Ways to Deploy a Large Size Solar Sail

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Outline

1. Objective of the study
2. Solar sails with a superconducting circular current-carrying loop or inflatable toroidal shell
   - *Current-carrying wire self-forces*
   - *Inflatable toroidal shell forces*
   - *Stress and strain in a circular membrane*
   - *Circular membrane attached to the current-carrying wire or* inflatable toroidal shell
3. Remarks
4. Conclusion

Detailed discussion and results can be found in articles

- Kezerashvili & Kezerashvili, *On deployment of solar sail with superconducting current-carrying wire.* Acta Astron. 189 (2021) 196.
- Kezerashvili & Kezerashvili, *Solar sail with superconducting circular current-carrying wire.* Adv. Space Res. 69 (2022) 664
- Kezerashvili, Kezerashvili & Starinova, *Solar sail with inflatable toroidal shell.* Acta Astron. 202 (2023) 17

Solar Sailing

A solar sail is a large sheet of low areal density material that captures and reflects the electromagnetic flux as a means of acceleration.

Studies on a solar sail have four primary foci:

- Finding low areal density material that allows to utilize the maximum acceleration due to the solar radiation pressure;
- Missions design for exploration of the Solar System and beyond using the solar sail;
- Maximizing the solar thrust through the increase of area of the solar sail made of a low areal density material;
- The development of mechanisms for the deployment and stretching the large size of solar sail membrane.

We focus on the last two objectives

Deployment Technology and Concerns

After a solar sail is released from the launch vehicle, it would need to be fully deployed to a functional sailcraft

Many different systems have been considered for the sail opening.

The deployment is usually performed by uniaxial mechanisms, such as

- extendable masts,
- telescopic, deployable, inflatable booms,
- composite booms,
- rigid linkage or reeling mechanisms.
- the centrifugal force that renders a spin-type deployment mechanism
- solar sail self-deployment using shape memory alloys (NiTi Shape Memory wires)

We list these approaches, but the recent literature on the subject is not limited by them.

Concerns

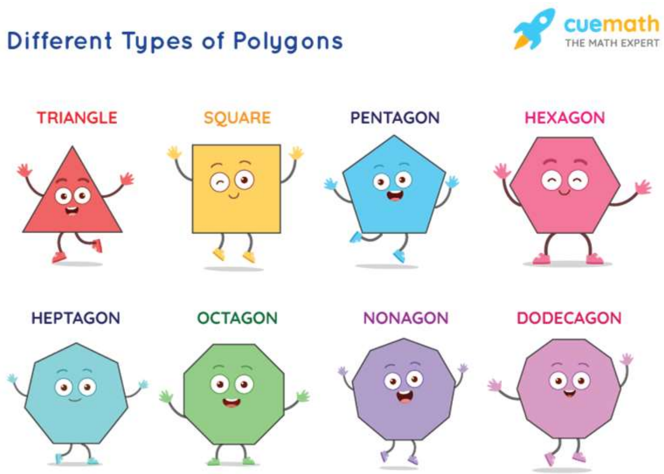
In the actual deployment technology of the solar sail, the main limitation is still the large weight of the system and the complexity of the deployment mechanism for the solar sail surface.

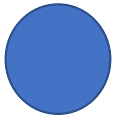
Generally, larger solar sails would require booms with large length-to-diameter ratios.

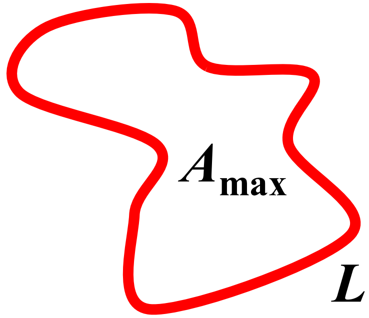
Some of the main concerns are boom buckling, jamming and other issues that could lead to deployment failure.

Booms developed for small sails may be insufficient to provide the needed structural support and mass effectiveness for large sails – Fu, Aerosp. Sci. Technol. 50, 281 (2016)

What Planar Surface Has the Maximum Area?







We use two equations:

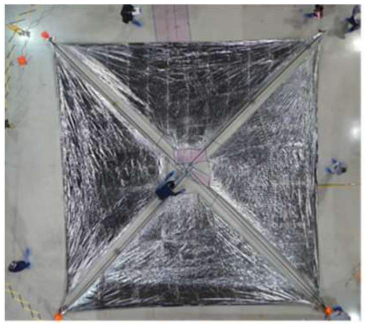
1. Maximum area enclosed by any given closed curve

$$\frac{1}{2}\int_0^{2\pi} r_{\max}^2 d\theta = A_{\max}$$

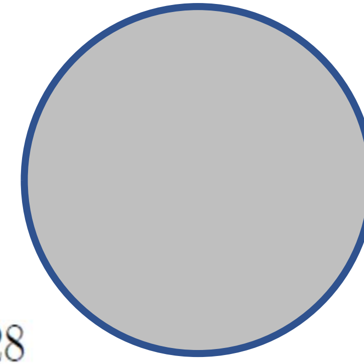
2. Perimeter described by such a curve

$$\int_0^{2\pi} \sqrt{r_{\max}^2 + \left(\frac{dr_{\max}}{d\theta}\right)^2} d\theta = L$$

