New York City College of Technology
Department of Mathematics

MAT 1375 Final Exam Review Problems[1]

1. Solve the inequality. Express the solution both on the number line and in interval notation. Use exact forms (such as fractions) instead of decimal approximations.

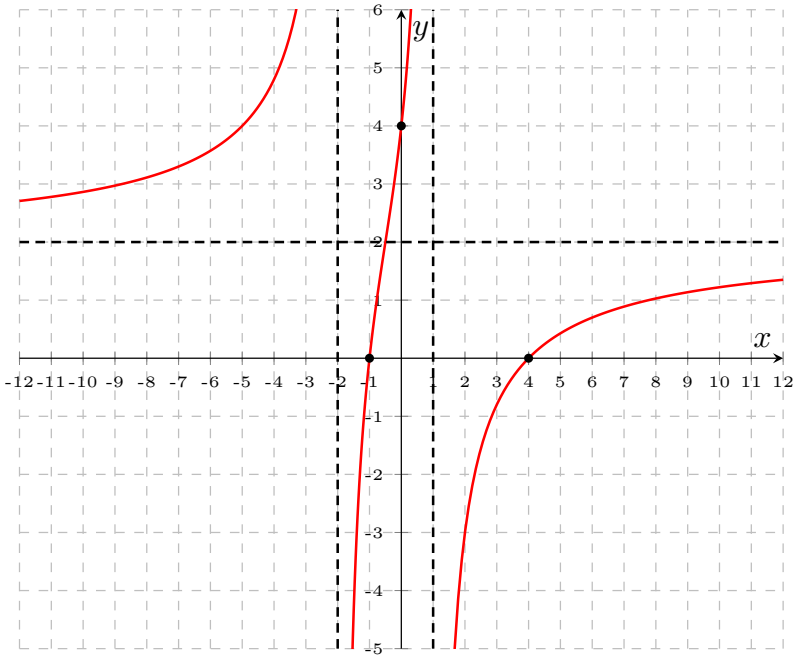
(a) $x^2 - 2x - 3 \geq 0$

(b) $6x - 2x^2 > 0$

(c) $\dfrac{3x-4}{9x+17} \leq 0$

(d) $\dfrac{6x+5}{7x-13} \geq 0$

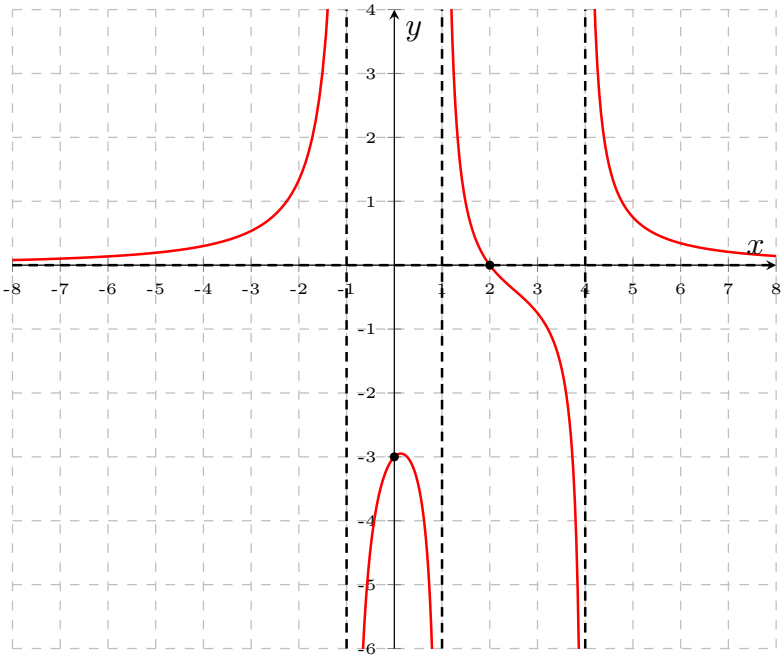
2. **(a)** A complete graph of the rational function $y = f(x)$ is displayed below. The numerator and denominator of f are polynomials of degree 2, and all asymptotes and intercepts of f are at integer values.



Find all intercepts, asymptotes, and the domain of f. Find a formula for the function f.

(b) A complete graph of the rational function $y = f(x)$ is displayed below. The numerator of f is a polynomial of degree 1, the denominator of f is a polynomial of degree 3, and all asymptotes and intercepts of f are at integer values.

