CHAPTER

0

An Arithmetic Review

INTRODUCTION

Number systems and methods to record patterns in their surroundings were developed by every culture in history. The Mayans in Central America had one of the most sophisticated number systems in the world in the twelfth century A.D. The Chinese number system dates from around 1200 B.C.E.

The oldest evidence of a number system is from Africa near modern-day Swaziland. Archeologists found a bone that was notched in a numerical pattern and dates from about 35,000 B.C.E.

The roots of algebra first appear in the 4,000-year-old Babylonian culture, in what is now Iraq. The Babylonians developed ways to record useful numerical relationships so that they would be easy to remember, easy to record, and helpful in solving problems. Some of the formulas developed by the Babylonians are still in use today.

You are about to embark on an exciting and useful endeavor: learning to use algebra to help you solve problems. It will take some time and effort, but do not be discouraged. Everyone can master this topic—people just like you have used it for many centuries!

CHAPTER 0 OUTLINE

0 pretest

Name ____________________

Section __________ Date __________

Answers

1. ______
2. ______
3. ______
4. ______
5. ______
6. ______ 7. ______
8. ______ 9. ______
10. ______ 11. ______
12. ______ 13. ______
14. ______ 15. ______
16. ______ 17. ______
18. ______
19. ______
20. ______
21. ______
22. ______
23. ______
24. ______ 25. ______
26. ______ 27. ______
28. ______ 29. ______
30. ______

This pretest provides a preview of the types of exercises you will encounter in each section of this chapter. The answers for these exercises can be found in the back of the text. If you are working on your own, or ahead of the class, this pretest can help you identify the sections in which you should focus more of your time.

0.1 **1.** List all the factors of 42.

2. For the group of numbers 2, 3, 6, 7, 9, 17, 18, 21, and 23, list the prime numbers and the composite numbers.

3. Find the prime factorization for each of the following numbers.
(a) 60 **(b)** 350

4. Find the greatest common factor (GCF) for each of the following groups of numbers.
(a) 12 and 32 **(b)** 24, 36, and 42

5. Find the least common multiple (LCM) for each of the following groups of numbers.
(a) 4, 5, and 10 **(b)** 36, 20, and 30

Perform the indicated operations.

0.2 **6.** $\frac{3}{5} \cdot \frac{25}{12}$ **7.** $\frac{6}{7} \quad \frac{12}{21}$ **8.** $\frac{5}{6} \quad \frac{3}{4}$

9. $\frac{17}{18} \quad \frac{5}{9}$ **0.3** **10.** 8.123 4.356 **11.** 7.16 3.19

0.2 **12.** $2\frac{2}{5} \cdot 5\frac{1}{4}$ **13.** $1\frac{5}{6} \quad 2\frac{4}{9}$ **0.3** **14.** 3.896 1.6

Evaluate the following expressions.

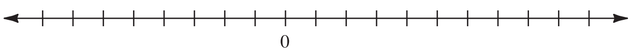
0.4 **15.** 21 3 5 **16.** 3 4 2^2 **17.** (18 9) 2 3^2

18. (15 12 5) 2^2

0.3 **19.** Write 23% as **(a)** a fraction and **(b)** a decimal.

20. Write 0.035 as a percent.

0.5 **21.** Represent the following integers on the number line shown: 6, 8, 4, 2, 10.



22. Place the following data set in ascending order: 5, 2, 4, 0, 1, 1.

23. Determine the maximum and minimum of the following data set:
4, 1, 5, 7, 3, 2.

Evaluate the following expressions.

0.5 **24.** $|\ \ 5|$ **25.** $|6|$ **26.** $|11 \quad 5|$

27. $|\ \ 11| \quad |5|$ **28.** $|4 \quad 5| \quad |6 \quad 3|$

Find the opposite of each of the following.

29. 16 **30.** 23

0.1 Prime Factorization and Least Common Multiples

< 0.1 Objectives >

1 > Find the factors of a whole number

2 > Determine whether a number is prime, composite, or neither

3 > Find the prime factorization for a number

4 > Find the GCF for two or more numbers

5 > Find the LCM for two or more numbers

Overcoming Math Anxiety

Over the first few chapters, we present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #1 Become familiar with your textbook.

Perform each of the following tasks.

1. Use the Table of Contents to find the title of Section 5.1.
2. Use the index to find the earliest reference to the term *mean.* (By the way, this term has nothing to do with the personality of either your instructor or the textbook author!)
3. Find the answer to the first Check Yourself exercise in Section 0.1.
4. Find the answers to the Self-Test for Chapter 1.
5. Find the answers to the odd-numbered exercises in Section 0.1.
6. In the margin notes for Section 0.1, find the definition for the term *relatively prime.*

Now you know where some of the most important features of the text are. When you have a moment of confusion, think about using one of these features to help you clear up that confusion.

How would you organize the following list of objects: cow, dog, daisy, fox, lily, sunflower, cat, tulip?

Although there are many ways to organize the objects, most people would break them into two groups, the animals and the flowers. In mathematics, we call a group of things that have something in common a **set.**

Definition

Set

A **set** is a collection of distinct objects that are grouped together into a single unit. Each member of a set is called an *element*.

We generally describe a set in one of two ways:

List the elements of the set.

Describe the rule(s) used to determine whether a given object is a member of the set.

We usually use braces to enclose the elements of a set when we are listing them:

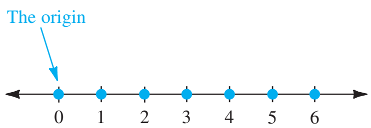
{cow, dog, fox, cat}   or   {daisy, lily, sunflower, tulip}

Of course, in mathematics many (but not all) of the sets we are interested in are sets of numbers.