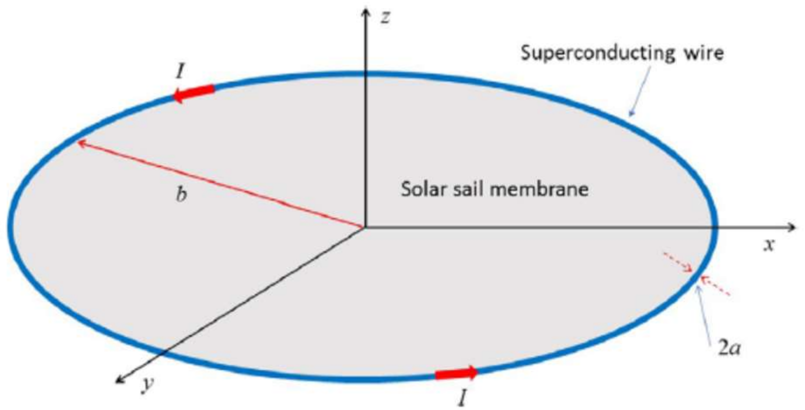
Application of the calculus of variation leads to the circle



$$\sqrt{\frac{2}{\pi}} = 1.128$$



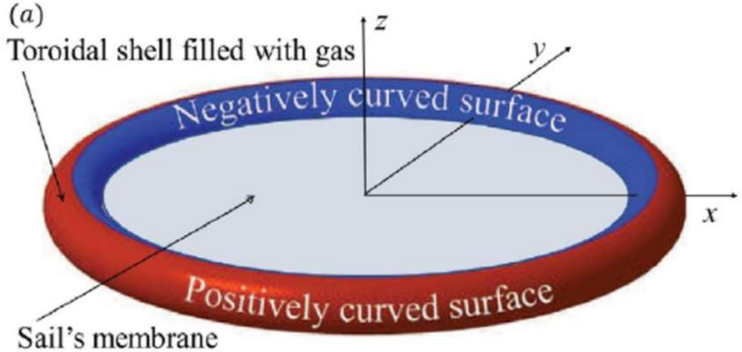
Solar sail with superconducting circular current-carrying loop

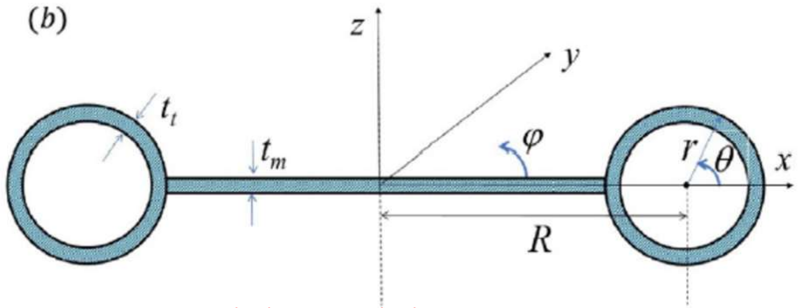


A schematic of the circular solar sail of radius *b* attached to a superconducting circular wire with cross-sectional radius *a* carrying a steady-state current *I* and laying in the *x* - *y* plane. The solar sail is stretched by self-forces generated by the magnetic field induced by the current-carrying wire on itself. The radius *b* is significantly larger than the cross-sectional radius of the wire a : *b* >>*a*.

Kezerashvili & Kezerashvili Advances in Space Research 69 (2022) 664

Solar sail with inflatable toroidal shell





$R \gg r$

Kezerashvili, Kezerashvili & Starinova, Acta Astronautica 202 (2023) 17

Magnetic field of the circular current-carrying wire

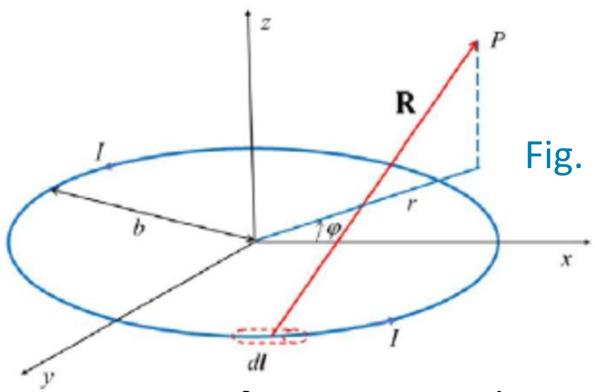


Fig. 1.

For a system of any currents the vector potential *A* and magnetic induction *B* in the surrounding space become (Landau and Lifshitz, 1984; Jackson, 1998)

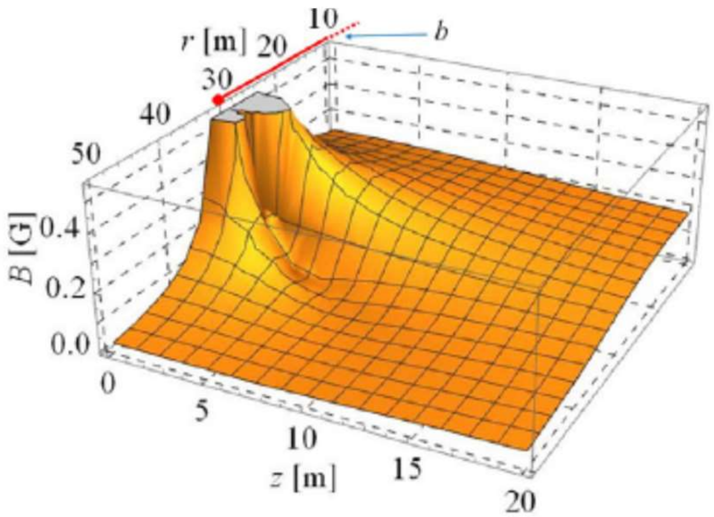
$$\mathbf{A} = \frac{\mu_0 I}{4\pi} \oint \frac{d\mathbf{l}}{R}, \qquad \mathbf{B} = \frac{\mu_0 I}{4\pi} \oint \frac{d\mathbf{l} \times \mathbf{R}}{R^3} \qquad (1)$$

By introducing the cylindrical coordinates *r; ϕ; z* (Fig. 1), axial symmetry of the system reduces Eq. (1) to a single scalar. The exact analytical solutions are

$$A_\varphi = \frac{\mu_0 I}{\pi k}\left(\frac{b}{r}\right)^{1/2}\left[\left(1 - \frac{1}{2}k^2\right)K(k) - E(k)\right],$$