[1] Revised by Professor Tradler in Spring 2023.



Find all intercepts, asymptotes, and the domain of f. Find a formula for the function f.

3. Find the difference quotient $\dfrac{f(x+h)-f(x)}{h}$ (assume $h \neq 0$).

(a) $f(x) = x^2 + 5x + 2$

(b) $f(x) = 2x^2 - 3x$

(c) $f(x) = -2x^2 + 4x + 1$

4. For each polynomial, find a real number C so that the polynomial has the indicated root. For this C, find all remaining roots of the polynomial algebraically and write the roots in simplest radical form. Sketch a complete graph of the polynomial, indicating the roots.

(a) $f(x) = x^3 + 5x^2 + x + C$ has a root at $x = -2$

(b) $f(x) = x^3 + 3x^2 - 16x + C$ has a root at $x = 3$

(c) $f(x) = 2x^3 - 4x^2 - 3x + C$ has a root at $x = -1$

5. Find the magnitude and the direction angle.

(a) $v = \langle 2, -2\sqrt{3} \rangle$

(b) $v = \langle -3, -3 \rangle$

(c) $v = \langle -5\sqrt{3}, 5 \rangle$

6. Find the quotient or product and write the result in standard complex form.

(a) $\dfrac{6(\cos 225^\circ + i \sin 225^\circ)}{3(\cos 135^\circ + i \sin 135^\circ)}$

(b) $2(\cos 120^\circ + i \sin 120^\circ) \cdot 4(\cos 90^\circ + i \sin 90^\circ)$

7. Let $u = \ln x$ and $v = \ln y$, where $x, y > 0$. Write the following expression in terms of u and v.

(a) $\ln\left(x^8 \cdot \sqrt[3]{y^4}\right)$

(b) $\ln\left(\dfrac{\sqrt{x^5}}{y^3}\right)$

(c) $\ln\left(\sqrt{x^3 \cdot \sqrt[4]{y}}\right)$

8. Find the domain, asymptotes, and x-intercepts of the function, and then sketch its graph.

(a) $f(x) = \log(x-4)$

(b) $f(x) = \log(5-x)$

(c) $f(x) = -\ln(x+2)$

9. State the amplitude, period and phase shift, and then sketch one complete cycle of the graph. Label all maxima, minima and x-intercepts.

(a) $y = 3\cos(2x+\pi)$

(b) $y = 2\sin(4x-\pi)$

(c) $y = -4\sin\left(x - \dfrac{\pi}{2}\right)$

10. Find all exact solutions in radians.

(a) $2\sin^2 x = \sin x$

(b) $2\cos^2 x - \cos x - 1 = 0$

(c) $\tan^2 x - \tan x = 0$

11. In 2022, the population of a colony is $10,000$, and is decreasing exponentially at 1.5% per year.

(a) What will the population be after 5 years?

(b) In what year will there be half of the population left?

12. In 2021, the population of a city is $80,000$ people, and is growing at a rate of 4% per year.

(a) What will the population be in 2033?

(b) In what year will the population be triple?

13. Find the inverse for the following function.

(a) $y = 3 - 4x$

(b) $y = \dfrac{4}{x-3}$

(c) $y = \dfrac{2}{8x+5}$

(d) $y = \dfrac{x-1}{x+2}$

14. **(a)** Find the sum of the first 70 terms of the arithmetic sequence: $22, 19, 16, 13, \ldots$

(b) Find the sum of the first 95 terms of the arithmetic sequence: $-17, -12, -7, -2, \ldots$

(c) Find the sum of the first 777 terms of the arithmetic sequence: $3, 9, 15, 21, \ldots$

15. Find the exact sum of the infinite geometric sequence.

(a) $\dfrac{1}{2}, -\dfrac{1}{4}, \dfrac{1}{8}, \ldots$

(b) $32, -16, 8, -4, \ldots$

(c) $3, 2, \dfrac{4}{3}, \dfrac{8}{9}, \ldots$

(d) $-54, -18, -6, -2, \ldots$

16. Solve for x.

(a) $3^{5x+2} = 9^{x-4}$

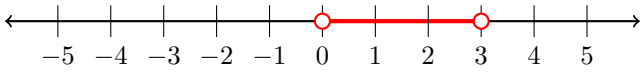
(b) $5^{x+3} = 7^{x}$

(c) $6^{-x} = 2^{4x+5}$

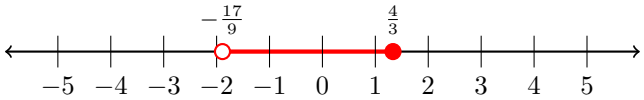
Answers:

