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New York City College of Technology  
Department of Mathematics

**MAT 1375 Final Exam Review Problems<sup>1</sup>**

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1. Solve the inequality. Express the solution both on the number line and in interval notation. Use exact forms (such as fractions) instead of decimal approximations.

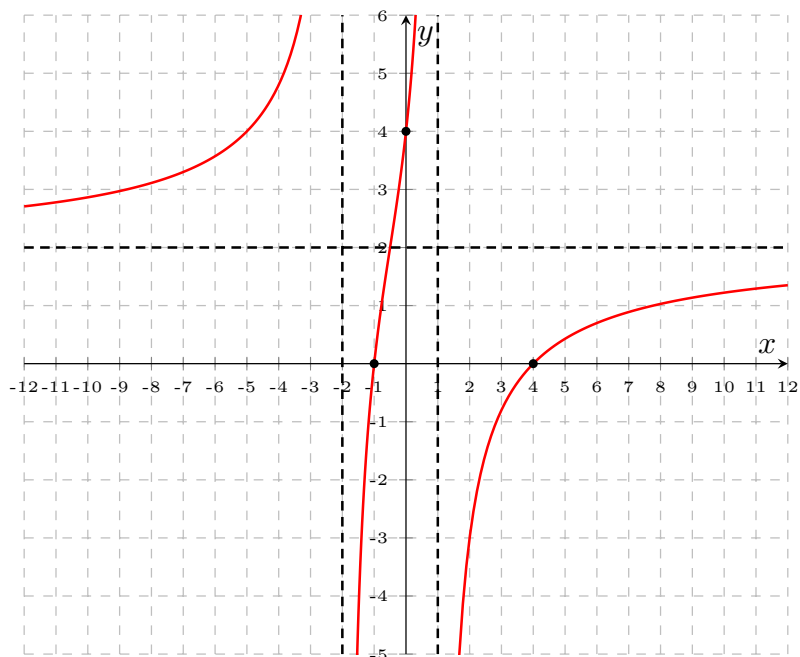
(a)  $x^2 - 2x - 3 \geq 0$

(b)  $6x - 2x^2 > 0$

(c)  $\frac{3x - 4}{9x + 17} \leq 0$

(d)  $\frac{6x + 5}{7x - 13} \geq 0$

2. (a) A complete graph of the rational function  $y = f(x)$  is displayed below. The numerator and denominator of  $f$  are polynomials of degree 2, and all asymptotes and intercepts of  $f$  are at integer values.

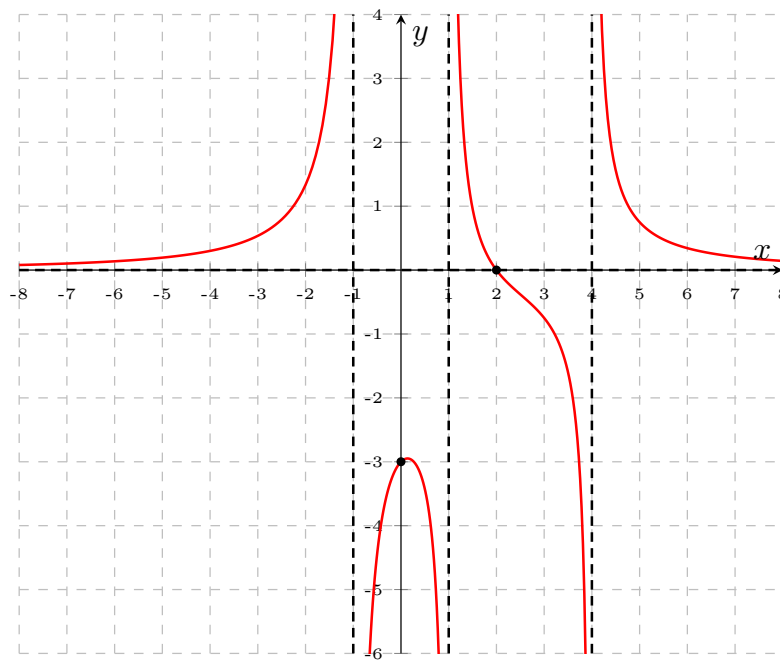


Find all intercepts, asymptotes, and the domain of  $f$ . Find a formula for the function  $f$ .

