda_p,\, \phi_p,\, \lambda_{fg},\, \phi_{fg},\, a_h,\, b_h,\, a_k,\, b_k,\, \lambda_{hk},\, \phi_{hk},\, L_0 \text{ and } L_f} \left\{ \text{Time of Flight} \quad T = \int_{L_0}^{L_f} \frac{\mathrm{d}t}{\mathrm{d}L} \mathrm{d}L \right\}$$

Boundary constraints (pos, vel) ⇨ $\tilde{p}_0$, $\tilde{p}_f$, $\tilde{f}_0$, $\tilde{f}_f$, $\tilde{g}_0$, $\tilde{g}_f$, $\tilde{h}_0$, $\tilde{h}_f$, $\tilde{k}_0$, and $\tilde{k}_f$

- Solar sail constraints:

$$\begin{cases} 1. \ : \ \max\limits_t(\|\boldsymbol{a}\| - a_{\max}) < 0 \\ 2. \ : \ \min\limits_t(a_r) > 0 \\ 3. \ : \ \max\limits_t\left(\left|a_r - a_c\left(\frac{r_\oplus}{r}\right)^2 \cos^3\alpha\right|\right) \boxed{< \epsilon} \\ 4. \ : \ \max\limits_t\left(\left|a_t - a_c\left(\frac{r_\oplus}{r}\right)^2 \cos^2\alpha \sin\alpha\right|\right) \boxed{< \epsilon} \end{cases}$$

Constraint satisfaction

Shape-based trajectory design for solar sailing

1. Shape-based trajectory:
    - Genetic Algorithm + gradient-based
    - 3 methods:
        1. Solar sail shape + all constraints 1-4
        2. Solar sail shape + constraints 1-2
        3. Linear-trigonometric shape (LT, Pascale, Vasile) + constraint 1

1. $: \max_t(\|\boldsymbol{a}\| - a_{\max}) < 0$
2. $: \min_t(a_r) > 0$
3. $: \max_t(|a_r - a_c(\frac{r_\oplus}{r})^2 \cos^3\alpha|) \boxed{< \epsilon}$
4. $: \max_t(|a_t - a_c(\frac{r_\oplus}{r})^2 \cos^2\alpha \sin\alpha|) \boxed{< \epsilon}$

2. Optimisation with full dynamics:
    - Multiple-shooting
    - Shape-based solution used as initial guess

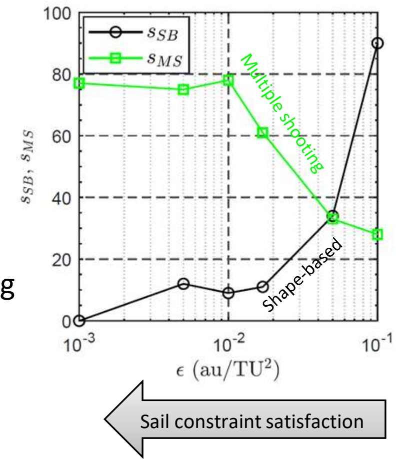
Performance evaluation

- Method 1:

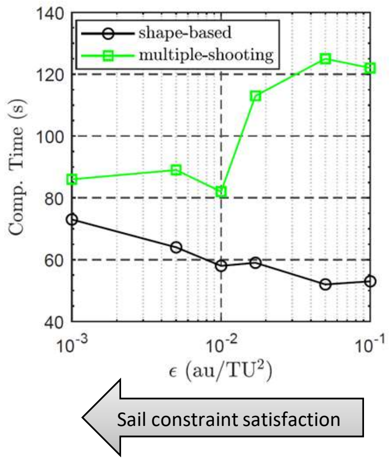
— Shape-based
— Multiple shooting

Earth - 2002 DU$_3$ $a_c = 0.3$ mm/s^2

Success rate



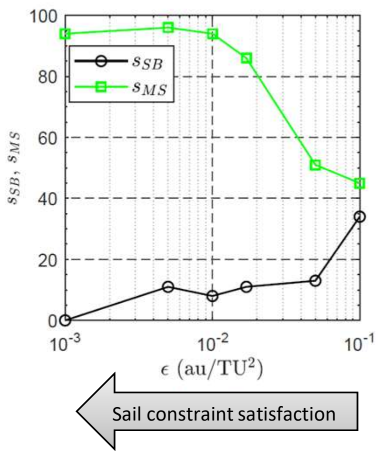
Comp. time



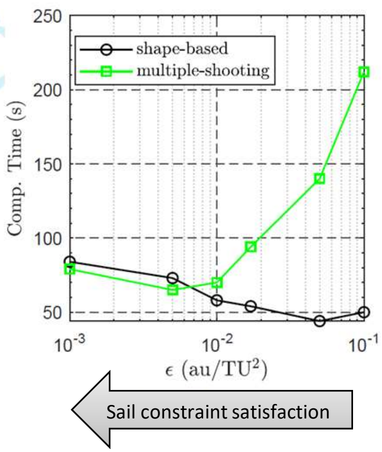
	Shape-based		Multiple shooting	
	Success rate	Comp. time	Success rate	Comp. time
Method 2:	93 %	58 s	38 %	104 s
Method 3:	50 %	52 s	16 %	187 s

Earth - 2007 MK$_{13}$ $a_c = 0.3$ mm/s^2

Success rate



Comp. time



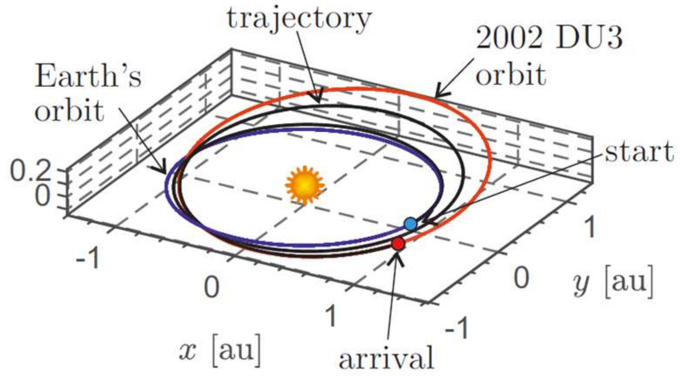
	Shape-based		Multiple shooting	
	Success rate	Comp. time	Success rate	Comp. time
Method 2:	100 %	45 s	46 %	221 s
Method 3:	100 %	16 s	2 %	284 s