The numbers used to count things—1, 2, 3, 4, 5, and so on—are called the **natural** (or **counting**) **numbers.** The **whole numbers** consist of the natural numbers and zero—0, 1, 2, 3, 4, 5, and so on. They can be represented on a number line like the one shown. Zero (0) is considered the origin.

NOTE

The centered dot indicates multiplication.



The number line continues forever in both directions.

Any whole number can be written as a product of two whole numbers. For example, we say that 12 3 4. We call 3 and 4 **factors** of 12.

Definition

Factor

A **factor** of a whole number is another whole number that *divides exactly* into that number. This means that the division has a remainder of 0.

Example 1 **Finding Factors**

< Objective 1 >

List all factors of 18.

3 6 18 — Because 3 6 18, 3 and 6 are factors (or divisors) of 18.

2 9 18 — 2 and 9 are also factors of 18.

1 18 18 — 1 and 18 are factors of 18.

1, 2, 3, 6, 9, and 18 are all the factors of 18.

NOTES

3 and 6 can also be called *divisors* of 18. They divide 18 exactly.

This is a complete list of the factors. There are no other whole numbers that divide 18 exactly. Note that the factors of 18, except for 18 itself, are all *smaller* than 18.

Check Yourself 1*

List all factors of 24.

* Check Yourself answers appear at the end of each section in this book.

Listing factors leads us to an important classification of whole numbers. Any whole number larger than 1 is either a *prime* or a *composite* number.

Definition

Prime Number

A **prime number** is any whole number greater than 1 that has only 1 and itself as factors.

NOTE

A whole number greater than 1 always has itself and 1 as factors. Sometimes these are the *only* factors. For instance, 1 and 3 are the only factors of 3.

As examples, 2, 3, 5, and 7 are prime numbers. Their only factors are 1 and themselves.

To check whether a number is prime, one approach is simply to divide the smaller primes, 2, 3, 5, 7, and so on, into the given number. If no factors other than 1 and the given number are found, the number is prime.

The Sieve of Eratosthenes is an easy method for identifying prime numbers.

NOTE

How large can a prime number be? There is no largest prime number!

As of December 2005, the largest known prime number is $2^{30,402,457}$ 1. If you are curious, this is a number with 9,152,052 *digits*. Of course, a computer was used to find this number and verify that it is prime.

By the time you read this, someone may very well have found an even larger prime number.

Step by Step

The Sieve of Eratosthenes

Step 1 Write down a sequence of counting numbers, beginning with the number 2. In the example below, we stop at 50.

Step 2 Start at the number 2. Delete every *second* number after the 2. Each of the deleted numbers has 2 as a factor. This means that each deleted number is a composite number.

Step 3 Move to the number 3. Delete every *third* number after 3 (some numbers will already have been deleted). Each deleted number is divisible by 3, so each deleted number is not prime.

Step 4 Move to the next undeleted number, which is 5 (you should already have deleted 4). Delete every fifth number after 5.

Step 5 Continue this process, deleting every seventh number after 7, and so on.

Step 6 When you have finished, the numbers that remain are prime.

NOTE

The mathematician, geographer, and astronomer Eratosthenes was born in Cyrene (now Shahhat, Libya) in 276 B.C.E. He worked in Athens, Greece, and Alexandria, Egypt. He died in 194 B.C.E.

Among other contributions, Eratosthenes computed the size of Earth and devised our system of latitude and longitude.

	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~	~~16~~	17	~~18~~	19	~~20~~
~~21~~	~~22~~	23	~~24~~	~~25~~	~~26~~	~~27~~	~~28~~	29	~~30~~
31	~~32~~	~~33~~	~~34~~	~~35~~	~~36~~	37	~~38~~	~~39~~	~~40~~
41	~~42~~	43	~~44~~	~~45~~	~~46~~	47	~~48~~	~~49~~	~~50~~

The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.

Example 2 Identifying Prime Numbers

< Objective 2 >

Which of the numbers 17, 29, and 33 are prime?

17 is a prime number. 1 and 17 are the only factors.

29 is a prime number. 1 and 29 are the only factors.

33 is *not* prime. 1, 3, 11, and 33 are all factors of 33.

Note: For two-digit numbers, if the number is *not* a prime, it will have one or more of the numbers 2, 3, 5, or 7 as factors.

Check Yourself 2

Which of the following numbers are prime numbers?

2, 6, 9, 11, 15, 19, 23, 35, 41

NOTE

This definition tells us that a composite number has factors other than 1 and itself.

We can now define a second class of whole numbers.

Definition

Composite Number A **composite number** is any whole number greater than 1 that is not prime.

Example 3 Identifying Composite Numbers

Which of the following numbers are composite: 18, 23, 25, and 38?

18 is a composite number. 1, 2, 3, 6, 9, and 18 are all factors of 18.

23 is not a composite number. 1 and 23 are the only factors. This means that 23 is a *prime number.*

25 is a composite number. 1, 5, and 25 are factors.

38 is a composite number. 1, 2, 19, and 38 are factors.

Check Yourself 3

Which of the following numbers are composite numbers?

2, 6, 10, 13, 16, 17, 22, 27, 31, 35

By the definitions of prime and composite numbers:

Property

0 and 1 The whole numbers 0 and 1 are neither prime nor composite.

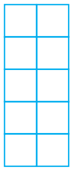
To **factor a number** means to write the number as a product of whole-number factors.