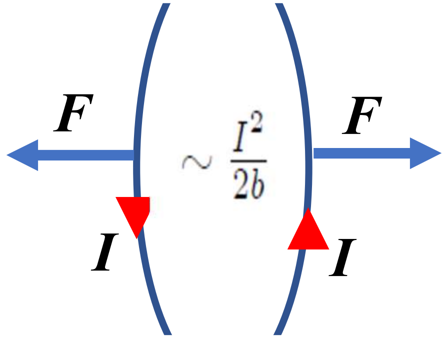
$$B_r = \frac{\mu_0 I}{2\pi}\frac{z}{r\left[(b+r)^2 + z^2\right]^{1/2}}\left[-K(k) + \frac{b^2 + r^2 + z^2}{(b-r)^2 + z^2}E(k)\right],$$

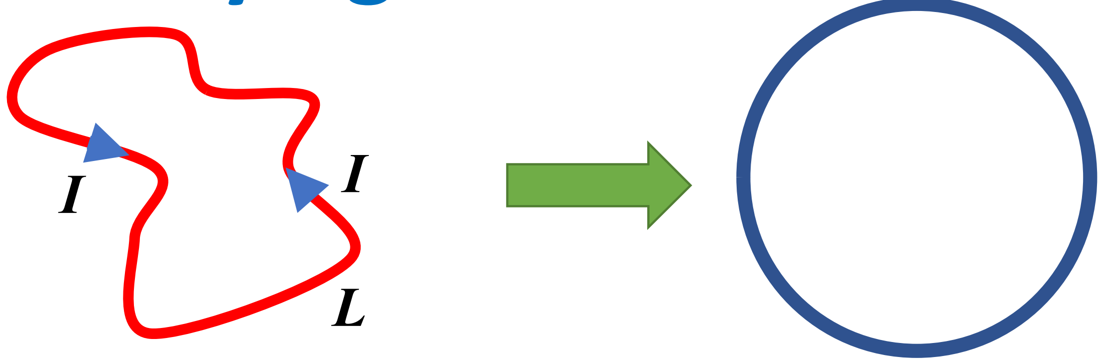
$$B_z = \frac{\mu_0 I}{2\pi}\frac{1}{\left[(b+r)^2 + z^2\right]^{1/2}}\left[K(k) + \frac{b^2 - r^2 - z^2}{(b-r)^2 + z^2}E(k)\right].$$



The magnetic field of the circular current-carrying wire as a function of *r* and *z*. The logarithmic singularity of the magnetic field at *r* = *b* is indicated by the point.

Current-carrying wire self-forces





The total energy of the magnetic field of a single wire carrying a constant current *I* is $U = \frac{1}{2}LI^2,$

The forces acting on the wire are $F = \left(\frac{\partial U}{\partial q}\right)_I = \frac{1}{2}I^2\left(\frac{\partial L}{\partial q}\right)_I$

The latter means that the forces acting on the wire will tend to increase wire self-inductance *L*. Therefore, the size of the area bound by wire increases to the maximum under the action of the magnetic self-field

The self-inductance of a current-carrying circular wire of radius *b* with a circular cross-section of radius *a* << *b* is

$$L = \mu_0 b\left(\ln\frac{8b}{a} - \frac{7}{4}\right)$$

For type I superconductors and type II superconductors with the magnetic field below the first critical field

$$L = \mu_0 b\left(\ln\frac{8b}{a} - 2\right)$$

Both expressions are logarithmically accurate and leads to an important conclusion: in equilibrium, a flexible closed current loop of any shape will tend to take the shape spanning the maximum area A for the fixed length of the wire L, which is the area of the circle.

In other words, the equilibrium shape of the flexible current-carrying wire is a perfect circle due to the magnetic force self-action.

Force acting on the unit length of superconducting current loop

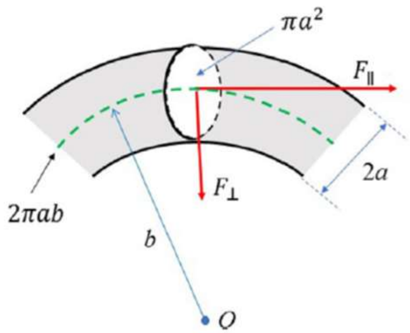
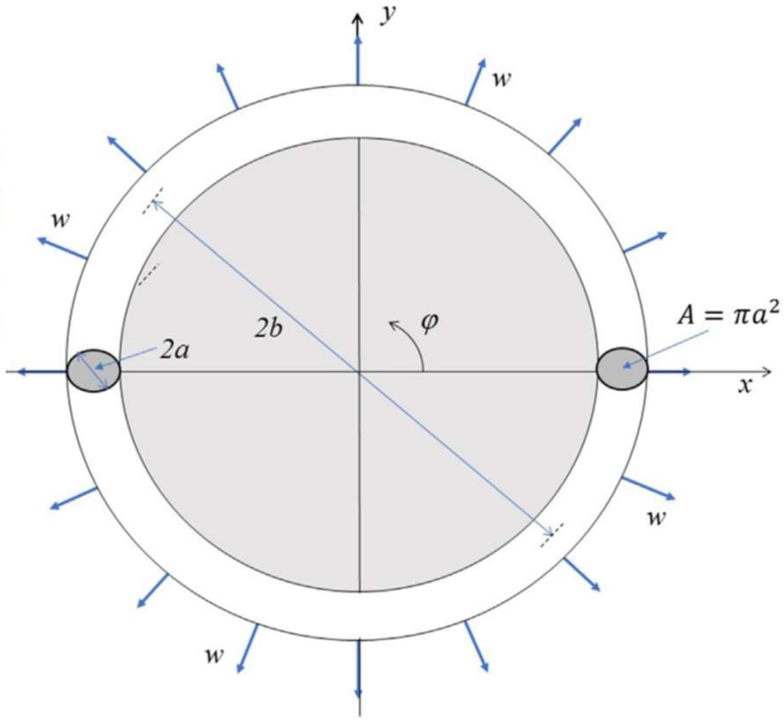