1. **(a)** $(-\infty, -1] \cup [3, \infty)$

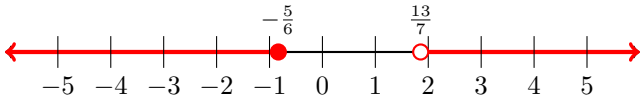
(b) $(0, 3)$



(c) $\left(-\frac{17}{9}, \frac{4}{3}\right]$



(d) $\left(-\infty, -\frac{5}{6}\right] \cup \left(\frac{13}{7}, \infty\right)$



2. **(a)** x-intercepts: $(-1, 0), (4, 0)$; y-intercept: $(0, 4)$
Vertical asymptotes: $x = 1$, $x = -2$; horizontal asymptote: $y = 2$
Domain: $D = \mathbb{R} \setminus \{-2, 1\}$
Function: $f(x) = \dfrac{2(x-4)(x+1)}{(x-1)(x+2)}$

(b) x-intercept: $(2, 0)$; y-intercept: $(0, -3)$
Vertical asymptotes: $x = -1$, $x = 1$, $x = 4$; horizontal asymptote: $y = 0$
Domain: $D = \mathbb{R} \setminus \{-1, 1, 4\}$
Function: $f(x) = \dfrac{6(x-2)}{(x-4)(x-1)(x+1)}$

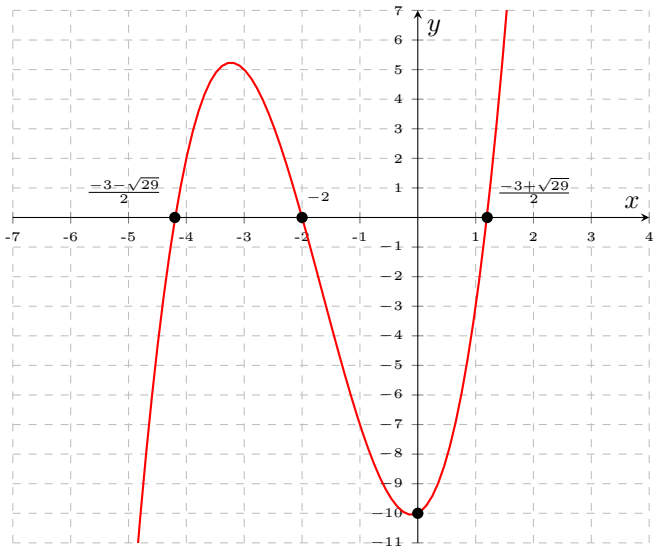
3. **(a)** $2x + h + 5$

(b) $4x + 2h - 3$

(c) $-4x - 2h + 4$

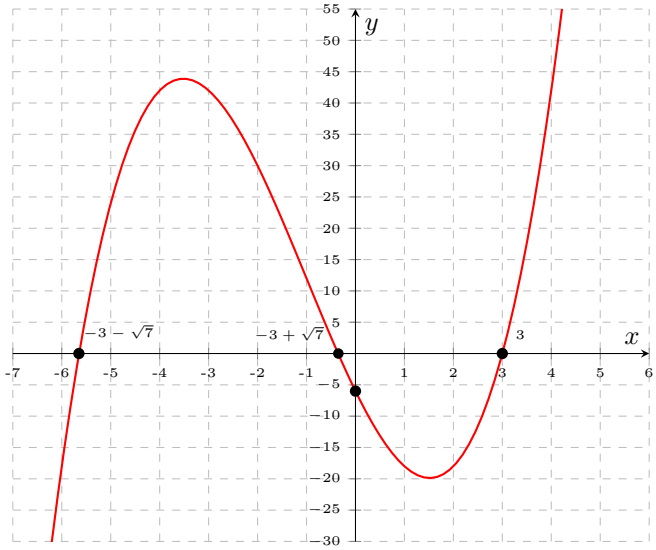
4. **(a)** $C = -10$
$f(x) = x^3 + 5x^2 + x - 10 = (x+2)(x^2 + 3x - 5)$
Roots: $-2, \dfrac{-3-\sqrt{29}}{2}, \dfrac{-3+\sqrt{29}}{2}$