Example 4 Factoring a Composite Number

< Objective 3 >

Factor the number 10.

10 2 5

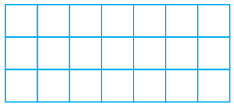


The order in which you write the factors does not matter, so 10 5 2 would also be correct.

Of course, 10 10 1 is also a correct statement. However, in this section we are interested in products that use factors other than 1 and the given number.

Factor the number 21.

21 3 7



Check Yourself 4

Factor 35.

In writing composite numbers as a product of factors, there may be several different possible factorizations.

Example 5 Factoring a Composite Number

NOTE

There have to be at least two different factorizations, because a composite number has factors other than 1 and itself.

Find three ways to factor 72.

72 8 9
  6 12
  3 24

Check Yourself 5

Find three ways to factor 42.

We now want to write composite numbers as a product of their **prime factors.** Look again at the first factored line of Example 5. The process of factoring can be continued until all the factors are prime numbers.

Example 6 Factoring a Composite Number into Prime Factors

NOTE

This is often called a **factor tree.**

72 8 9
 2 4 3 3
2 2 2 3 3

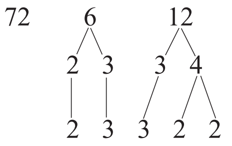
4 is still not prime, and so we continue by factoring 4.

72 is now written as a product of prime factors.

> **NOTES**
>
> Finding the prime factorization of a number is a skill that is used when adding fractions.
>
> Because 2 3 6 3 2, the order in which we write the factors does not matter. As a matter of convention, we usually write the factors in size order.

When we write 72 as 2 2 2 3 3, no further factorization is possible. This is called the **prime factorization** of 72.

Now, what if we start with the second factored line from the same example, 72 6 12?



Continue to factor 6 and 12.

Continue again to factor 4. Other choices for the factors of 12 are possible. The end result is always the same.

No matter which factor pair you begin with, you will always finish with the same set of prime factors. In this case, the factor 2 appears three times and the factor 3 appears twice. The order in which we write the factors does not matter.

Check Yourself 6

We could also write

72 2 36

Continue the factorization.

Property

The Fundamental Theorem of Arithmetic

There is exactly one prime factorization for any composite number.

The method shown in Example 6 always works. However, an easier method for factoring composite numbers exists. This method is particularly useful when factoring large numbers, in which case factoring with a number tree becomes unwieldy.

Property

Factoring by Division

To find the prime factorization of a number, divide the number by a series of primes until the final quotient is a prime number.

Example 7 Finding Prime Factors

> **NOTE**
>
> The prime factorization is the product of all the prime divisors and the final quotient.

To write 60 as a product of prime factors, divide 2 into 60 for a quotient of 30. Continue to divide by 2 again for the quotient 15. Because 2 does not divide exactly into 15, we try 3. Because the quotient 5 is prime, we are done.

$$2\overline{)60}^{\,30} \rightarrow 2\overline{)30}^{\,15} \rightarrow 3\overline{)15}^{\,5}\quad \text{Prime}$$

Our factors are the prime divisors and the final quotient. We have

60 2 2 3 5

Check Yourself 7

Complete the process to find the prime factorization of 90.

$$2\overline{)90}^{\,45} \rightarrow\ ?\overline{)45}^{\,?}$$

Remember to continue until the final quotient is prime.

Writing composite numbers in their completely factored form can be simplified if we use a format called **continued division.**

Example 8 Finding Prime Factors Using Continued Division

NOTE

In each short division, we write the quotient *below* rather than above the dividend. This is just a convenience for the next division.

Use the continued-division method to divide 60 by a series of prime numbers.

Primes

$$\begin{array}{r} 2\overline{)60} \\ 2\overline{)30} \\ 3\overline{)15} \\ 5 \end{array}$$

Stop when the final quotient is prime.

To write the factorization of 60, we include each divisor used and the final prime quotient. In our example, we have

60 2 2 3 5

Check Yourself 8

Find the prime factorization of 234.

NOTE

Again the factors of 20, other than 20 itself, are less than 20.

We know that a factor or a divisor of a whole number divides that number exactly. The factors or divisors of 20 are

1, 2, 4, 5, 10, and 20

Each of these numbers divides 20 exactly, that is, with no remainder.

Next, we look at common factors or divisors. A **common factor** or **divisor** of two numbers is any factor that divides both the numbers exactly.

Example 9 Finding Common Factors

< Objective 4 >

Look at the numbers 20 and 30. Is there a common factor for the two numbers?

First, we list the factors. Then, we circle the ones that appear in both lists.

Factors

20: 1, 2, 4, 5, 10, 20

30: 1, 2, 3, 5, 6, 10, 15, 30

We see that 1, 2, 5, and 10 are common factors of 20 and 30. Each of these numbers divides both 20 and 30 exactly.

Our later work with fractions will require that we find the greatest common factor of a group of numbers.

Definition

Greatest Common Factor

The **greatest common factor (GCF)** of a group of numbers is the *largest* number that divides each of the given numbers exactly.

In the first part of Example 9, the common factors of the numbers 20 and 30 were listed as

1, 2, 5, 10 Common factors of 20 and 30

The GCF of the two numbers is 10, because 10 is the *largest* of the four common factors.

Check Yourself 9

List the factors of 30 and 36, and then find the GCF.

The method of Example 9 also works in finding the GCF of a group of more than two numbers.

Example 10 Finding the GCF by Listing Factors

> **NOTE**
>
> Looking at the three lists, we see that 1, 2, 3, and 6 are common factors.