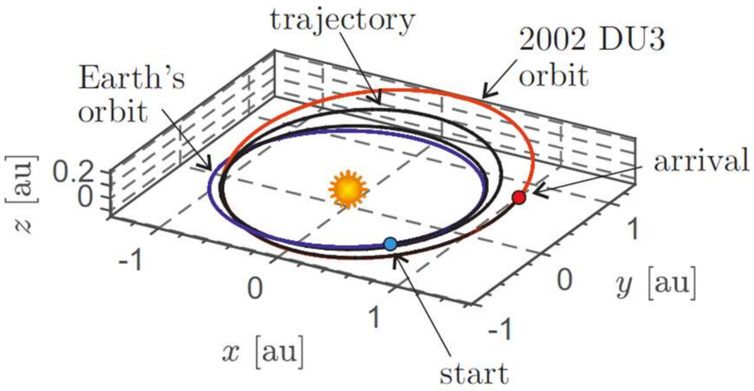
Results

a_c = 0.2 mm/s^2
Method 1
ε = 10^{-2}

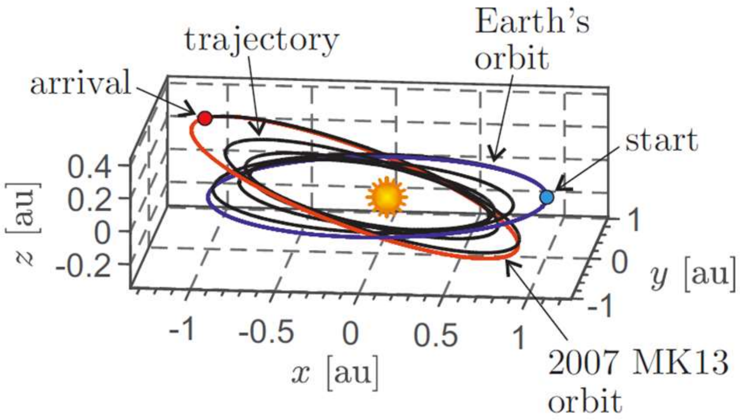
Shape-based | **Multiple shooting (optimal)**