Graph of $f(x) = x^3 + 5x^2 + x - 10$



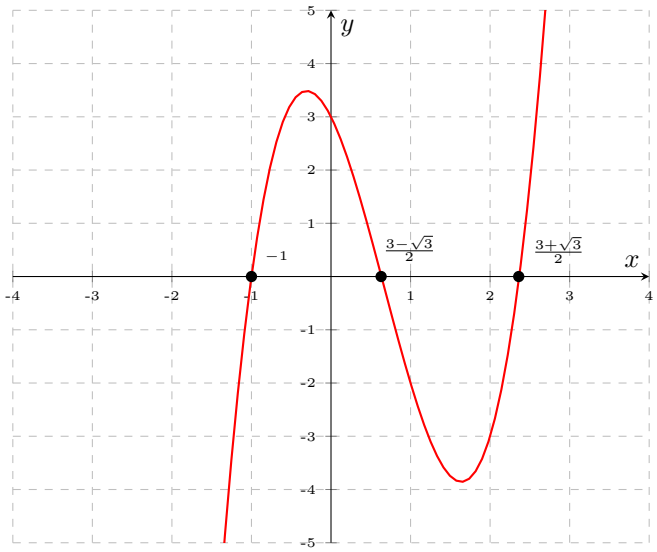
(b) $C = -6$
$f(x) = x^3 + 3x^2 - 16x - 6 = (x-3)(x^2 + 6x + 2)$
Roots: $-3 - \sqrt{7}$, $-3 + \sqrt{7}$, 3

Graph of $f(x) = x^3 + 3x^2 - 16x - 6$



(c) $C = 3$
$f(x) = 2x^3 - 4x^2 - 3x + 3 = (x+1)(2x^2 - 6x + 3)$
Roots: -1, $\dfrac{3-\sqrt{3}}{2}$, $\dfrac{3+\sqrt{3}}{2}$