Find the GCF of 24, 30, and 36. We list the factors of each of the three numbers.

24: 1, 2, 3, 4, 6, 8, 12, 24

30: 1, 2, 3, 5, 6, 10, 15, 30

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

6 is the GCF of 24, 30, and 36.

Check Yourself 10

Find the GCF of 16, 24, and 32.

> **NOTE**
>
> If there are no common prime factors, the GCF is 1.

The process shown in Example 10 is very time-consuming when larger numbers are involved. A better approach to the problem of finding the GCF of a group of numbers uses the prime factorization of each number.

Step by Step

Finding the GCF

Step 1 Write the prime factorization for each of the numbers in the group.
Step 2 Locate the prime factors that appear in every prime factorization.
Step 3 The GCF is the *product* of all the common prime factors.

Example 11 Finding the GCF

Find the GCF of 20 and 30.

Step 1 Write the prime factorizations of 20 and 30.

20 2 2 5
30 2 3 5

Step 2 Find the prime factors common to each number.

20 2 2 5
30 2 3 5

2 and 5 are the common prime factors.

Step 3 Form the product of the common prime factors.

2 5   10

10 is the GCF.

Check Yourself 11

Find the GCF of 30 and 36.

To find the GCF of a group of more than two numbers, we use the same process.

Example 12 Finding the GCF

Find the GCF of 24, 30, and 36.

24 2 2 2 3
30 2 3 5
36 2 2 3 3

2 and 3 are the prime factors common to *all three numbers.*

2 3   6 is the GCF.

Check Yourself 12

Find the GCF of 15, 30, and 45.

Sometimes, two numbers have no common factors other than 1.

Example 13 Finding the GCF

NOTE

If two numbers, such as 15 and 28, have no common factor other than 1, they are called **relatively prime.**

Find the GCF of 15 and 28.

15 3 5
28 2 2 7

There are no common prime factors listed. But remember that 1 is a factor of every whole number.

1 is the GCF.

Check Yourself 13

Find the GCF of 30 and 49.

Another idea that will be important in our work with fractions is the concept of *multiples.* Every whole number has an associated group of multiples.

Definition

Multiples

The **multiples** of a number are the product of that number with the natural numbers 1, 2, 3, 4, 5, ...

Example 14 Listing Multiples

< Objective 5 >

NOTE

Other than 3 itself, the multiples of 3 are all larger than 3.

List the multiples of 3.

The multiples of 3 are

3 · 1, 3 · 2, 3 · 3, 3 · 4, . . .

The three dots indicate that the list continues without stopping.

or

3, 6, 9, 12, . . .

An easy way of listing the multiples of 3 is to think of *counting by threes*.

Check Yourself 14

List the first seven multiples of 4.

You may see the relationship between factors and multiples. Saying "12 is a multiple of 3" is the same as saying "3 is a factor of 12."

Sometimes we need to find common multiples of two or more numbers.

Definition

Common Multiples

If a number is a multiple of each of a group of numbers, it is called a **common multiple** of the numbers; that is, it is a number that is evenly divisible by all the numbers in the group.

Example 15 Finding Common Multiples

NOTE

15, 30, 45, and 60 are multiples of *both* 3 and 5.

NOTE

We can say that both 3 and 5 are common factors of 15, 30, 45, and 60.

Find four common multiples of 3 and 5.

Some common multiples of 3 and 5 are

15, 30, 45, 60

Check Yourself 15

List the first six multiples of 6. Then look at your list from Check Yourself 14 and list some common multiples of 4 and 6.

In our later work with fractions, we will use the *least common multiple* of a group of numbers.

Definition

Least Common Multiple

The **least common multiple** (LCM) of a group of numbers is the *smallest* number that is a multiple of each number in the group.

It is possible to simply list the multiples of each number and then find the LCM by inspection.

Example 16 Finding the LCM

NOTE

48 is also a common multiple of 6 and 8, but we are looking for the *smallest* such number.

NOTE

For instance, if a number appears three times in the factorization of a number, it will be included at least three times in forming the least common multiple.

Find the LCM of 6 and 8.

Multiples

6: 6, 12, 18, 24, 30, 36, 42, 48, . . .

8: 8, 16, 24, 32, 40, 48, . . .

We see that 24 is the smallest number common to both lists. So 24 is the LCM of 6 and 8.

Check Yourself 16

Find the LCM of 20 and 30 by listing the multiples of each number.

The technique of Example 16 will work for any group of numbers. However, it becomes tedious for larger numbers. Here is an easier approach.

Step by Step

Finding the LCM

Step 1 Write the prime factorization for each of the numbers in the group.

Step 2 List the prime factors that occur the greatest number of times in any one prime factorization.

Step 3 Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.

Some students prefer to line up the factors to help remember the process of finding the LCM of a group of numbers.

Example 17 Finding the LCM

> **NOTE**
>
> Line up the *like* factors vertically.

To find the LCM of 10 and 18, we factor:

10	2			5
18	2	3	3	
	2	3	3	5

Bring down the factors.

2 and 5 appear, at most, one time in any one factorization. 3 appears twice in one factorization.

2 3 3 5 90

So 90 is the LCM of 10 and 18.

Check Yourself 17

Use the method of Example 17 to find the LCM of 24 and 36.

The procedure is the same for a group of more than two numbers.

Example 18 Finding the LCM

> **NOTE**
>
> The different factors that appear are 2, 3, and 5.

To find the LCM of 12, 18, and 20, we factor:

12	2	2	3		
18	2		3	3	
20	2	2			5
	2	2	3	3	5

2 and 3 appear twice in one factorization; 5 appears just once.