Fig. 3. A schematic for the action of forces $F_\parallel$ and $F_\perp$ on the circular wire element. The force $F_\parallel$ acts on the cross-section area πa^2 along the axis of the wire. For visibility, the cross-section of the wire is given in 3D format. The force $F_\perp$ acts on the area $2\pi ab$ perpendicular to the plane of the figure

$$F_\parallel = \frac{1}{2} I^2 \frac{\partial L}{\partial(2\pi b)} = \frac{\mu_0}{4\pi} I^2 \left(\ln\frac{8b}{a} - \frac{3}{4}\right) = \pi a^2 \sigma_\parallel,$$

$$F_\perp = \frac{1}{2} I^2 \frac{\partial L}{\partial a} = -\frac{1}{2}\frac{\mu_0 b}{a} I^2 = 2\pi ab\sigma_\perp,$$



$$\sigma_\parallel = \frac{\mu_0}{4\pi^2}\frac{I^2}{a^2}\left(\ln\frac{8b}{a} - \frac{3}{4}\right),$$

$$\sigma_\perp = -\frac{\mu_0}{2\pi}\frac{I^2}{a^2}.$$

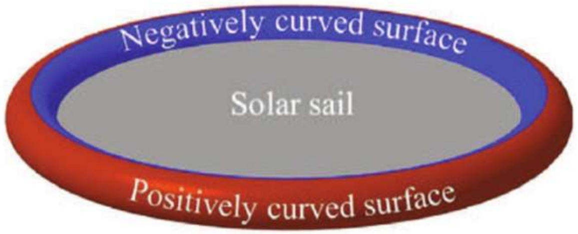
$$\Delta b_w = \frac{wb^2}{\pi E_w a^2}.$$

The stresses $\sigma_\parallel$ and $\sigma_\perp$ will strain the wire.

$$w = \frac{\mu_0}{4\pi}\frac{I^2}{b}\left(\ln\frac{8b}{a} - \frac{3}{4} + 2\pi v_w\right)$$

The magnitude of the radial uniformly distributed force w per unit length of the wire circular loop

Force acting on the unit length of the toroidal shell



The magnitude of the force F_t acting on the unit length of the toroidal shell is

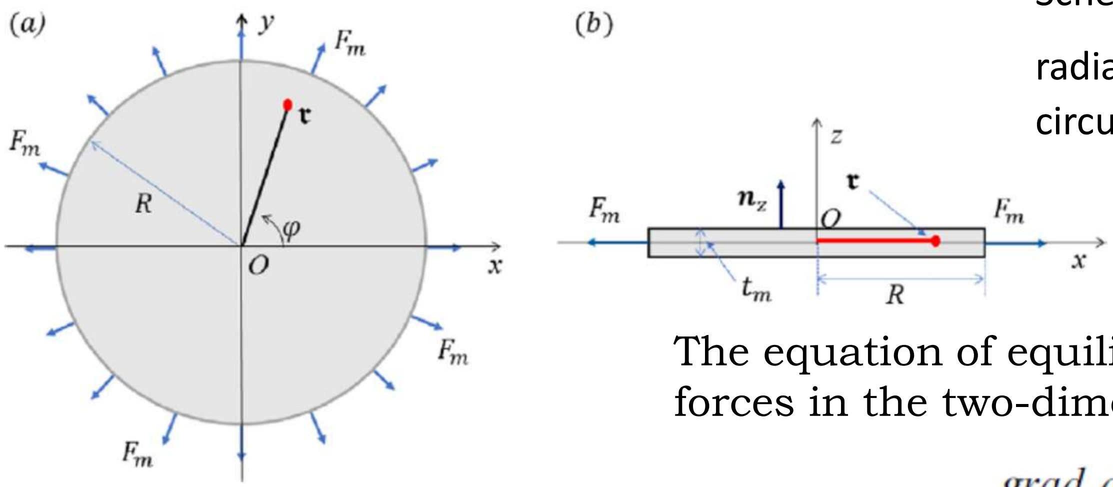
$$F_t = \frac{\pi P r^2}{R}$$

The pressure *P* is defined by van der Waals equation $\left(P + a\frac{n^2}{V^2}\right)(V - nb) = nR_gT.$

Combining these two equations one can find the in-plane tensile force per unit length in the membrane produced by the inflatable torus shell filled with a gas

$$F_t = n\frac{\pi r^2 R_g T}{2\pi^2 r^2 R^2 - nbR} - n^2\frac{a}{4\pi^3 r^2 R^3}.$$

Stress and strain of circular membrane



Schematics for the action of the uniformly distributed radial force F_m per unit of circumferential length of the circular membrane. (a) top view. (b) side view.

The equation of equilibrium in the absence of the body forces in the two-dimensional vector form is (Landau & Lifshitz, 1986):

$$grad\ div\ \mathbf{u} - \frac{1}{2}(1 - \nu_m) curl\ curl\ \mathbf{u} = 0,$$

where $\boldsymbol{u}$ is the displacement vector, ν_m is the membrane Poisson ratio, and all the vector operators are two-dimensional

Finally, the radial deformation, strain and strass are

$$\Delta R_m = \frac{F_m}{t_m} \frac{(1 - \nu_m)}{E_m} R,$$

$$\frac{\Delta R_m}{R} = \frac{F_m}{t_m} \frac{(1 - \nu_m)}{E_m},$$

$$\sigma_{\tau m} = \frac{F_m}{t_m}.$$

$$F_m = F_t \left(1 + 2\pi \frac{r}{R} \frac{t_t}{t_m} \frac{E_t}{E_m} (1 - \nu_m)\right)^{-1}$$