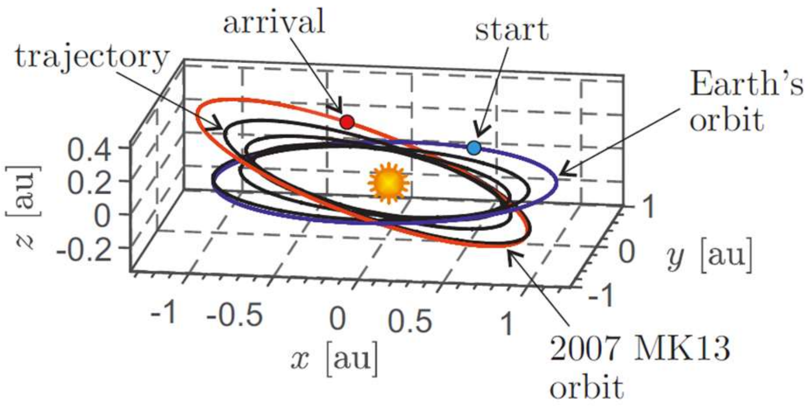
Earth – 2002 DU$_3$

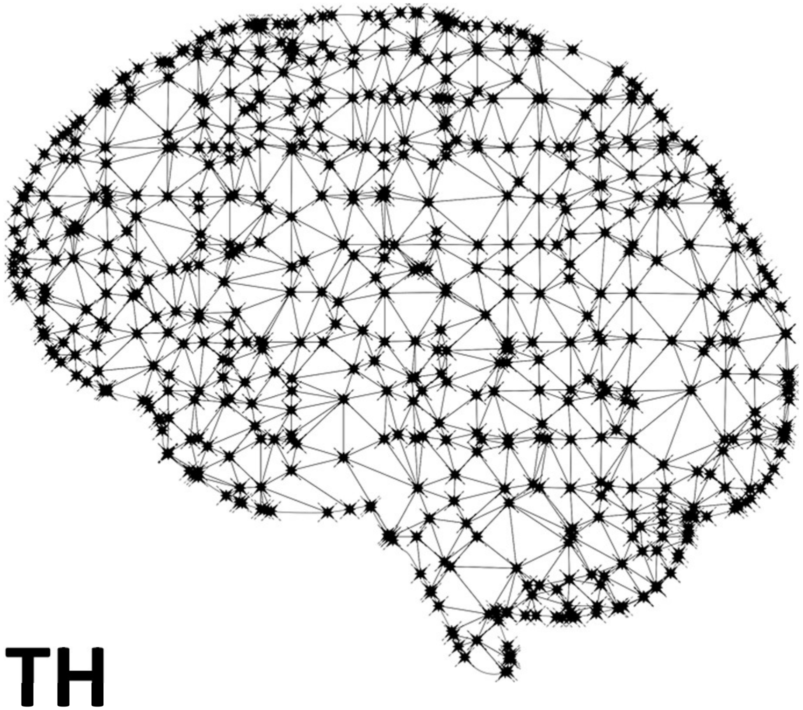




Earth – 2007 MK$_{13}$



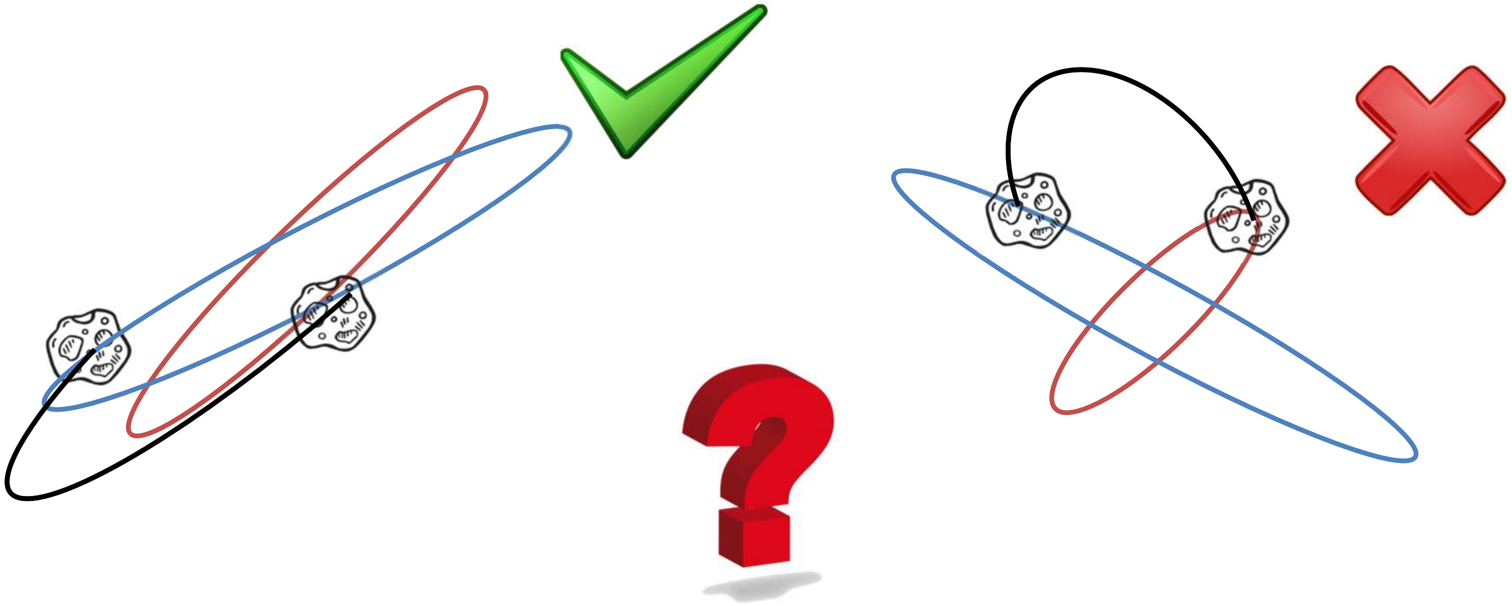




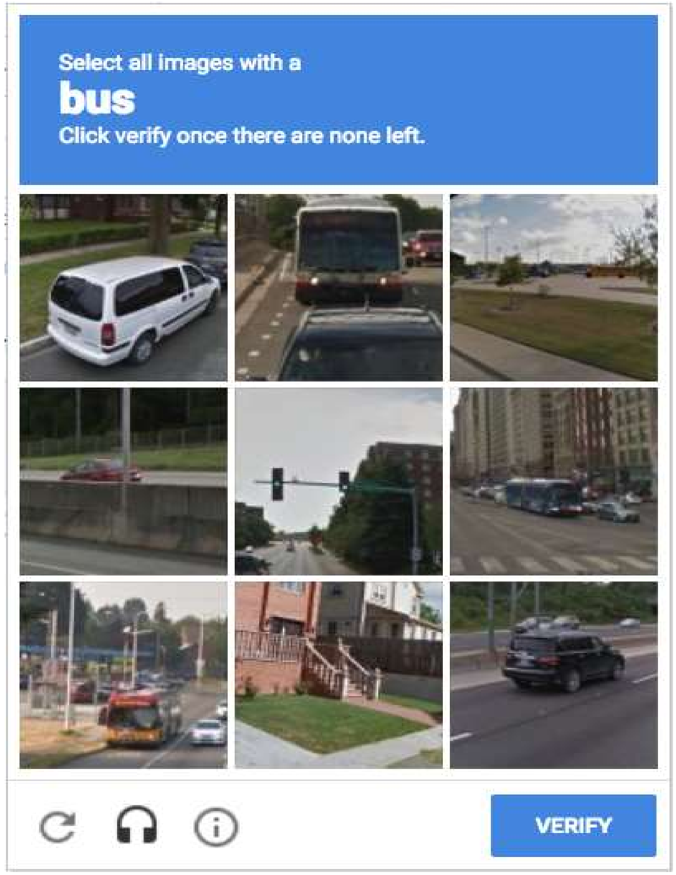
SAIL TRAJECTORY DESIGN WITH ARTIFICIAL NEURAL NETWORKS

An even faster trajectory estimation

- Can a machine to “learn” to estimate the “cost” of transfers between asteroids?