2 2 3 3 5 180

So 180 is the LCM of 12, 18, and 20.

Check Yourself 18

Find the LCM of 3, 4, and 6.

Check Yourself ANSWERS

1. 1, 2, 3, 4, 6, 8, 12, and 24 **2.** 2, 11, 19, 23, and 41 are prime numbers.
3. 6, 10, 16, 22, 27, and 35 are composite numbers. **4.** 5 7
5. 2 21, 3 14, 6 7 **6.** 2 2 2 3 3
7. $2\overline{)90}^{\,45} \rightarrow 3\overline{)45}^{\,15} \rightarrow 3\overline{)15}^{\,5}$ **8.** 2 3 3 13

90 2 3 3 5

9. 30: 1, 2, 3, 5, 6, 10, 15, 30

36: 1, 2, 3, 4, 6, 9, 12, 18, 36

The GCF is 6.

10. 16: 1, 2, 4, 8, 16

24: 1, 2, 3, 4, 6, 8, 12, 24

32: 1, 2, 4, 8, 16, 32

The GCF is 8.

11. 30 2 3 5

36 2 2 3 3

The GCF is 2 3 6.

12. 15 **13.** The GCF is 1; 30 and 49 are relatively prime.
14. The first seven multiples of 4 are 4, 8, 12, 16, 20, 24, and 28.
15. 6, 12, 18, 24, 30, 36; some common multiples of 4 and 6 are 12, 24, and 36.
16. The multiples of 20 are 20, 40, 60, 80, 100, 120, . . . ; the multiples of 30 are 30, 60, 90, 120, 150, . . . ; the LCM of 20 and 30 is 60, the smallest number common to both lists.
17.

24	2	2	2	3		
36	2	2		3	3	
	2	2	2	3	3	72

18. 12

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.1

(a) The centered dot in the expression 3 4 indicates ________________.

(b) A composite number is any whole number greater than 1 that is not ________________.

(c) A pair of numbers that have no common factor other than 1 are called ________________ prime.

(d) Saying “12 is a ________________ of 3” is the same as saying “3 is a factor of 12.”

0.1 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

Name ____________

Section ________ Date ________

Answers

1. ____ 2. ____
3. ____
4. ____
5. ____ 6. ____
7. ____
8. ____
9. ____
10. ____
11. ____ 12. ____
13. ____
14. ____
15. ____
16. ____
17. ____ 18. ____
19. ____ 20. ____
21. ____ 22. ____

< Objective 1 >

List the factors of each of the following numbers.

1. 8 **2.** 6

3. 10 **4.** 12

5. 15 **6.** 21

7. 24 **8.** 32

9. 64 > Videos **10.** 66

11. 13 **12.** 37

< Objective 2 >

Use the following list of numbers for exercises 13 and 14.

0, 1, 15, 19, 23, 31, 49, 55, 59, 87, 91, 97, 103, 105

13. Which of the given numbers are prime? > Videos

14. Which of the given numbers are composite?

15. List all the prime numbers between 30 and 50.

16. List all the prime numbers between 55 and 75.

< Objective 3 >

Find the prime factorization of each number.

17. 20 **18.** 22

19. 30 **20.** 35

21. 51 **22.** 24

23. 63
24. 94
25. 70
26. 90
27. 88
28. 100
29. 130
30. 66
31. 315
32. 400
33. 225
34. 132
35. 189 > Videos
36. 330

In later mathematics courses, you often will want to find factors of a number with a given sum or difference. The following problems use this technique.

37. Find two factors of 24 with a sum of 10.

38. Find two factors of 15 with a difference of 2.

39. Find two factors of 30 with a difference of 1.

40. Find two factors of 28 with a sum of 11.

< Objective 4 >

Find the GCF for each of the following groups of numbers.

41. 4 and 6
42. 6 and 9
43. 10 and 15
44. 12 and 14
45. 21 and 24
46. 22 and 33
47. 20 and 21
48. 28 and 42
49. 18 and 24 > Videos
50. 35 and 36
51. 45, 60, and 75
52. 36, 54, and 180
53. 12, 36, and 60
54. 15, 45, and 90
55. 105, 140, and 175
56. 32, 80, and 112
57. 25, 75, and 150
58. 36, 72, and 144

Answers

23. ______ 24. ______
25. ______ 26. ______
27. ______ 28. ______
29. ______ 30. ______
31. ______
32. ______
33. ______
34. ______
35. ______
36. ______
37. ______ 38. ______
39. ______ 40. ______
41. ______ 42. ______
43. ______ 44. ______
45. ______ 46. ______
47. ______ 48. ______
49. ______ 50. ______
51. ______ 52. ______
53. ______ 54. ______
55. ______ 56. ______
57. ______ 58. ______

Answers

59. ____
60. ____
61. ____
62. ____
63. ____
64. ____
65. ____
66. ____
67. ____
68. ____
69. ____
70. ____
71. ____
72. ____
73. ____
74. ____
75. ____
76. ____

< Objective 5 >

Find the LCM for each of the following groups of numbers. Use whichever method you wish.