Solar sail with inflatable toroidal shell

$$w_m = w\left(1 + \frac{\pi a^2}{tb} \frac{E_w}{E_m} (1 - \nu_m)\right)^{-1}$$

Solar sail with superconducting current loop

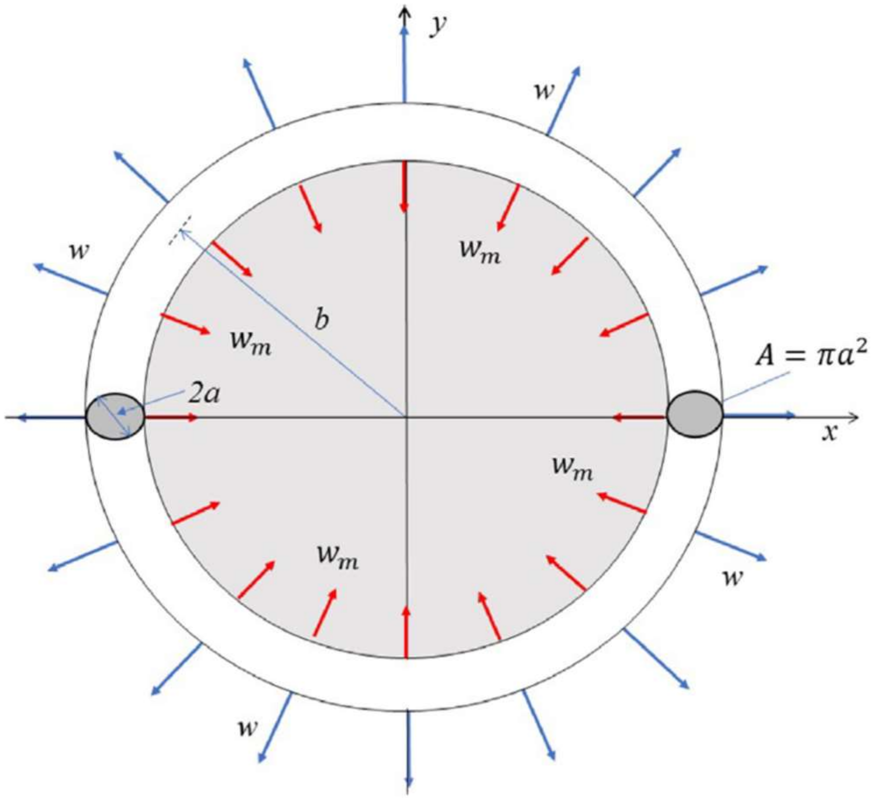


Fig. 6. Schematics for the action of uniform radial forces $w$and w_m per unit of circumferential length on the circular current-carrying wire. The force opposite in direction and equal in magnitude to w_m acts on the circular membrane and is not shown. For visibility, the cross-section of the wire is given in 3D format. The figure is not drawn to scale: $b \gg a$.

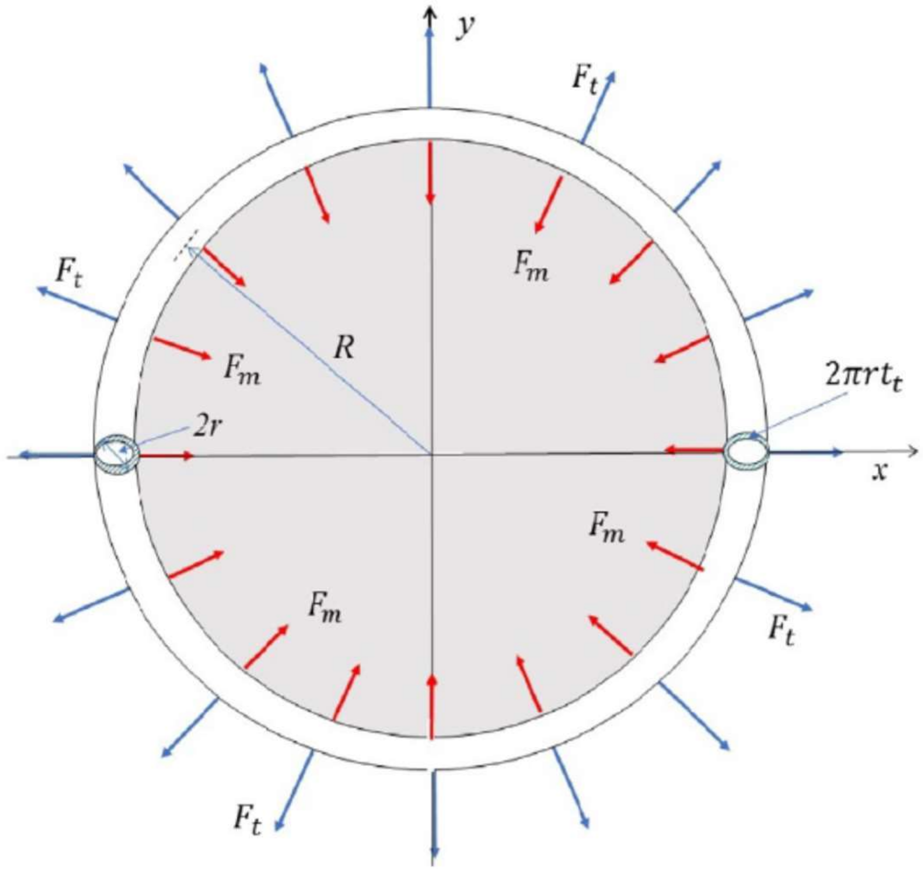
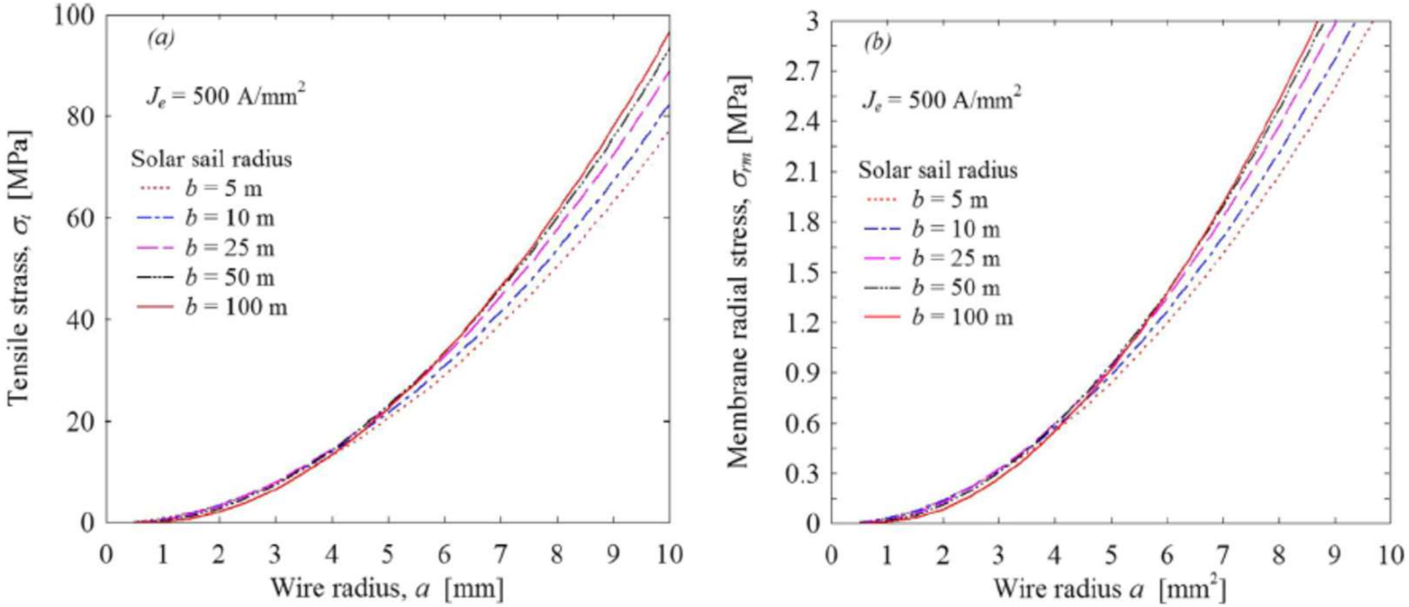