- (b) A complete graph of the rational function  $y = f(x)$  is displayed below. The numerator of  $f$  is a polynomial of degree 1, the denominator of  $f$  is a polynomial of degree 3, and all asymptotes and intercepts of  $f$  are at integer values.

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<sup>1</sup>Revised by Professor Tradler in Spring 2024.



Find all intercepts, asymptotes, and the domain of  $f$ . Find a formula for the function  $f$ .

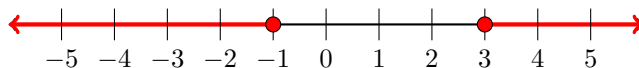
3. Find the difference quotient  $\frac{f(x+h) - f(x)}{h}$  (assume  $h \neq 0$ ).
  - (a)  $f(x) = x^2 + 5x + 2$
  - (b)  $f(x) = 2x^2 - 3x$
  - (c)  $f(x) = -2x^2 + 4x + 1$
4. For each polynomial, find a real number  $C$  so that the polynomial has the indicated root. For this  $C$ , find all remaining roots of the polynomial algebraically and write the roots in simplest radical form. Sketch a complete graph of the polynomial, indicating the roots.
  - (a)  $f(x) = x^3 + 5x^2 + x + C$  has a root at  $x = -2$
  - (b)  $f(x) = x^3 + 3x^2 - 16x + C$  has a root at  $x = 3$
  - (c)  $f(x) = 2x^3 - 4x^2 - 3x + C$  has a root at  $x = -1$
5. Find the magnitude and the direction angle.
  - (a)  $v = \langle 2, -2\sqrt{3} \rangle$
  - (b)  $v = \langle -3, -3 \rangle$
  - (c)  $v = \langle -5\sqrt{3}, 5 \rangle$
6. Find the quotient or product and write the result in standard complex form.
  - (a)  $\frac{6(\cos 225^\circ + i \sin 225^\circ)}{3(\cos 135^\circ + i \sin 135^\circ)}$
  - (b)  $2(\cos 120^\circ + i \sin 120^\circ) \cdot 4(\cos 90^\circ + i \sin 90^\circ)$
7. Let  $u = \ln x$  and  $v = \ln y$ , where  $x, y > 0$ . Write the following expression in terms of  $u$  and  $v$ .
  - (a)  $\ln(x^8 \cdot \sqrt[3]{y})$
  - (b)  $\ln\left(\frac{\sqrt[4]{x}}{y^3}\right)$
  - (c)  $\ln(\sqrt{x^5} \cdot y^7)$