59. 12 and 15 **60.** 12 and 21 **61.** 18 and 36

62. 25 and 50 **63.** 25 and 40 **64.** 10 and 14

65. 3, 5, and 6 **66.** 2, 8, and 10 **67.** 18, 21, and 28

> Videos

68. 8, 15, and 20 **69.** 20, 30, and 40 **70.** 12, 20, and 35

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

71. Prime numbers that differ by two are called *twin primes.* Examples are 3 and 5, 5 and 7, and so on. Find one pair of twin primes between 85 and 105.

chapter 0 > Make the Connection

72. The following questions refer to twin primes (see exercise 71).

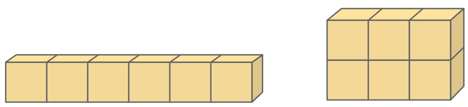
(a) Search for, and make a list of several pairs of twin primes in which the primes are greater than 3.

chapter 0 > Make the Connection

(b) What do you notice about each number that lies *between* a pair of twin primes?

(c) Write an explanation for your observation in part **(b).**

73. Obtain (or imagine that you have) a quantity of square tiles. Six tiles can be arranged in the shape of a rectangle in two different ways:



(a) Record the dimensions of the rectangles shown.

(b) If you use seven tiles, how many different rectangles can you form?

(c) If you use ten tiles, how many different rectangles can you form?

(d) What kind of number (of tiles) permits *only one* arrangement into a rectangle? *More than* one arrangement?

74. The number 10 has four factors: 1, 2, 5, and 10. We can say that 10 has an even number of factors. Investigate several numbers to determine which numbers have an *even number* of factors and which numbers have an *odd number* of factors.

75. A natural number is said to be *perfect* if it is equal to the sum of its divisors, except itself.

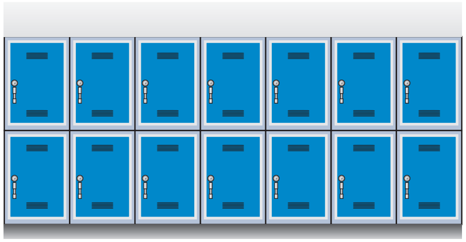
(a) Show that 28 is a perfect number.

(b) Identify another perfect number less than 28.

chapter 0 > Make the Connection

76. Find the smallest natural number that is divisible by all of the following: 2, 3, 4, 6, 8, 9.

77. Suppose that a school has 1,000 lockers and that they are all closed. A person passes through, opening every other locker, beginning with locker 2. Then another person passes through, changing every third locker (closing it if it is open, opening it if it is closed), starting with locker 3. Yet another person passes through, changing every fourth locker, beginning with locker 4. This process continues until 1,000 people pass through.



(a) At the end of this process, which locker numbers are closed?

(b) Write an explanation for your answer to part **(a).** (Hint: It may help to attempt exercise 74 first.)

Answers

77. ____________

Answers

We provide the answers for the odd-numbered problems at the end of each exercise set.

1. 1, 2, 4, 8 **3.** 1, 2, 5, 10 **5.** 1, 3, 5, 15
7. 1, 2, 3, 4, 6, 8, 12, 24 **9.** 1, 2, 4, 8, 16, 32, 64 **11.** 1, 13
13. 19, 23, 31, 59, 97, 103 **15.** 31, 37, 41, 43, 47 **17.** 2 2 5
19. 2 3 5 **21.** 3 17 **23.** 3 3 7 **25.** 2 5 7 **27.** 2 2 2 11
29. 2)130
5)65
13
130 2 5 13
31. 3 3 5 7 **33.** 3 3 5 5
35. 3)189
3)63
3)21
7
189 3 3 3 7
37. 4, 6 **39.** 5, 6 **41.** 2 **43.** 5 **45.** 3
47. 1 **49.** 6 **51.** 15 **53.** 12 **55.** 35 **57.** 25 **59.** 60
61. 36 **63.** 200 **65.** 30 **67.** 252 **69.** 120
71. 101, 103 **73.** Above and Beyond **75.** Above and Beyond
77. Above and Beyond

0.2 Fractions and Mixed Numbers

< **0.2 Objectives** >

1 > Simplify a fraction

2 > Multiply and divide fractions

3 > Add and subtract fractions

4 > Write fractions as mixed numbers

5 > Multiply and divide mixed numbers

6 > Add and subtract mixed numbers

This section provides a review of the basic arithmetic operations—addition, subtraction, multiplication, and division—with fractions and mixed numbers.

In Section 0.1, we identified the set of whole numbers as the set consisting of the numbers 0, 1, 2, 3, and so on. In this section, we look at the set of positive numbers that can be written as fractions.

There are two types of fractions that we examine here: proper fractions and improper fractions. **Proper fractions** are those fractions that are less than 1, such as $\frac{1}{2}$ and $\frac{2}{3}$ (the numerator is less than the denominator). **Improper fractions** are those fractions that are greater than or equal to 1, such as $\frac{7}{2}$ and $\frac{19}{5}$ (the numerator is greater than the denominator).

> **NOTE**
>
> The set of numbers that can be written as fractions is called the set of *rational numbers.*

Every whole number can be written in fraction form, $\frac{a}{b}$, in which the denominator $b \quad 0$. In fact, there are many fraction forms for each number. This is because the fraction bar can be interpreted as division. For example, we can write $2 \quad 2$ as $\frac{2}{2}$. Of course, this is another way of writing the whole number 1. Any fraction in which the numerator and the denominator are the same is a representation of the number 1 because any nonzero number divided by itself is 1.

$$1 \quad \frac{2}{2} \qquad 1 \quad \frac{12}{12} \qquad 1 \quad \frac{257}{257}$$

These fractions are called **equivalent fractions** because they all represent the same number.

To determine whether two fractions are equivalent or to find equivalent fractions, we use the **Fundamental Principle of Fractions.** The Fundamental Principle of Fractions arises from the idea that multiplying any number by 1 does not change the number.