Fig. 5. (Color online) Schematics for the action of uniform radial forces F_t and F_m per unit of circumferential length on the toroidal shell. The force opposite in direction and equal in magnitude to F_m acts on the circular membrane and is not shown. For visibility, the cross-section of the toroidal shell is given in 3D format. The figure is not to scale: $R \gg r$ and $r \gg t_t$.

Numerical Results

Obtained analytical expressions can be applied to a wide range of solar sail sizes. We present the numerical calculations for the sail of radii 5 m to150 m made of CP1 membrane of thickness of 3.5 μm attached to Bi–2212 superconducting wire with the cross-section radii of 0.5 mm to 10 mm. Calculations are performed for the engineering current densities of 100 A/mm^2 to 1000 A/mm^2.

The tensile stress and membrane radial stress vs wire radius



Yield strength $10^{-2} E_m$ MPa

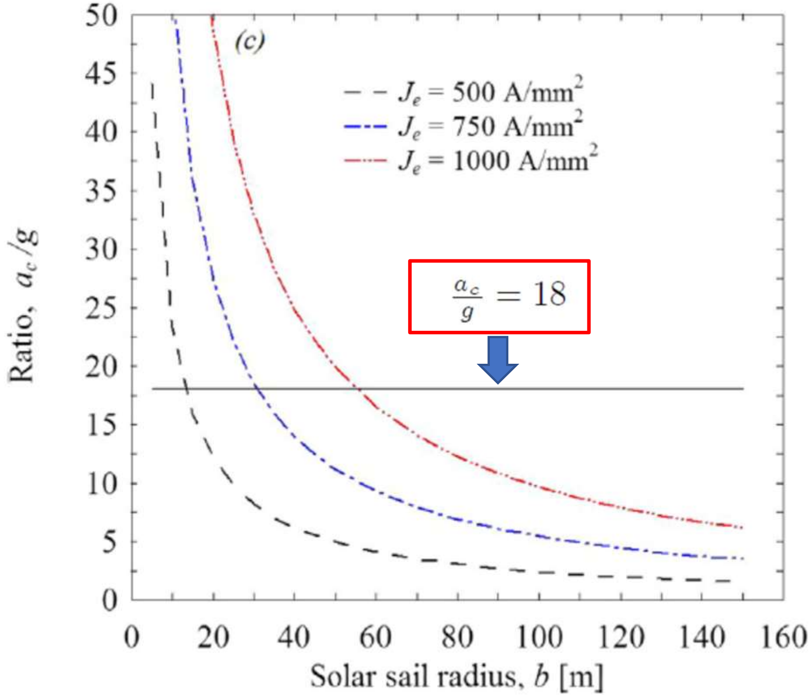
~ 22 MPa

Dependence of the tensile stress (a), membrane radial stress (b), on the wire radius. Calculations are performed for the current density J_e = 500 A/mm^2 and different sizes of solar sail.