8. Find the domain, asymptotes, and  $x$ -intercepts of the function, and then sketch its graph.
  - (a)  $f(x) = \ln(3x - 7)$
  - (b)  $f(x) = \log(11 - 6x)$
  - (c)  $f(x) = \log_2(7x + 5)$
9. State the amplitude, period and phase shift, and then sketch one complete cycle of the graph. Label all maxima, minima and  $x$ -intercepts.
  - (a)  $y = 3 \cos(2x + \pi)$
  - (b)  $y = 2 \sin(4x - \pi)$
  - (c)  $y = -4 \sin\left(x - \frac{\pi}{2}\right)$
10. Find all exact solutions in radians.
  - (a)  $2 \sin^2 x + \sqrt{3} \sin x = 0$
  - (b)  $2 \cos^2 x - \cos x - 1 = 0$
  - (c)  $\tan^2 x - \tan x = 0$
11. In 2022, the population of a colony is 10,000, and is decreasing exponentially at 1.5% per year.
  - (a) What will the population be after 5 years?
  - (b) In what year will there be half of the population left?
12. In 2021, the population of a city is 80,000 people, and is growing at a rate of 4% per year.
  - (a) What will the population be in 2033?
  - (b) In what year will the population be triple?
13. Find the inverse for the following function.
  - (a)  $y = 3 - 4x$
  - (b)  $y = \frac{4}{7x - 3}$
  - (c)  $y = \frac{2x}{8x + 5}$
  - (d)  $y = \frac{x - 1}{x + 2}$
14.
  - (a) Find the sum of the first 70 terms of the arithmetic sequence: 22, 19, 16, 13, ...
  - (b) Find the sum of the first 95 terms of the arithmetic sequence: -17, -12, -7, -2, ...
  - (c) Find the sum of the first 777 terms of the arithmetic sequence: 3, 9, 15, 21, ...
15. Find the exact sum of the infinite geometric sequence.
  - (a)  $\frac{1}{2}, -\frac{1}{4}, \frac{1}{8}, \dots$
  - (b) 32, -16, 8, -4, ...
  - (c)  $3, 2, \frac{4}{3}, \frac{8}{9}, \dots$
  - (d) -54, -18, -6, -2, ...
16. Solve for  $x$ .
  - (a)  $3^{5x+2} = 9^{x-4}$
  - (b)  $5^{x+3} = 7^x$
  - (c)  $6^{-x} = 2^{4x+5}$

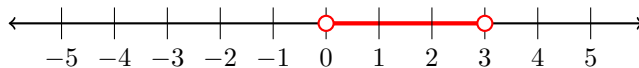
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**Answers:**

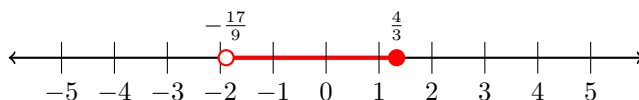
1. (a)  $(-\infty, -1] \cup [3, \infty)$



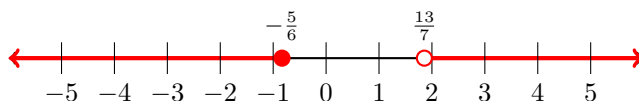
- (b)  $(0, 3)$



- (c)  $\left(-\frac{17}{9}, \frac{4}{3}\right]$



- (d)  $\left(-\infty, -\frac{5}{6}\right] \cup \left(\frac{13}{7}, \infty\right)$



2. (a)  $x$ -intercepts:  $(-1, 0), (4, 0)$ ;  $y$ -intercept:  $(0, 4)$   
Vertical asymptotes:  $x = 1, x = -2$ ; horizontal asymptote:  $y = 2$

