1. Find $P(X < 3 \mid X > 1)$ if:
   a. $X$ is a Poisson distribution with $\lambda = 1$.
   b. $X$ is a normal distribution with $\mu = 1$ and $\sigma^2 = 4$.

2. Let $Y_j$ be the price of a stock on the $j^{th}$ day and $Y_j - Y_{j-1} = X_j$ be the difference in stock prices between consecutive days. Assume for $j \geq 1$, the $X_j$'s are i.i.d. random variables with mean $\mu = 0$ and variance $\sigma^2 = 256$.
   a. Show that $\sum_{j=1}^{365} X_j = \sum_{j=1}^{365} (Y_j - Y_{j-1}) = Y_{365} - Y_1$.
   b. Find the probability that the price of the stock will be more than $100$ in $365$ days if it is $75$ now.

3. If two distinct numbers are chosen from the set $\{1, 2, 3, 4, 5, 6\}$ randomly, and without replacement. Find the probability density function and expectation of the absolute value of the difference of the two numbers.

4. The moment generating function of a distribution is given by $m(t) = \left( e^{t/5} + 4/5 \right)^{25}$. Identify the distribution and find the value of its mean, variance and $P(X \geq 1)$.

5. Let $X$ and $Y$ be independent random variables with $E(X) = \mu_1$, $V(X) = \sigma_1^2$, $E(Y) = \mu_2$ and $V(Y) = \sigma_2^2$, where $\mu_1, \mu_2, \sigma_1^2$ and $\sigma_2^2$ are positive constants that are less than one. Evaluate the following and give your answers in terms of $\mu_1, \mu_2, \sigma_1^2$ and $\sigma_2^2$.
   i. $E(X + 2Y)$  
   ii. $V(X + 2Y)$  
   iii. $E(XY)$  
   iv. $V(XY)$

6. If a pair of fair dice is rolled, what is the probability that a sum of $3$ is rolled before a sum of $7$ is rolled?

7. $f(x)$ is a probability density function with $f(x) = \begin{cases} \frac{c}{x^3} & \text{for } (1, \infty) \\ 0 & \text{elsewhere} \end{cases}$.
   Find the value of $c$ and evaluate: i. $P\left(\frac{1}{2} < x < 2\right)$;  
   ii. $P\left(-\frac{1}{2} \leq x \leq \frac{1}{2}\right)$
8. $X$ is a Poisson distribution with mean $\lambda$. If $P(X = 2) = \frac{\lambda^2}{4}$. Find $\lambda$ and the $P(X \geq 2)$.

9. In an experiment, probes land at random in the interval $(0,1)$ on the $x$–axis.
   a. What is the probability that a probe lands in the interval $(3/4,1)$?
   b. If three probes operate independently and lands, what is the probability that exactly two probes land in the interval $(3/4,1)$?

10. If $A$ and $B$ are independent events, show that
   a. $A$ and $\bar{B}$ are also independent events.
   b. $\bar{A}$ and $B$ are also independent events.

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### Solutions to the Sample MA 572 Final Examination Spring 2005

1. a. $P(X < 3 \mid X > 1) = \frac{1}{2(e-2)}$  
   b. $P(X < 3 \mid X > 1) = 0.683$

2. a. $\sum_{j=1}^{365} (Y_j - Y_{j-1}) = (Y_1 - Y_0) + (Y_2 - Y_1) + \ldots + (Y_{364} - Y_{365}) + (Y_{365} - Y_0) = Y_{365} - Y_0$,
   this sum telescopes.
   b. $P(\sum_{j=1}^{365} X_j > 25) \approx P\left(z > \frac{25}{16\sqrt{365}}\right) = P(z > 0.0817) = 0.47$ or 47%.

3. Let $X$ be the absolute value of the difference. $P(X = i) = \frac{6-i}{15}$, $i = 1, 2, 3, 4, 5$
   
   $E(X) = \sum_{i=1}^{5} i \left(\frac{6-i}{15}\right) = \frac{7}{3}$

4. This is a Binomial distribution with $p = \frac{1}{5}, q = \frac{4}{5}$ and $n = 25$. $\mu = np = 5$,
   
   $\sigma^2 = npq = 4$. $P(X \geq 1) = 1 - \left(\frac{4}{5}\right)^{25} \approx 0.996$

5. a. $\mu_1 + 2\mu_2$  
   b. $\sigma_1^2 + 4\sigma_2^2$  
   c. $\mu_1 \mu_2$  
   d. $\sigma_1^2 \sigma_2^2 + \sigma_1^2 \mu_2^2 + \mu_1^2 \sigma_2^2$

6. $\frac{1}{4}$
7. \( c = 2 \)
   i. \( 3/4 \) ii. 0

8. \( \lambda = \ln 2 \) and \( P(X \geq 2) = \ln \sqrt{e/2} \)

9. a. 1/4 b. 9/64

10. a. Use the disjoint union \( A = (A \cap \overline{B}) \cup (A \cap B) \)
    b. Use the disjoint union \( \overline{B} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{B}) \) and the result from part a.