New York City College Of Technology
Department of Mathematics
Sample MAT 1572 Final Examination, Fall 2007
By Prof. S. Singh
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1. Find $P(X < 3 \mid X > 1)$ if:
   a. X is a Poisson distribution with $\lambda = 1$.
   b. X is a normal distribution with $\mu = 1$ and $\sigma^2 = 4$.

2. Let Y_j be the price of a stock on the j^{th} day and $Y_j - Y_{j-1} = X_j$ be the difference in stock prices between consecutive days. Assume for $j \geq 1$, the X_j's are i.i.d. random variables with mean $\mu = 0$ and variance $\sigma^2 = 256$.
   a. Show that $\sum_{j=1}^{365} X_j = \sum_{j=1}^{365} (Y_j - Y_{j-1}) = Y_{365} - Y_0$.
   b. Find the probability that the price of the stock will be more than \$100 in 365 days if it is \$75 now.

3. If two distinct numbers are chosen from the set $\{1, 2, 3, 4, 5, 6\}$ randomly, and without replacement. Find the probability density function and expectation of the absolute value of the difference of the two numbers.

4. The moment generating function of a distribution is given by $m(t) = \left(e^t/5 + 4/5\right)^{25}$. Identify the distribution and find the value of its mean, variance and $P(X \geq 1)$.

5. Let X and Y be independent random variables with $E(X) = \mu_1$, $V(X) = \sigma_1^2$, $E(Y) = \mu_2$ and $V(Y) = \sigma_2^2$, where μ_1, μ_2, σ_1^2 and σ_2^2 are positive constants that are less than one. Evaluate the following and give your answers in terms of μ_1, μ_2, σ_1^2 and σ_2^2.
   i. $E(X + 2Y)$ ii. $V(X + 2Y)$ iii. $E(XY)$ iv. $V(XY)$

6. If a pair of fair dice is rolled, what is the probability that a sum of 3 is rolled before a sum of 7 is rolled?

7. $f(x)$ is a probability density function with $f(x) = \begin{cases} c/x^3 & \text{for } (1, \infty) \\ 0 & \text{elsewhere} \end{cases}$.
   Find the value of c and evaluate: i. $P(1/2 < x < 2)$. ii. $P(-1/2 \leq x \leq 1/2)$

8. X is a Poisson distribution with mean λ. If $P(X=2)=\dfrac{\lambda^2}{4}$. Find λ and the $P(X \geq 2)$.

9. In an experiment, probes land at random in the interval $(0,1)$ on the $x-$axis.
   a. What is the probability that a probe lands in the interval $(3/4,1)$?
   b. If three probes operate independently and lands, what is the probability that exactly two probes land in the interval $(3/4,1)$?

10. If A and B are independent events, show that
   a. A and $\bar{B}$ are also independent events.
   b. $\bar{A}$ and $\bar{B}$ are also independent events.

Solutions to the Sample MA 572 Final Examination Spring 2005

1. a. $P(X<3 \mid X>1)=\dfrac{1}{2(e-2)}$

   b. $P(X<3 \mid X>1)=0.683$

2. a. $\sum_{j=1}^{365}(Y_j - Y_{j-1}) = (Y_1 - Y_0)+(Y_2 - Y_1)+\ldots+(Y_{364}-Y_{365})+(Y_{365}-Y_0) = Y_{365}-Y_0$, this sum telescopes.

   b. $P\left(\sum_{j=1}^{365} X_j > 25\right) \approx P\left(z > \dfrac{25}{16\sqrt{365}}\right) = P(z>0.0817)=0.47$ or 47%.

3. Let X be the absolute value of the difference. $P(X=i)=\dfrac{6-i}{15}$, $i=1,2,3,4,5$

   $E(X)=\sum_{i=1}^{5} i\left(\dfrac{6-i}{15}\right)=\dfrac{7}{3}$

4. This is a Binomial distribution with $p=\dfrac{1}{5}, q=\dfrac{4}{5}$ and $n=25$. $\mu = np = 5$,

   $\sigma^2 = npq = 4$. $\quad P(X \geq 1) = 1-\left(\dfrac{4}{5}\right)^{25} \approx 0.996$

5. a. $\mu_1 + 2\mu_2$ b. $\sigma_1^2 + 4\sigma_2^2$ c. $\mu_1\mu_2$ d. $\sigma_1^2\sigma_2^2 + \sigma_1^2\mu_2^2 + \mu_1^2\sigma_2^2$

6. $\dfrac{1}{4}$

7. $c = 2$
i. $3/4$ ii. 0

8. $\lambda = \ln 2$ and $P(X \geq 2) = \ln\sqrt{e/2}$

9. a. $1/4$ b. $9/64$

10. a. Use the disjoint union $A = (A \cap \overline{B}) \cup (A \cap B)$

b. Use the disjoint union $\overline{B} = (A \cap \overline{B}) \cup (\overline{A} \cap \overline{B})$ and the result from part a.