Domain:  $D = \mathbb{R} \setminus \{-2, 1\}$

$$\text{Function: } f(x) = \frac{2(x-4)(x+1)}{(x-1)(x+2)}$$

- (b)  $x$ -intercept:  $(2, 0)$ ;  $y$ -intercept:  $(0, -3)$

Vertical asymptotes:  $x = -1, x = 1, x = 4$ ; horizontal asymptote:  $y = 0$

Domain:  $D = \mathbb{R} \setminus \{-1, 1, 4\}$

$$\text{Function: } f(x) = \frac{6(x-2)}{(x-4)(x-1)(x+1)}$$

3. (a)  $2x + h + 5$

- (b)  $4x + 2h - 3$

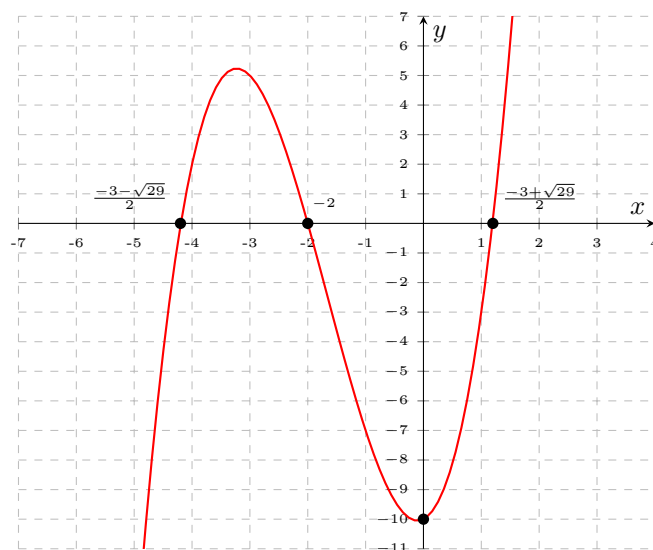
- (c)  $-4x - 2h + 4$

4. (a)  $C = -10$

$$f(x) = x^3 + 5x^2 + x - 10 = (x+2)(x^2 + 3x - 5)$$

$$\text{Roots: } -2, \frac{-3 - \sqrt{29}}{2}, \frac{-3 + \sqrt{29}}{2}$$

Graph of  $f(x) = x^3 + 5x^2 + x - 10$

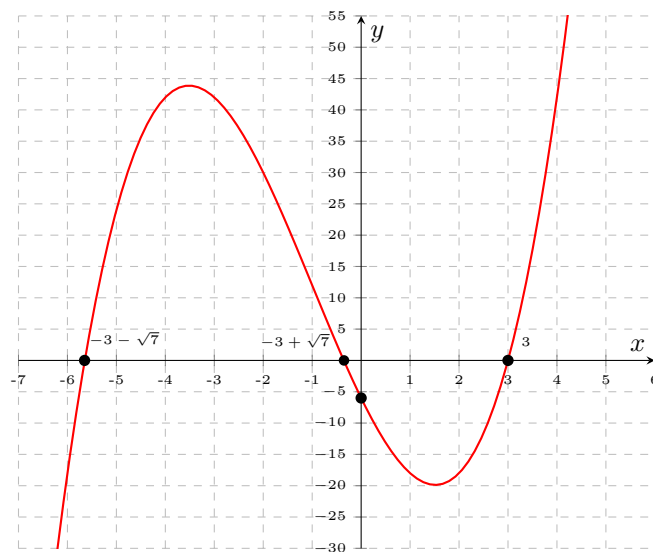


(b)  $C = -6$

$$f(x) = x^3 + 3x^2 - 16x - 6 = (x - 3)(x^2 + 6x + 2)$$

Roots:  $-3 - \sqrt{7}$ ,  $-3 + \sqrt{7}$ ,  $3$

Graph of  $f(x) = x^3 + 3x^2 - 16x - 6$