a_c/g and arial density vs wire radius

$$\frac{a_c}{g} = \frac{(w - w_m)}{\rho_w \pi a^2 g}$$

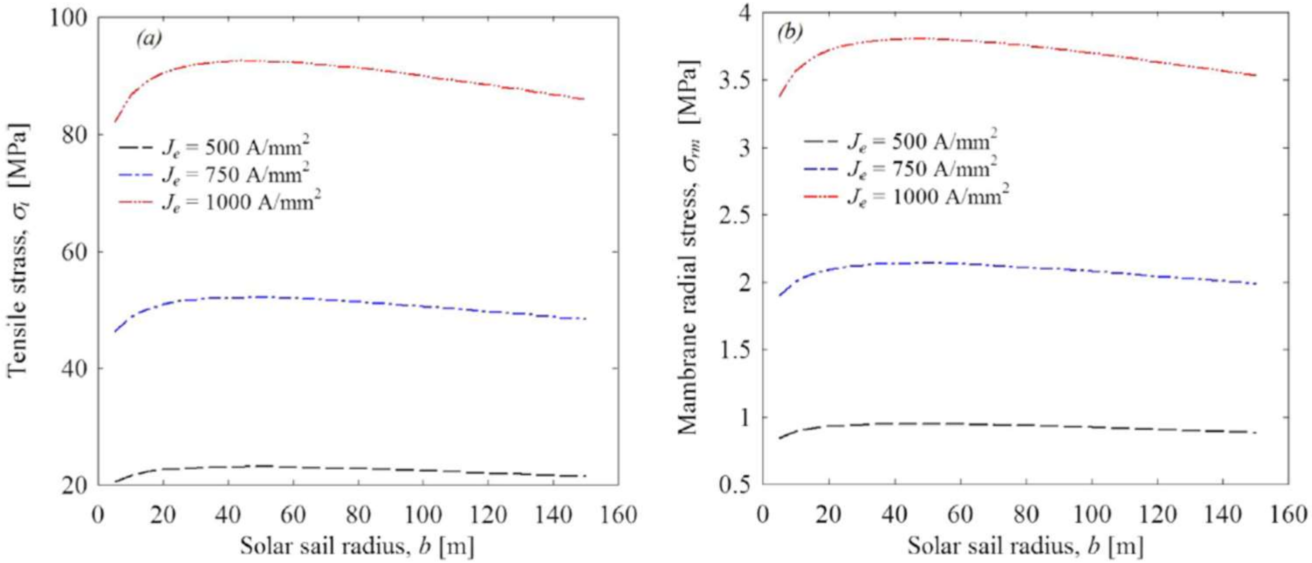
The greater the a_c/g ratio, the greater the chance of successful deployment of initially-folded superconductive wire and sail membrane.



Dependence of the ratio a_c/g (c) on the wire radius.
Calculations are performed for different current densities.

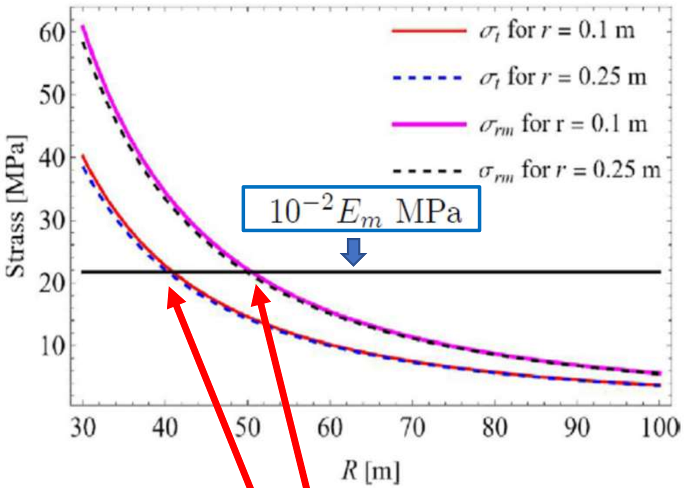
The horizontal line is the ratio **a_c/g = 18 which corresponds to successful deployment of initially-folded sail**

Tensile stress and membrane radial stress vs solar sail radius

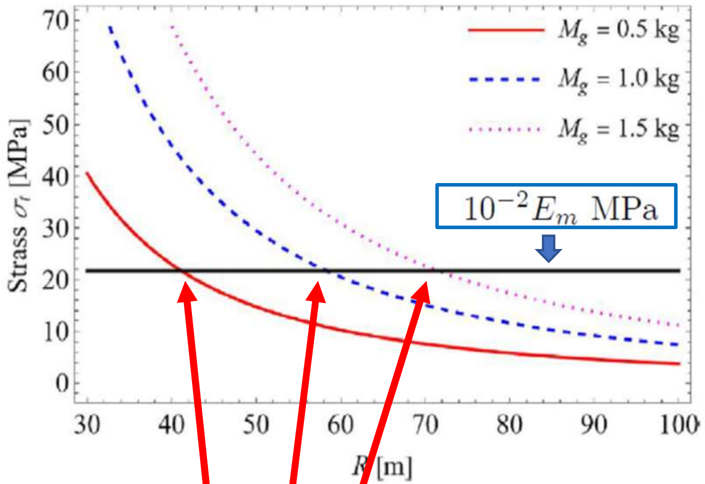


Dependence of the tensile stress (*a*), membrane radial stress (*b*), on the solar sail radius. Calculations are performed for the wire radius *a* = 5 mm and different current density J_e

Dependence of Stresses on Sail Radius



The dependence of the tensile stress of the toroidal shell σ_t and the membrane radial stress σ_{rm} on the membrane radius for different radii of the toroidal shell. The gas mass 0.750 kg