Property

The Fundamental Principle of Fractions

$\frac{a}{b} \quad \frac{a \cdot c}{b \cdot c}$ or $\frac{a \cdot c}{b \cdot c} \quad \frac{a}{b}$, in which neither b nor c is zero.

Example 1 Rewriting Fractions

< Objective 1 >

NOTE

Each representation is a numeral, or name for the number. Each number has many names.

Write three fractional representations for each number.

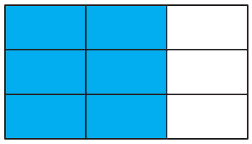
(a) $\frac{2}{3}$

We use the Fundamental Principle of Fractions to multiply the numerator and denominator by the same number.

$\frac{2}{3} \quad \frac{2 \cdot 2}{3 \cdot 2} \quad \frac{4}{6}$

$\frac{4}{6}$

$\frac{2}{3} \quad \frac{2 \cdot 3}{3 \cdot 3} \quad \frac{6}{9}$

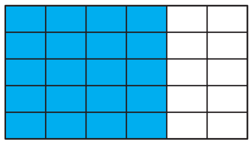


$\frac{6}{9}$

NOTE

In each case, we use the Fundamental Principle of Fractions with c equal to a different number.

$\frac{2}{3} \quad \frac{2 \cdot 10}{3 \cdot 10} \quad \frac{20}{30}$



$\frac{20}{30}$

(b) 5

$5 \quad \frac{5 \cdot 2}{1 \cdot 2} \quad \frac{10}{2}$

$5 \quad \frac{5 \cdot 3}{1 \cdot 3} \quad \frac{15}{3}$

$5 \quad \frac{5 \cdot 100}{1 \cdot 100} \quad \frac{500}{100}$

Check Yourself 1

Write three fractional representations for each number.

(a) $\frac{5}{8}$ **(b)** $\frac{4}{3}$ **(c)** 3

The simplest fractional representation for a number has the smallest numerator and denominator. Fractions written in this form are said to be **simplified.**

Example 2 Simplifying Fractions

Simplify each fraction.

(a) $\frac{22}{55}$ **(b)** $\frac{35}{45}$ **(c)** $\frac{24}{36}$

We first find the prime factors for the numerator and for the denominator.

(a) $\frac{22}{55} \quad \frac{2 \cdot 11}{5 \cdot 11}$

We then use the Fundamental Principle of Fractions.

$$\frac{22}{55} \quad \frac{2 \cdot 11}{5 \cdot 11} \quad \frac{2}{5}$$

(b) $\frac{35}{45} \quad \frac{7 \cdot 5}{3 \cdot 3 \cdot 5} \quad \frac{7 \cdot 5}{9 \cdot 5}$

Using the fundamental principle to remove the common factor of 5 yields

$$\frac{35}{45} \quad \frac{7}{9}$$

(c) $\frac{24}{36} \quad \frac{2 \cdot 2 \cdot 2 \cdot 3}{2 \cdot 2 \cdot 3 \cdot 3}$

Removing the common factor 2 2 3 yields

$$\frac{24}{36} \quad \frac{2}{3}$$

NOTE

Often, we use the convention of "canceling" a factor that appears in both the numerator and denominator to prevent careless errors.

$$\frac{7 \cdot 5}{3 \cdot 3 \cdot 5} \quad \frac{7 \cdot \cancel{5}}{3 \cdot 3 \cdot \cancel{5}}$$
$$\frac{7}{3 \cdot 3}$$
$$\frac{7}{9}$$

NOTE

With practice, you will be able to simplify fractions mentally.

$$\frac{\cancel{2} \cdot \cancel{2} \cdot 2 \cdot \cancel{3}}{\cancel{2} \cdot \cancel{2} \cdot 3 \cdot \cancel{3}} \quad \frac{2}{3}$$

Check Yourself 2

Simplify each fraction.

(a) $\frac{21}{33}$ **(b)** $\frac{15}{30}$ **(c)** $\frac{12}{54}$

The Fundamental Principle of Fractions is really based on the way in which we multiply fractions. To multiply a pair of fractions, we multiply the numerators—the result becomes the numerator of their product. Then, we multiply the denominators—the result becomes the denominator of the product.

Property

Multiplying Fractions

$$\frac{a}{b} \cdot \frac{c}{d} \quad \frac{a \cdot c}{b \cdot d}$$

When multiplying two fractions, rewrite them in factored form, and then simplify before multiplying. To multiply a fraction by a whole number, we rewrite the whole number as a fraction in which the denominator is 1.

Example 3 Multiplying Fractions

< Objective 2 >

RECALL

A product is the result of multiplication.

Find the product of the two fractions.

$$\frac{9}{2}\cdot\frac{4}{3}$$

$$\frac{9}{2}\cdot\frac{4}{3} \quad \frac{9\cdot 4}{2\cdot 3}$$

$$\frac{\cancel{3}\cdot 3\cdot \cancel{2}\cdot 2}{\cancel{2}\cdot\cancel{3}}$$

$$\frac{3\cdot 2}{1}$$

$$\frac{6}{1}$$ The denominator of 1 is not necessary.

$$6$$

Check Yourself 3

Multiply and simplify each pair of fractions.

(a) $\frac{3}{5}\cdot\frac{10}{7}$ (b) $\frac{12}{5}\cdot\frac{10}{6}$

Property

Dividing Fractions

$$\frac{a}{b} \quad \frac{c}{d} \quad \frac{a}{b}\cdot\frac{d}{c} \quad \frac{a\cdot d}{b\cdot c}$$

This rule says that to divide two fractions, invert the divisor (flip the second fraction) and multiply.