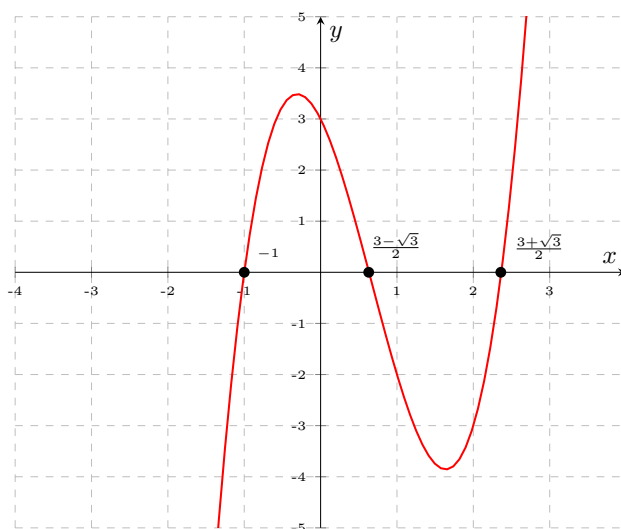


(c)  $C = 3$

$$f(x) = 2x^3 - 4x^2 - 3x + 3 = (x + 1)(2x^2 - 6x + 3)$$

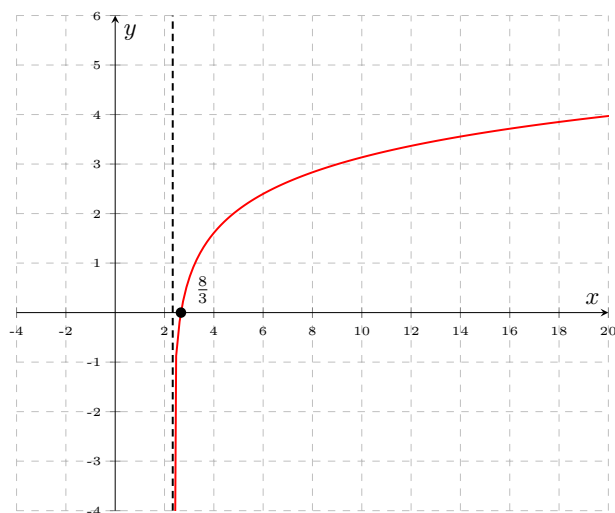
Roots:  $-1$ ,  $\frac{3 - \sqrt{3}}{2}$ ,  $\frac{3 + \sqrt{3}}{2}$

Graph of  $f(x) = 2x^3 - 4x^2 - 3x + 3$



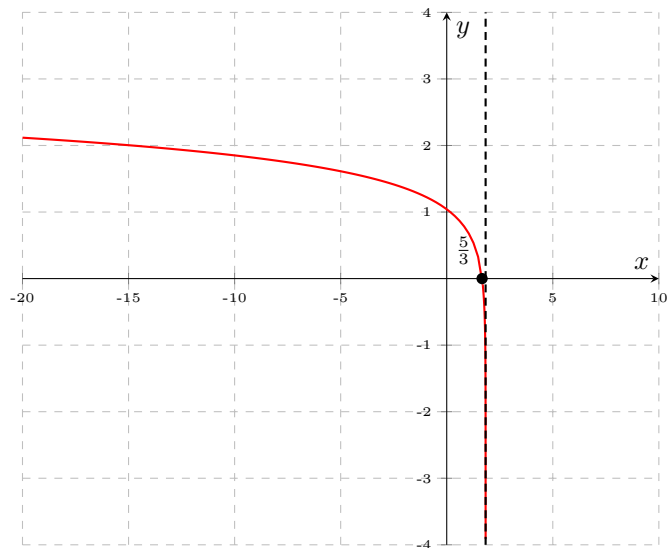
5. (a)  $\|v\| = 4$ ; direction angle:  $\theta = 300^\circ$   
 (b)  $\|v\| = 3\sqrt{2}$ ; direction angle:  $\theta = 225^\circ$   
 (c)  $\|v\| = 10$ ; direction angle:  $\theta = 150^\circ$
6. (a)  $2(\cos 90^\circ + i \sin 90^\circ) = 2i$   
 (b)  $8(\cos 210^\circ + i \sin 210^\circ) = -4\sqrt{3} - 4i$
7. (a)  $8u + \frac{1}{3}v$   
 (b)  $\frac{1}{4}u - 3v$   
 (c)  $\frac{5}{2}u + 7v$
8. (a) Domain:  $D = \left(\frac{7}{3}, \infty\right)$   
 Vertical asymptote:  $x = \frac{7}{3}$   
 x-intercept:  $\left(\frac{8}{3}, 0\right)$