Artificial Neural Networks



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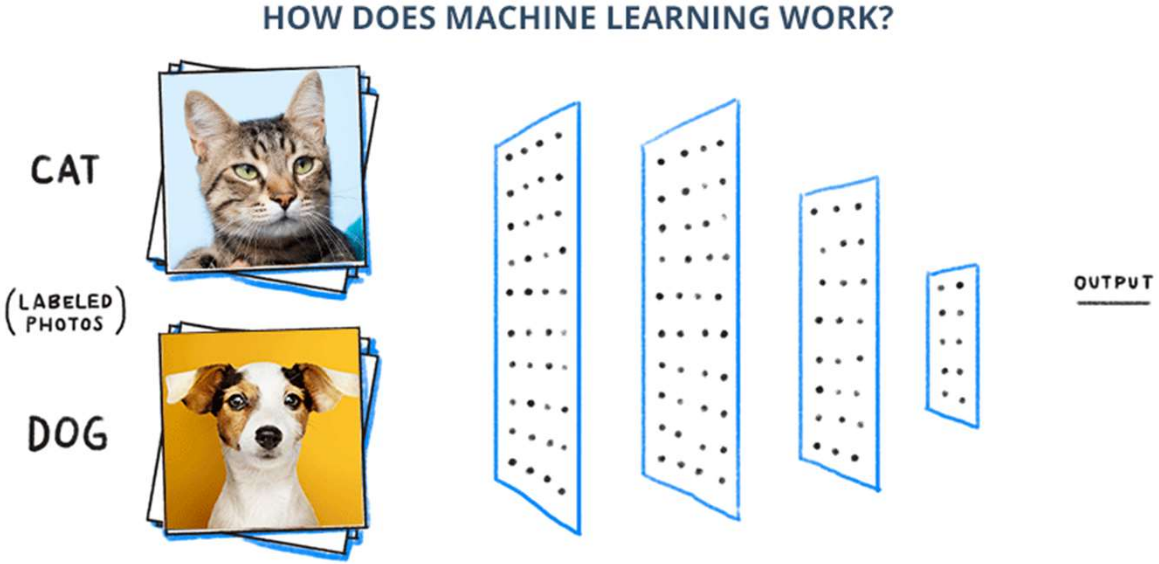
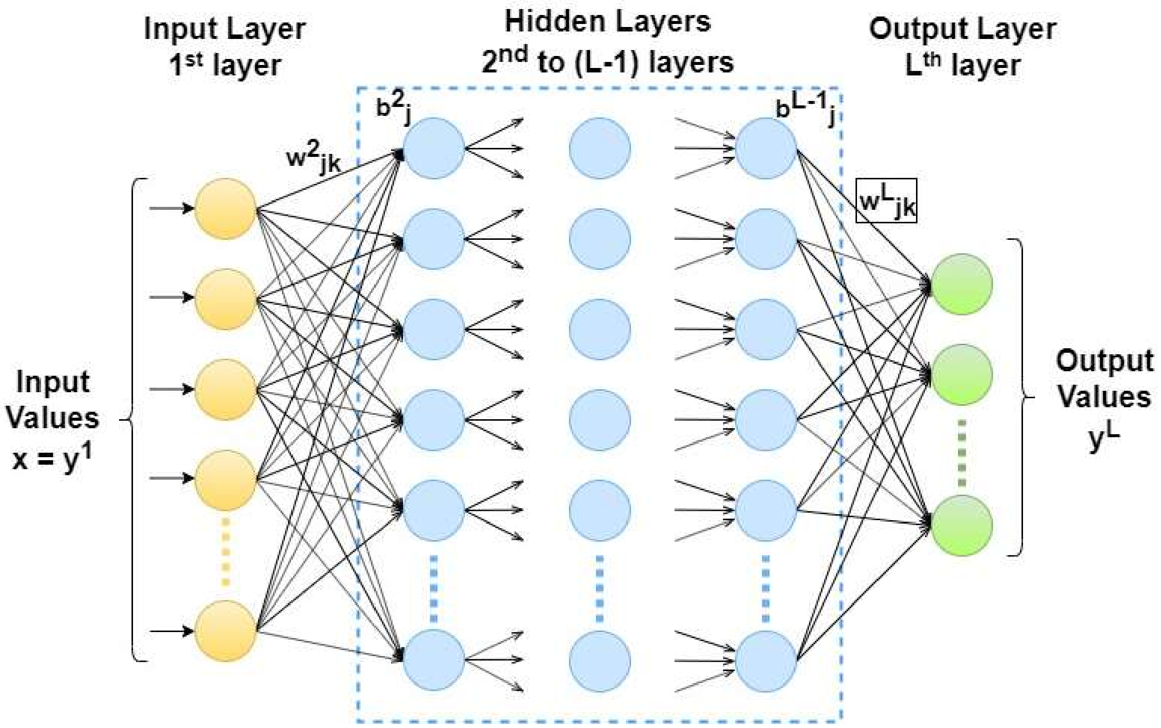
Click verify once there are none left.



Kate Libbie, QATestLab

Neural Network Design



TRAINING DATABASE

Collection of $(\mathbf{x}, \mathbf{t})_i$ with $i \in [1, N]$

Input

Target output
(from full model)

TRAINING

Find optimal w^l_{jk} and b^l_j

so that **Mean Square Error** (MSE) is minimised

$$\mathrm{MSE} = \frac{1}{N}\sum_{i=1}^{N} \|\mathbf{y}_i - \mathbf{t}_i\|$$

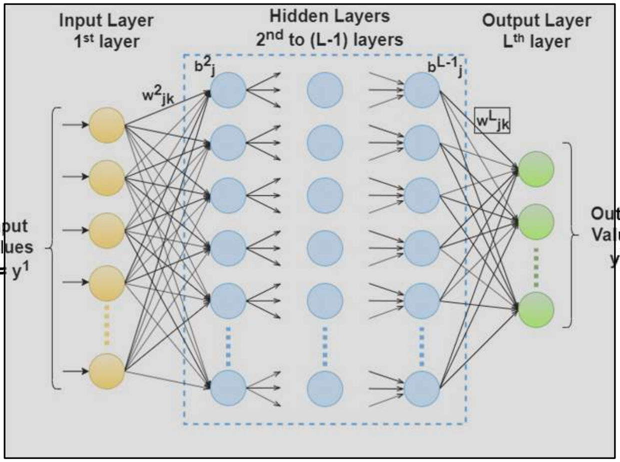
ANN output
(estimation)

Target output
(from full model)

Neural Network Design

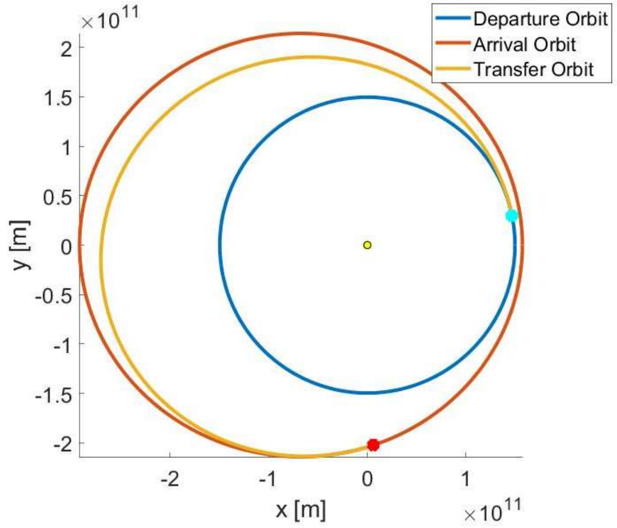
Departure object orbital parameters

Arrival object orbital parameters



TOF

Δv

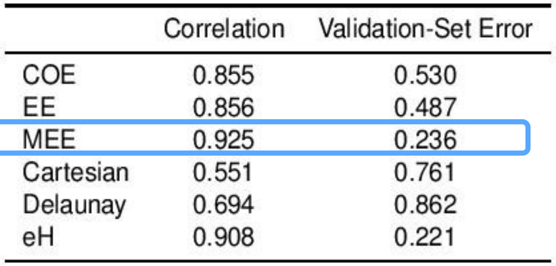


$$\mathbf{x} = [\overbrace{p_1,\ f_1,\ g_1,\ h_1,\ k_1,\ L_{1,0},}\ \overbrace{p_2,\ f_2,\ g_2,\ h_2,\ k_2,\ L_{2,0}}]$$