Graph of $f(x) = 2x^3 - 4x^2 - 3x + 3$



5. **(a)** $||v|| = 4$; direction angle: $\theta = 300°$

(b) $||v|| = 3\sqrt{2}$; direction angle: $\theta = 225°$

(c) $||v|| = 10$; direction angle: $\theta = 150°$

6. **(a)** $2(\cos 90° + i \sin 90°) = 2i$

(b) $8(\cos 210° + i \sin 210°) = -4\sqrt{3} - 4i$

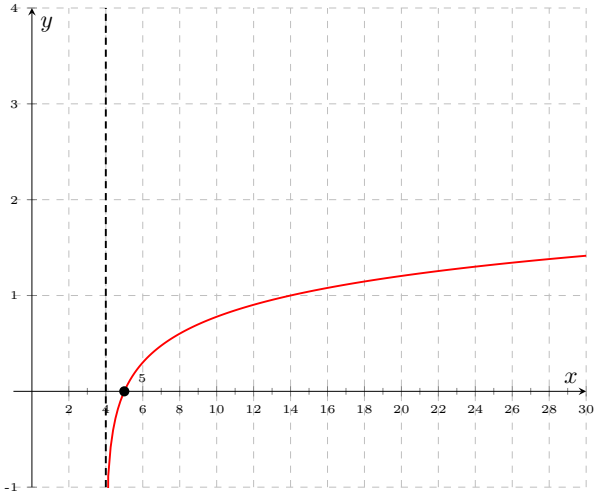
7. **(a)** $8u + \frac{4}{3}v$

(b) $\frac{5}{2}u - 3v$

(c) $\frac{3}{2}u + \frac{1}{8}v$

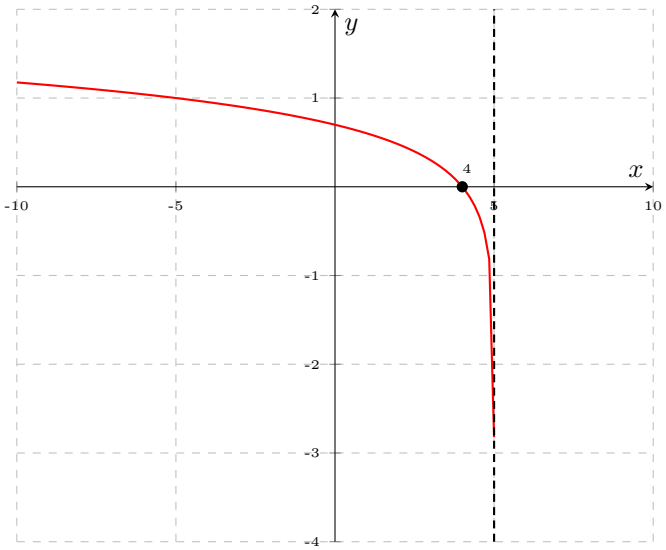
8. **(a)** Domain: $D = (4, \infty)$
Vertical asymptote: $x = 4$
x-intercept: $(5, 0)$

Graph of $f(x) = \log(x - 4)$



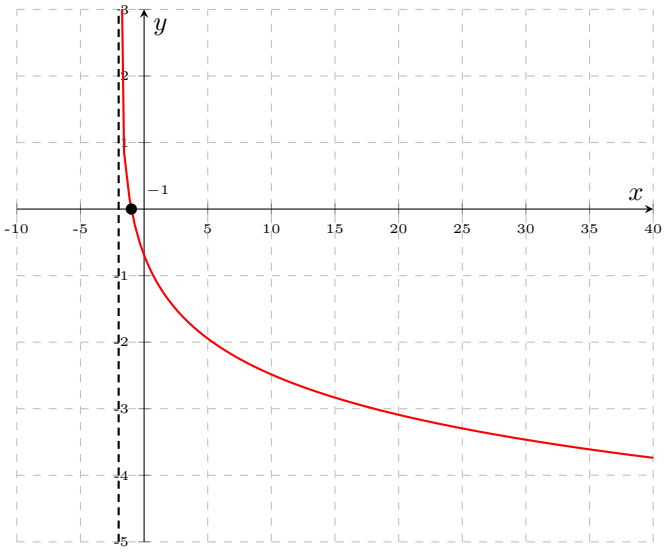
(b) Domain: $D = (-\infty, 5)$
Vertical asymptote: $x = 5$
x-intercept: $(4, 0)$

Graph of $f(x) = \log(5 - x)$



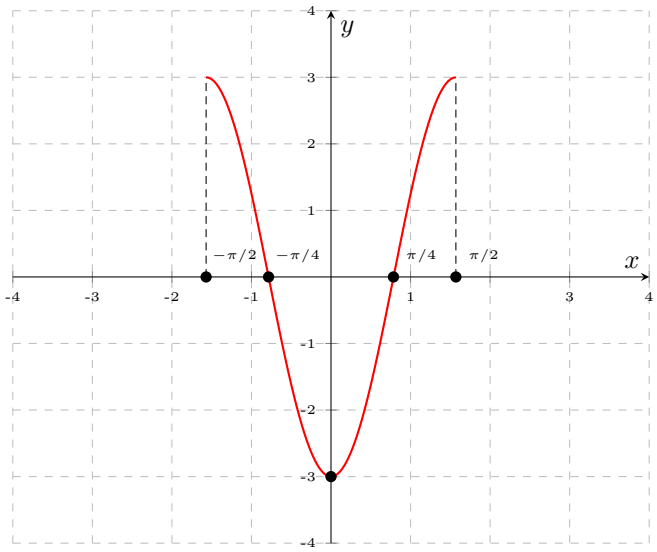
(c) Domain: $D = (-2, \infty)$
Vertical asymptote: $x = -2$
x-intercept: $(-1, 0)$