Graph of  $f(x) = \ln(3x - 7)$



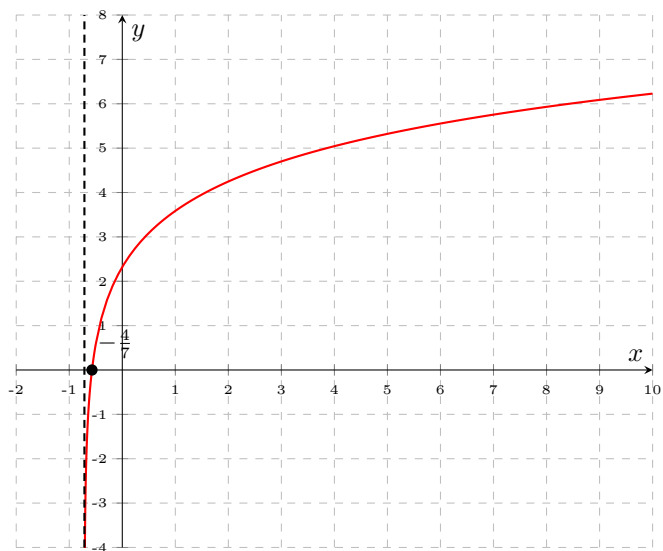
- (b) Domain:  $D = \left(-\infty, \frac{11}{6}\right)$   
 Vertical asymptote:  $x = \frac{11}{6}$   
 $x$ -intercept:  $\left(\frac{5}{3}, 0\right)$

Graph of  $f(x) = \log(11 - 6x)$



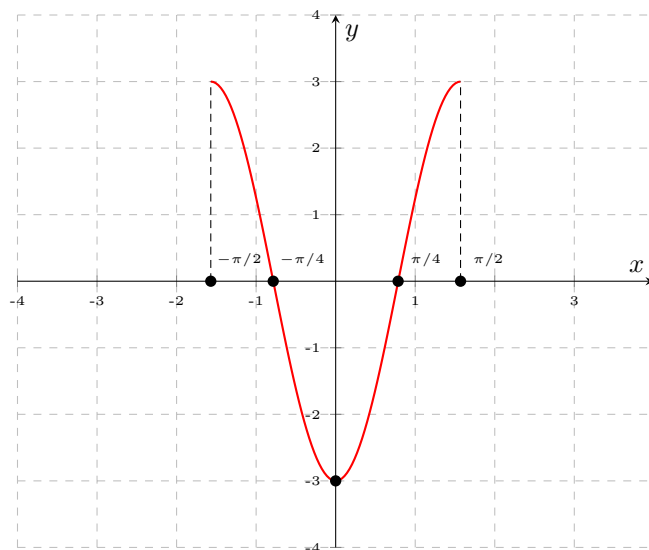
- (c) Domain:  $D = \left(-\frac{5}{7}, \infty\right)$   
 Vertical asymptote:  $x = -\frac{5}{7}$   
 $x$ -intercept:  $\left(-\frac{4}{7}, 0\right)$

Graph of  $f(x) = \log_2(7x + 5)$



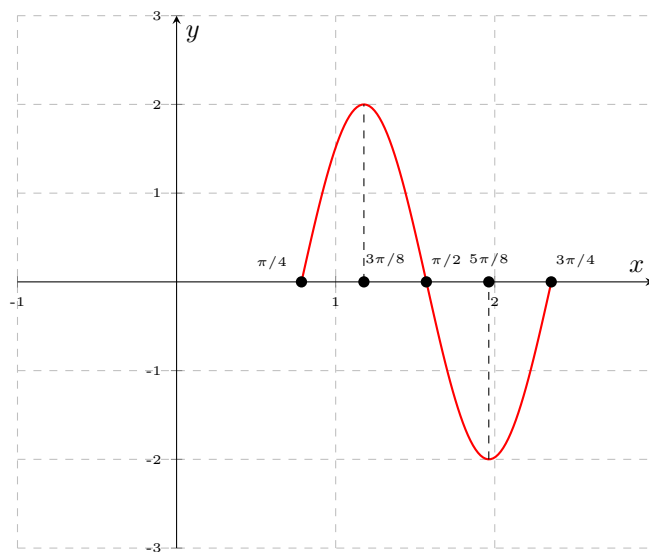
9. (a) Amplitude: 3; period:  $\pi$ ; phase shift:  $-\frac{\pi}{2}$ ; max:  $(\pm\frac{\pi}{2}, 3)$ ; min:  $(0, -3)$ ;  $x$ -intercepts:  $(\pm\frac{\pi}{4}, 0)$

