New York City College of Technology
Department of Mathematics.
MAT 2672- Sample Final Examination- Fall 2007
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1. In a study of exertion, a measure of heartbeats per minute was made. In a sample of 20 men doing manual labor, the sample mean and sample variance of the heartbeats were 180 and 20 respectively. For 20 men doing the same work using machinery the sample mean and sample-variance were 120 and 25 respectively. Devise an approach that would let you quantify the evidence for significant difference in average heart rate for the two groups.

2. a. If F is a random variable with the $f(5,5)$ distribution, find $P[F > 0.2]$.
   b. *A sample taken from a normally distributed population must behave normally.* Comment on the validity of this statement taking into account the sample size and the manner in which the sample is chosen.

3. X and Y are discrete random variables with joint density function $f(x,y) = \alpha y\left(2^{\sin\left(\frac{\pi x}{2}\right)}\right)$ for $x = 0,1$ and $y = 0,1,2$. Find α and $E\left(e^{xy}\right)$.

4. X and Y are continuous random variables with joint density function $f(x,y) = ce^{x^2+y^2}$ on the unit circle centered around the origin and zero elsewhere. Find c exactly.

5. Let $\begin{pmatrix} X_1 \\ X_2 \\ X_3 \end{pmatrix} \square N\left(\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 & 2 & 1 \\ 2 & 2 & 1 \\ 1 & 1 & 3 \end{pmatrix}\right)$. Find $P\left(X_1 - 2X_2 + X_3 > 15.4\right)$ and $P\left(\left(X_1 - 2X_2 + X_3\right)^2 > 15.4\right)$.

6. Let $\begin{pmatrix} X_1 \\ X_2 \end{pmatrix} \square N\left(\begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix}\right)$. Find $P\left(X_2 < X_1\right)$ and the distribution of $Z = \left(\frac{X_1 + X_2}{X_1}\right)^2$.

7. Consider the set of points $(1,-3),(2,3),(3,5),(6,6)$ and $(7,1)$. Find the line which best fits this data by solving $\hat{\beta} = \left(X^T X\right)^{-1} X^T y$, where the symbols have their usual meanings.

Solutions.

1. For the manual labor, the 90% confidence interval is: $\bar{X} \pm t_{0.05}(19).s/\sqrt{n} = [178.3, 181.7]$. For the work via machinery, the 90% confidence interval is: $[118.1,\ 121.9]$. The two intervals are far apart which gives strong evidence of a difference in heart rate.

2. a. 0.95

b. The statement can be true or false depending on the sample size and the manner in which the sample is chosen. Explain each case.

3. $\alpha = \frac{1}{9},\ E\left(e^{xy}\right) = \frac{3+2e+4e^2}{9}$.

4. $c = \frac{1}{\pi(e-1)}$

5. $P\left(X_1 - 2X_2 + X_3 > 15.4\right) = 0,\ P\left(\left(X_1 - 2X_2 + X_3\right)^2 > 15.4\right) = 0.05$

6. $P\left(X_2 < X_1\right) = 0.31,\ \left(\frac{X_1 + X_2}{X_1}\right)^2 \square\ \frac{4}{3}F_{1,1}(0.25)$.

7. $Y = (29/134) + (77/134)X$.