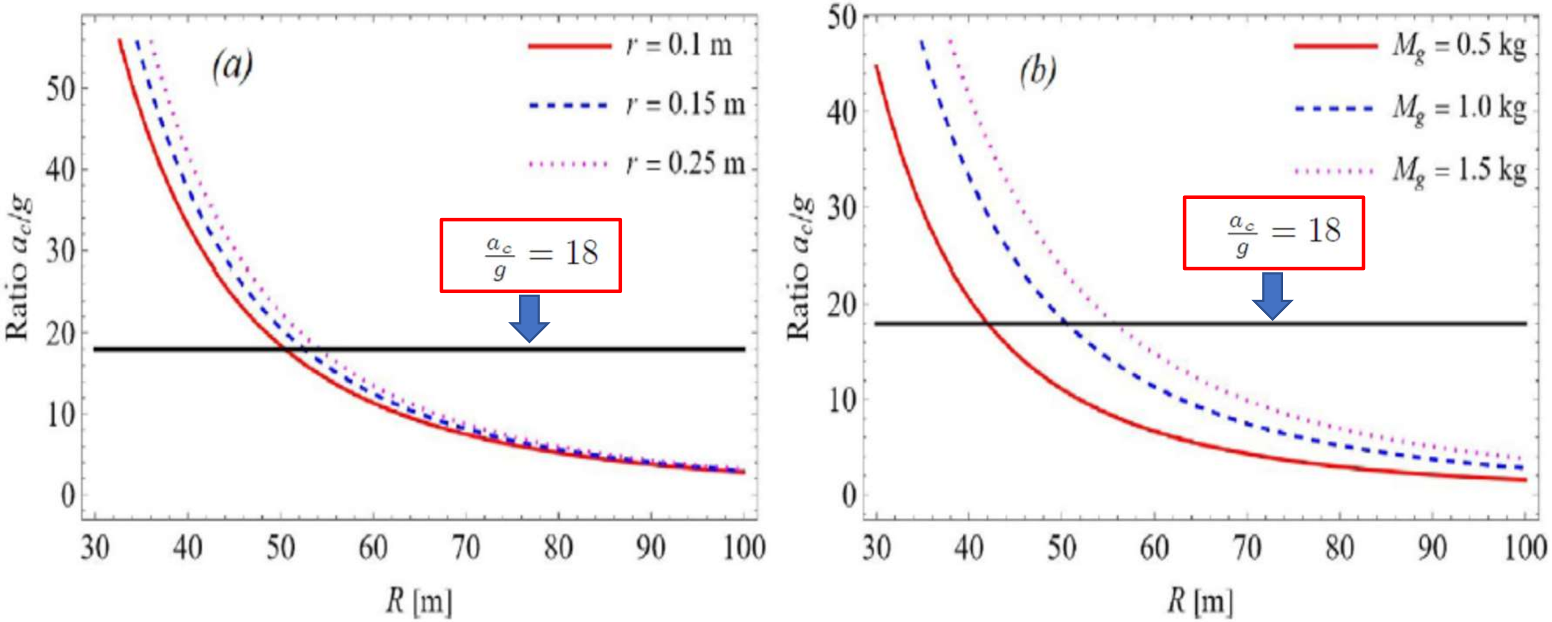
The dependence of the membrane radial stress on the membrane radius for different masses of the gas in the toroidal shell. Calculations performed for r = 0.1 m.

For the stable sails stresses of the toroidal shell and membrane must be less than the Yield strength $10^{-2}E_m$ MPa **~ 22 MPa**

The stable sails need to have radii greater than the radii of crossover points

$$\frac{a_c}{g} = \frac{(F_t - F_m)}{g(2\pi r t_t \rho_t + M_g / 2\pi R)}$$

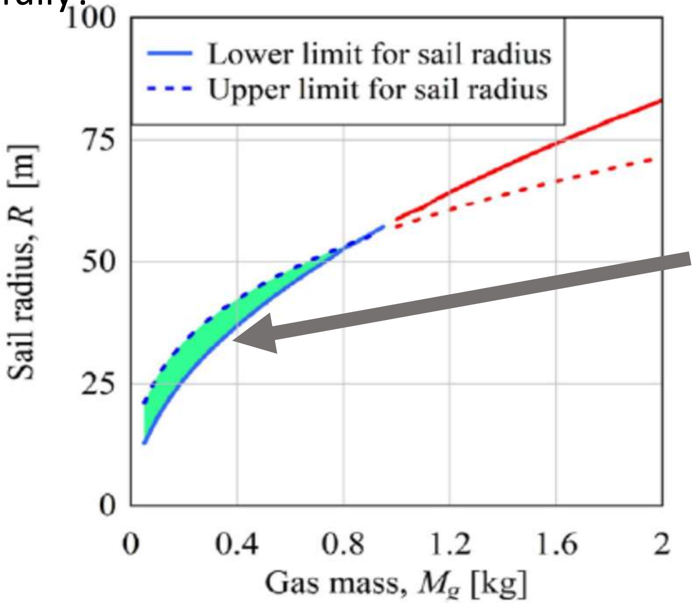
The greater ratio the greater the chance of successful deployment of initially folded toroidal shell and sail membrane to the open state of circular shape.



The dependence of the ratio on the radius of the solar sail membrane for (*a*) different radii of the toroidal shell for *Mg* = 1.0 kg and (*b*) different masses of the gas in the toroidal shell for *r* = 0.1 m.

Limitations for the radius of the sail

The logical question which arises now is the following: which value of the sail radius should be considered to deploy the sail successfully?



Limitations for the radius of the sail. The colored area shows the allowed radii of the sail's membrane for the radius of the toroidal shell ***r* = 0.1 m**.

The sails with the radius and the gas mass within the green shaded area are deployable and stable.

In summary, there are two limits for the radii of the sail:

i. the lower limit is due to the sail stability requirement demanding the stresses to be within limits of elasticity;

ii. ii. the upper limit is due to requirement of the successful deployment. The radius of the sail has to simultaneously satisfy both requirements being greater than the lower limit and less than the upper limit.

Concluding remarks

- Today all launched solar sails have a square shape and such design is related to the deployment mechanism. The world's first interplanetary solar sail, – the IKAROS, successfully deployed its 196 m^2 sail in 2010.
- NASA's first solar sail deployed in low earth orbit was NanoSail-D which had 9.3 m^2 of light-reflecting catching surface
- The LightSail-2 on July 23, 2019, deployed its 32 m^2 solar sail.

For comparison, our calculations show that

- a sail of ~ **1,963 m^2** area (25 m radius) with the attached wire of 5 mm cross-section radius is expected to be deployed by the current in the wire of the engineering density 750 A/mm^2
- a sail with radius of 55 m (~ **9500 m^2** area) attached to the toroidal shell of r = 0.1 m inflated by 0.75 kg of the hydrogen can be successfully deployed and stretched.

Conclusions

- We developed the theory of deployment of the circular solar sail attached to a superconducting current-carrying wire or an inflatable toroidal shell within the framework of classical electrodynamics and the theory of elasticity.

- We obtained the analytical expressions that can be applied to a wide range of materials for the superconducting wire, sail membrane and inflatable toroidal shell.

- The presented numerical example demonstrates the power of the developed theory to provide sound estimates of the stresses of the solar sail with attached superconducting circular current-carrying wire or inflatable toroidal shell.

Thank you for your attention!