Viavattene, G. and Ceriotti, M. (2020) Artificial Neural Network Design for Tours of Multiple Asteroids. In: 15th International Conference on Hybrid Artificial Intelligence Systems (HAIS 2020), Gijón, Spain, 11-13 Nov 2020, pp. 751-762. ISBN 9783030617042 (doi:10.1007/978-3-030-61705-9_63)

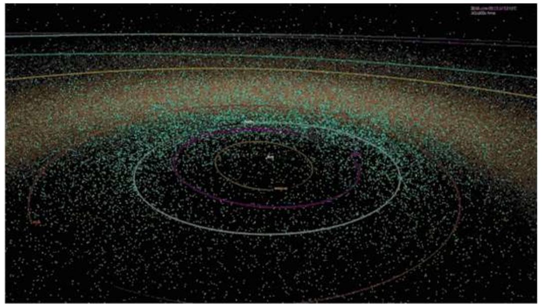
Additional challenges

- **Orbit parameterisation**

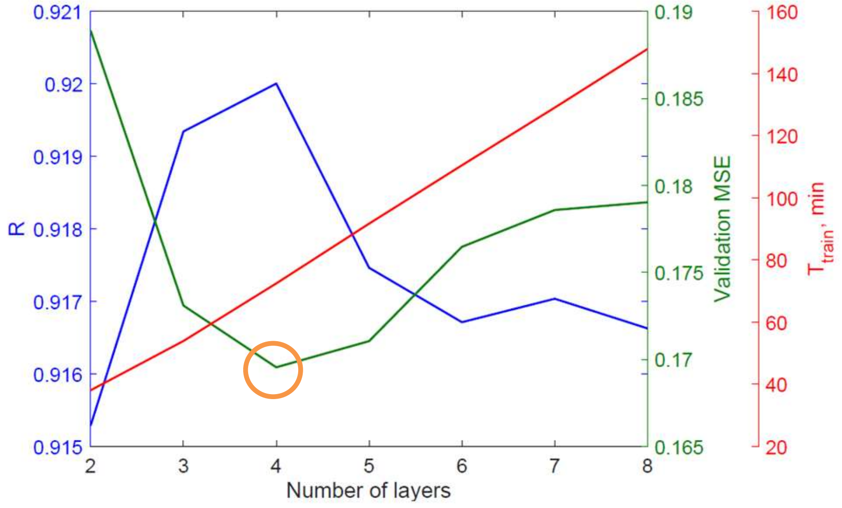


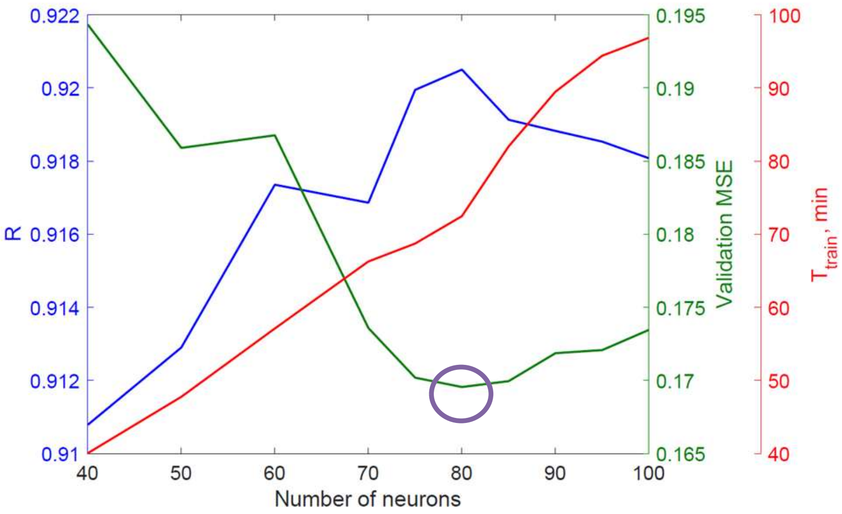
		Correlation	Validation-Set Error
Classical Orbital Elements	COE	0.855	0.530
Equinoctial Elements	EE	0.856	0.487
Modified Equinoctial Elements	MEE	0.925	0.236
Cartesian Coordinates	Cartesian	0.551	0.761
Delaunay Elements	Delaunay	0.694	0.862
Eccentricity and angular momentum vector	eH	0.908	0.221