Graph of  $f(x) = 3\cos(2x + \pi)$



- (b) Amplitude: 2; period:  $\frac{\pi}{2}$ ; phase shift:  $\frac{\pi}{4}$ ; max:  $(\frac{3\pi}{8}, 2)$ ; min:  $(\frac{5\pi}{8}, -2)$ ;  $x$ -intercepts:  $(\frac{\pi}{4}, 0)$ ,  $(\frac{\pi}{2}, 0)$ ,  $(\frac{3\pi}{4}, 0)$

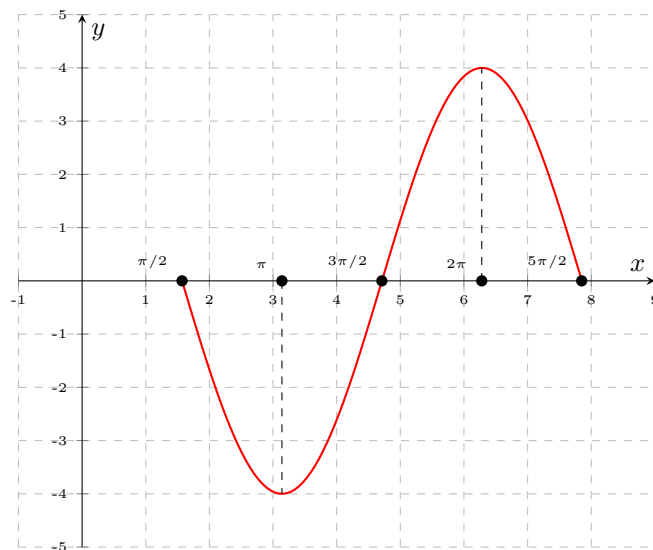
Graph of  $f(x) = 2\sin(4x - \pi)$



- (c) Amplitude: 4; period:  $2\pi$ ; phase shift:  $\frac{\pi}{2}$ ; max:  $(2\pi, 4)$ ; min:  $(\pi, -4)$ ;  $x$ -intercepts:  $(\frac{\pi}{2}, 0)$ ,  $(\frac{3\pi}{2}, 0)$ ,  $(\frac{5\pi}{2}, 0)$



Graph of  $f(x) = -4\sin(x - \pi/2)$



10. (a)  $x = 2n\pi$  or  $x = \pi + 2n\pi$  or  $x = -\frac{\pi}{3} + 2n\pi$  or  $x = \frac{4\pi}{3} + 2n\pi$ , where  $n = 0, \pm 1, \pm 2, \dots$   
 (b)  $x = \pm\frac{2\pi}{3} + 2n\pi$  or  $x = 2n\pi$ , where  $n = 0, \pm 1, \pm 2, \dots$   
 (c)  $x = n\pi$  or  $x = \frac{\pi}{4} + n\pi$ , where  $n = 0, \pm 1, \pm 2, \dots$
11. (a) The population will be approximately 9,277 people.  
 (b) Sometime in the year 2068 half of the population will be left.
12. (a) The population will be approximately 129,286 people.  
 (b) It will be triple in the year 2048.
13. (a)  $f^{-1}(x) = \frac{3-x}{4}$   
 (b)  $f^{-1}(x) = \frac{3x+4}{7x}$   
 (c)  $f^{-1}(x) = \frac{5x}{2-8x}$   
 (d)  $f^{-1}(x) = \frac{2x+1}{1-x}$
14. (a) -5705  
 (b) 20710  
 (c) 1811187
15. (a)  $\frac{1}{3}$   
 (b)  $\frac{64}{3}$   
 (c) 9  
 (d) -81
16. (a)  $x = -\frac{10}{3}$   
 (b)  $x = \frac{3\log(5)}{\log(7) - \log(5)}$   
 (c)  $x = \frac{-5\log(2)}{\log(6) + 4\log(2)}$