Graph of $f(x) = -\ln(x + 2)$



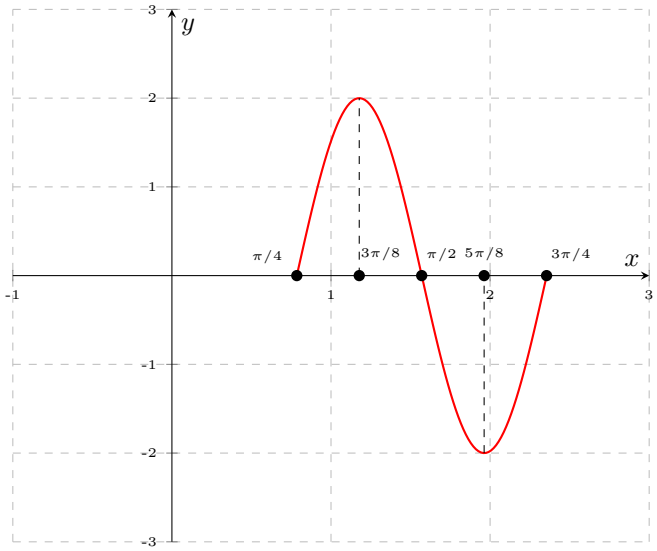
9. **(a)** Amplitude: 3; period: π; phase shift: $-\dfrac{\pi}{2}$; max: $\left(\pm\dfrac{\pi}{2}, 3\right)$; min: $(0, -3)$; x-intercepts: $\left(\pm\dfrac{\pi}{4}, 0\right)$

Graph of $f(x) = 3\cos(2x + \pi)$



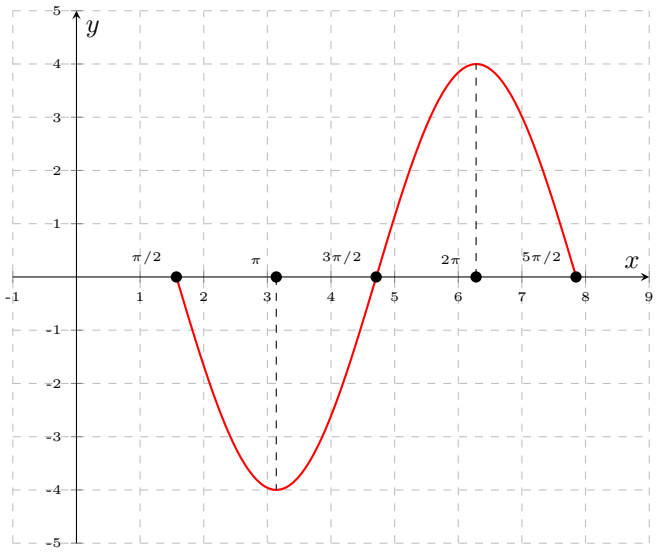
(b) Amplitude: 2; period: $\frac{\pi}{2}$; phase shift: $\frac{\pi}{4}$; max: $\left(\frac{3\pi}{8}, 2\right)$; min: $\left(\frac{5\pi}{8}, -2\right)$; x-intercepts: $\left(\frac{\pi}{4}, 0\right)$, $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{4}, 0\right)$

Graph of $f(x) = 2\sin(4x - \pi)$



(c) Amplitude: 4; period: 2π; phase shift: $\frac{\pi}{2}$; max: $(2\pi, 4)$; min: $(\pi, -4)$; x-intercepts: $\left(\frac{\pi}{2}, 0\right)$, $\left(\frac{3\pi}{2}, 0\right)$, $\left(\frac{5\pi}{2}, 0\right)$

Graph of $f(x) = -4\sin(x - \pi/2)$



10. **(a)** $x = n\pi$ or $x = (-1)^n \dfrac{\pi}{6} + n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$

(b) $x = \pm\dfrac{2\pi}{3} + 2n\pi$ or $x = 2n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$

(c) $x = n\pi$ or $x = \dfrac{\pi}{4} + n\pi$, where $n = 0, \pm 1, \pm 2, \ldots$

11. **(a)** The population will be approximately $9,272$ people.

(b) Sometime in the year 2067 half of the population will be left.

12. **(a)** The population will be approximately $128,083$ people.

(b) It will be triple in the year 2049.

13. **(a)** $f^{-1}(x) = \dfrac{3 - x}{4}$

(b) $f^{-1}(x) = \dfrac{4}{x} + 3$

(c) $f^{-1}(x) = \dfrac{1}{4x} - \dfrac{5}{8}$

(d) $f^{-1}(x) = \dfrac{2x + 1}{1 - x}$

14. **(a)** -5705

(b) 20710

(c) 1811187

15. **(a)** $\dfrac{1}{3}$

(b) $\dfrac{64}{3}$

(c) 9

(d) -81

16. **(a)** $x = -\dfrac{10}{3}$

(b) $x = \dfrac{3\log(5)}{\log(7) - \log(5)}$

(c) $x = \dfrac{-5\log(2)}{\log(6) + 4\log(2)}$