- **Dependence on time** (phasing problem)
  - Orbital transfer ≠ Rendezvous



ANN architecture design

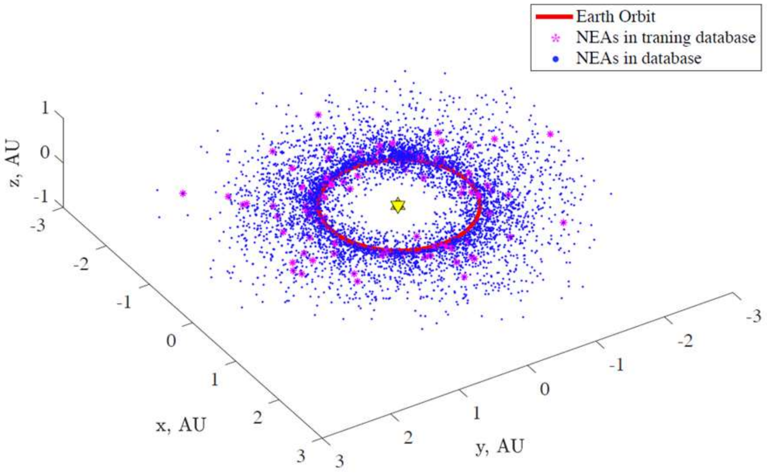


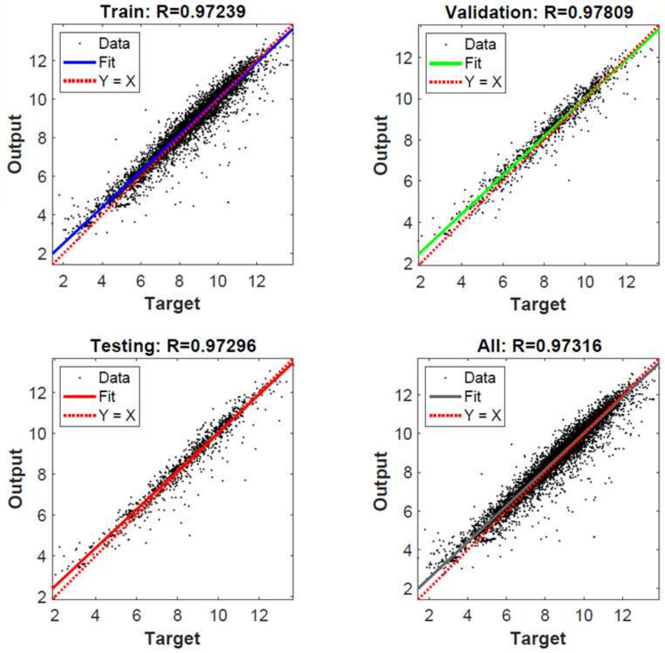


ANN parameter	Search space	Optimal value
No. hidden layers	[2, 8]	4
No. neurons	[40, 100]	80
Learning algorithm	Levenberg-Marquardt Resilient back-propagation Scaled conjugate gradient Gradient descent	Levenberg-Marquardt
Activation function	tansig, sigmoid, ReLu	Sigmoid

ANN Training

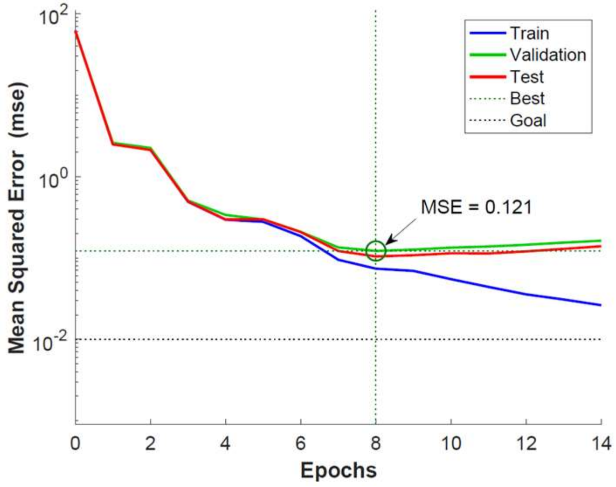
- 100 NEA in training database + Earth





- 10,100 low-thrust transfers
  - Training set (70%)
  - Validation set (15%)
  - Test set (15%)
- Exponential sinusoid shape with $a_{\max} = 0.1$ mm/s^2

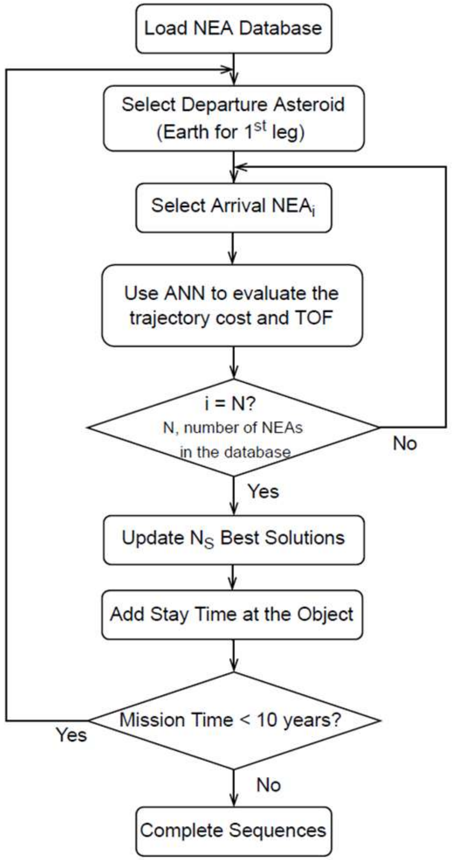
R = 0.97
MSE = 0.1211



Sequence search with ANN

Multiple asteroid rendezvous

- 6,286 asteroids
  - ~300 PHAs, ~1,450 NHATS
  - Excluded highly inclined ($i \geq 20^\circ$) and eccentric ($e \geq 0.4$)
- Launch date fixed: 2035/01/01
  - Systematic scan of launch window could be done
- Stay time at asteroid: 100 days
- Only best 200 transfers with lowest ToF stored
  - Limits exponential growth
- 2 sequences selected for full optimisation
  - OPC solver GPOPS-II [Patterson, Rao]
  - Ideal solar sail with $a_c = 0.2$ mm/s^2



Asteroid sequence tree graph

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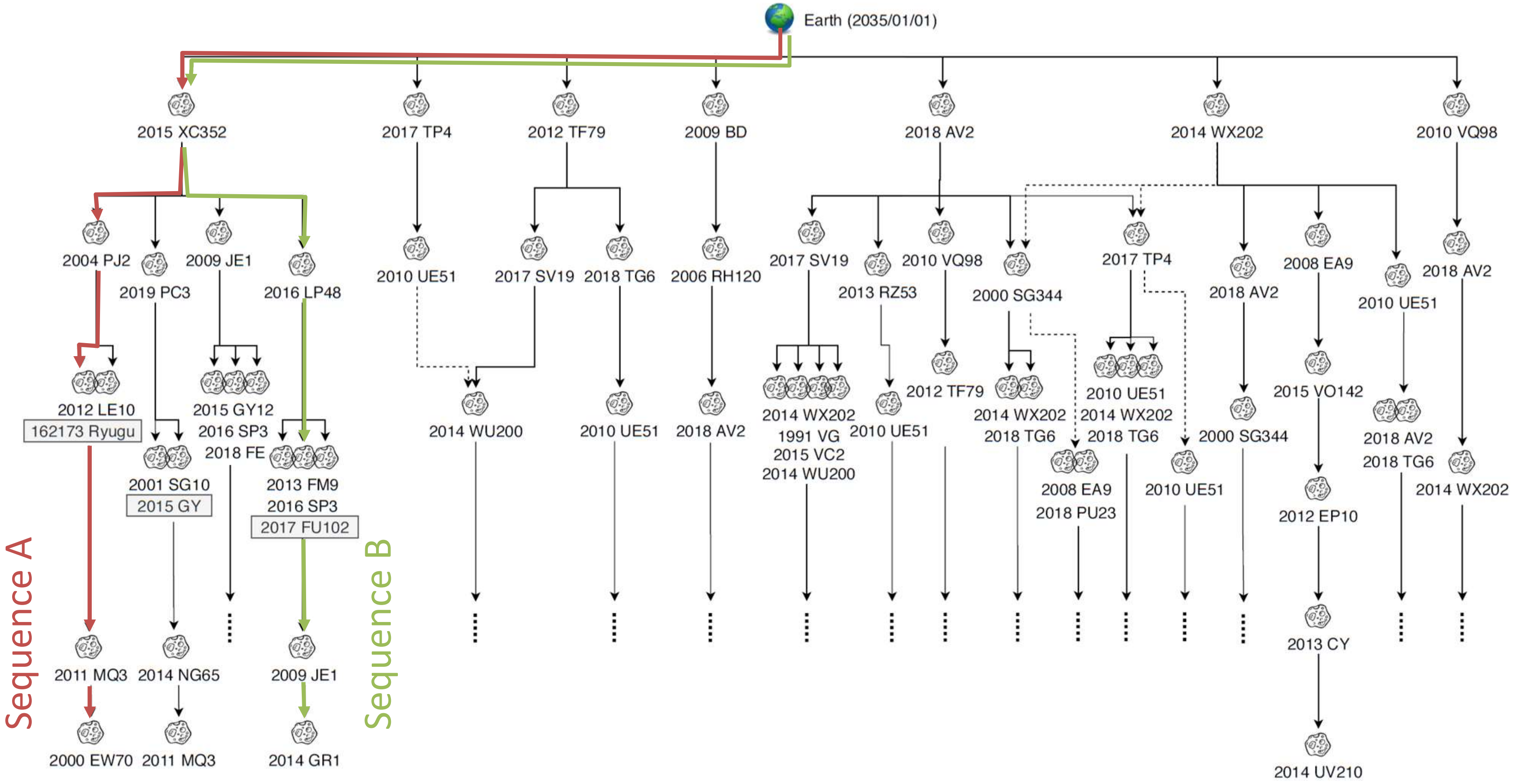
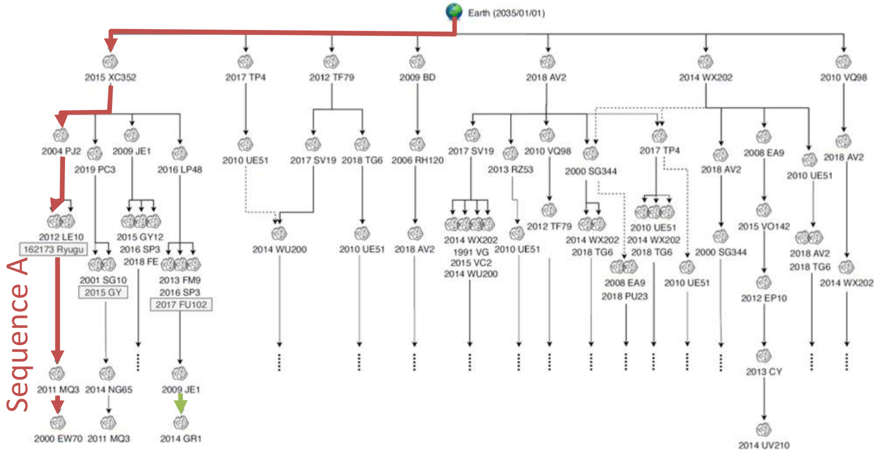
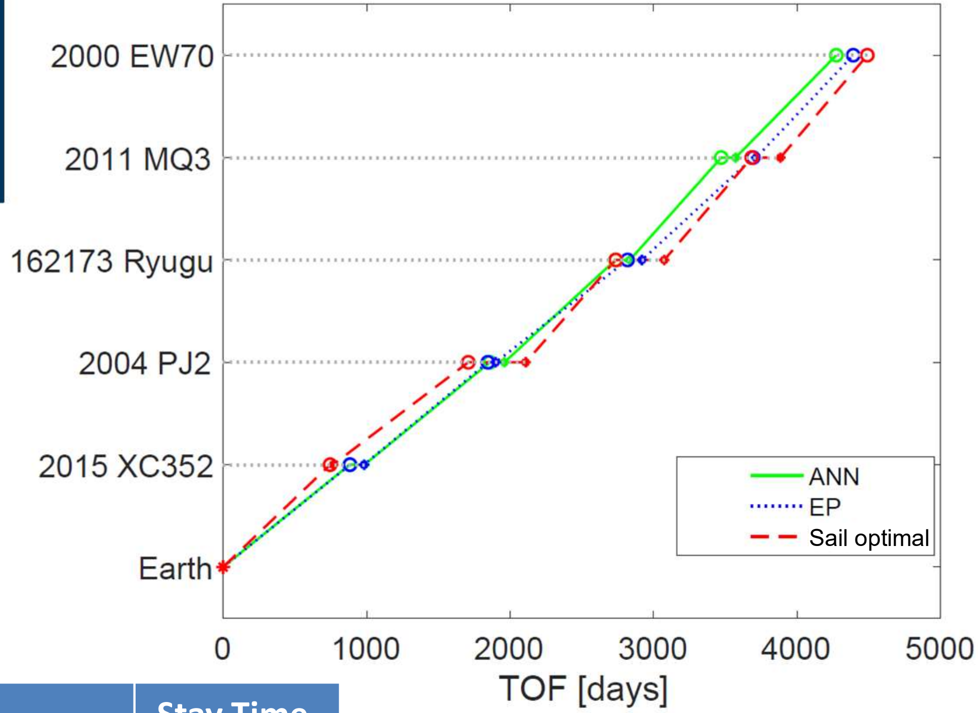


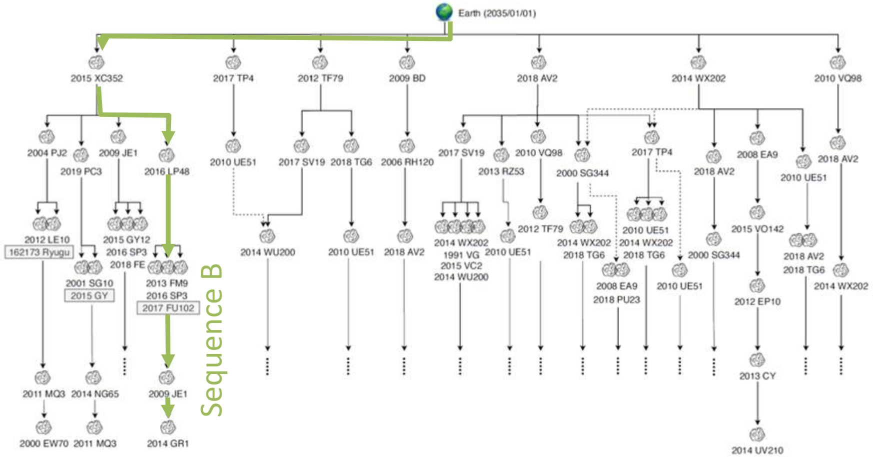
Sequence A

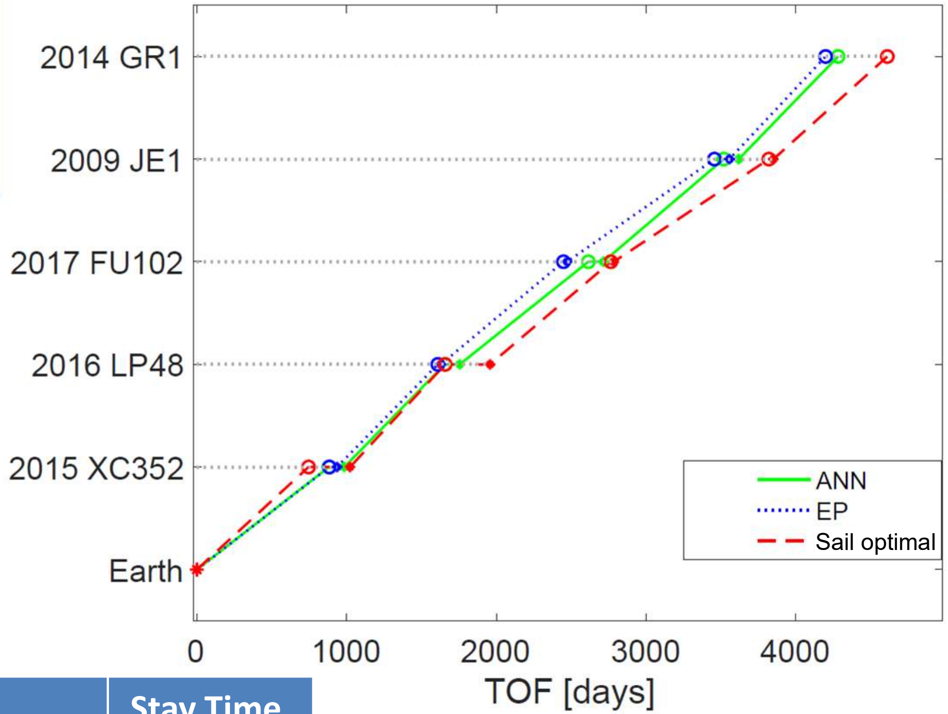




Leg	Departure	Arrival	TOF, days		Stay Time, days
			ANN estimate	Optimal	
Earth - 2015 XC352	2035-02-08	2037-04-04	882	746	20
2015 XC352 - 2004 PJ2	2037-04-24	2039-12-15	878	943	400
2004 PJ2 - 162173 Ryugu	2037-04-24	2042-10-08	775	628	338
162173 Ryugu - 2011MQ3	2043-09-20	2045-05-13	637	609	200
2011MQ3 - 2000 EW70	2045-11-29	2047-07-28	702	606	-

Sequence B





Leg	Departure	Arrival	TOF, days		Stay Time, days
			ANN estimate	Optimal	
Earth - 2015 XC352	2035-02-08	2037-04-04	882	746	277
2015 XC352 - 2016 LP48	2037-11-28	2039-08-26	673	635	300
2016 LP48 - 2017 FU102	2040-06-21	2042-09-05	859	806	20
2017 FU102 - 2009 JE1	2042-09-26	2045-07-27	984	1035	31
2009 JE1 - 2014 GR1	2045-08-27	2047-09-26	642	760	-

Performance analysis

Error in Time of Flight estimation

$$\mathcal{E}_{TOF} = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{|TOF_{i,opt} - TOF_{i,ANN}|}{TOF_{i,opt}}\right) = 13.4\ \%$$

ANN within sequence search algorithm

200 best sequences in less than 8 hours

~100x faster than using shape-based trajectories*

ANN **not** trained on fully-optimal solar sail trajectories

*compared to the method used in: A. Peloni, M. Ceriotti, and B. Dachwald. Solar-Sail Trajectory Design for a Multiple Near-Earth-Asteroid Rendezvous Mission. Journal of Guidance, Control, and Dynamics, 39(12):2712–2724, Sep 2016

Conclusions

- **Shape-based methods** can provide an approximate thrust profile
  - Still require optimisation
  - Trade-off between accuracy of solution and computational time
  - A better shape-based solution eases the follow-up optimisation, but takes longer

- **Artificial neural networks** can provide a very fast estimate of transfer "cost"
  - Require training
  - The position-time problem (phasing of bodies) is not fully solved

- **Future work**
  - Train ANN with full solar sail trajectory model

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Thank you!

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