Example 4 Dividing Fractions

Find the quotient.

$$\frac{7}{3} \quad \frac{5}{6}$$

$$\frac{7}{3} \quad \frac{5}{6} \quad \frac{7}{3}\cdot\frac{6}{5} \quad \frac{7\cdot 6}{3\cdot 5}$$

$$\frac{7\cdot 2\cdot 3}{3\cdot 5} \quad \frac{7\cdot 2}{5}$$

$$\frac{14}{5}$$

Check Yourself 4

Find the result from dividing the two fractions.

$$\frac{9}{2} \quad \frac{3}{5}$$

NOTE

Find the LCM of the set of denominators (as described in Section 0.1). The result is the LCD.

The next property tells us how to add fractions when they have the same denominator.

Property

Adding Fractions

$$\frac{a}{b} \quad \frac{c}{b} \quad \frac{a \quad c}{b}$$

When adding two fractions with different denominators, find the **least common denominator (LCD)** first. The LCD is the smallest number that both denominators evenly divide. After rewriting the fractions with this denominator, add the numerators, and then simplify the result.

Example 5 Adding Fractions

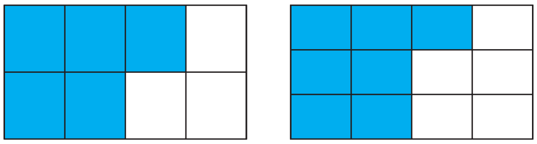
< Objective 3 >

RECALL

A sum is the result of addition.

Find the sum of the two fractions.

$$\frac{5}{8} \quad \frac{7}{12}$$



The fractions $\frac{5}{8}$ and $\frac{7}{12}$ have different denominators. In order to add them, we need to find equivalent fractions that have the same denominator. To find the least common denominator (LCD) of these fractions, we find the LCM of their denominators 8 and 12.

From Section 0.1, we know that the LCM of 8 and 12 is 24, so we rewrite each fraction as an equivalent fraction with a denominator of 24.

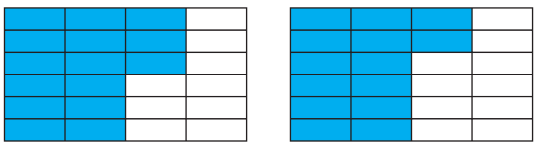
RECALL

To find equivalent fractions, multiply each fraction by 1.

$$\frac{5}{8}\cdot\frac{3}{3} \quad \frac{15}{24} \qquad 24 \quad 8 \quad 3$$

$$\frac{7}{12}\cdot\frac{2}{2} \quad \frac{14}{24} \qquad 24 \quad 12 \quad 2$$

$$\frac{5}{8} \quad \frac{7}{12} \quad \frac{15}{24} \quad \frac{14}{24} \quad \frac{29}{24}$$ This fraction cannot be simplified.



Check Yourself 5

Find the sum for each pair of fractions.

(a) $\frac{4}{5}$ $\frac{7}{9}$ (b) $\frac{5}{6}$ $\frac{4}{15}$

Property

Subtracting Fractions

$$\frac{a}{b} \quad \frac{c}{b} \quad \frac{a \quad c}{b}$$

Subtracting fractions is treated exactly like adding them, except the numerator becomes the difference of the two numerators.

Example 6 Subtracting Fractions

Find the difference.

$$\frac{7}{9} \quad \frac{1}{6}$$

The LCD is 18. We rewrite the fractions with that denominator.

$$\frac{7}{9} \quad \frac{14}{18}$$

$$\frac{1}{6} \quad \frac{3}{18}$$

$$\frac{7}{9} \quad \frac{1}{6} \quad \frac{14}{18} \quad \frac{3}{18} \quad \frac{11}{18}$$

This fraction cannot be simplified.

RECALLS

The difference is the result of subtraction.

We cannot simplify $\frac{11}{18}$ because 11 and 18 are relatively prime.

Check Yourself 6

Find the difference $\frac{11}{12}$ $\frac{5}{8}$.

Another way to write an improper fraction is as a *mixed number.*

Definition

Mixed Number

A **mixed number** is the sum of a whole number and a proper fraction.

For our later work it will be important to be able to change back and forth between improper fractions and mixed numbers. Because an improper fraction represents a number that is greater than or equal to 1, we have the following rule:

Property

Writing Improper Fractions as Mixed Numbers

An improper fraction can always be written as either a mixed number or a whole number.

To do this, remember that you can think of a fraction as indicating division. The numerator is divided by the denominator. This leads us to the following rule:

Step by Step

Writing an Improper Fraction as a Mixed Number

Step 1 Divide the numerator by the denominator.

Step 2 If there is a remainder, write the remainder over the original denominator.

Example 7 Writing a Fraction as a Mixed Number

< Objective 4 >

Write $\frac{17}{5}$ as a mixed number.

Divide 17 by 5.

$$5\overline{)17}\quad\text{quotient } 3,\quad \frac{15}{2}$$

$$\frac{17}{5} = 3\frac{2}{5}$$

Remainder

Original denominator

Quotient

NOTE

Both forms are correct, but in subsequent courses you will find that improper fractions are preferred to mixed numbers.

Check Yourself 7

Write $\frac{32}{5}$ as a mixed number.

In order to write a mixed number as an equivalent improper fraction, we write the whole-number part as an equivalent fraction with the same denominator as the fraction part. We then add the two fractions. This is illustrated in Example 8.

Example 8 Writing a Mixed Number as an Improper Fraction

(a) Write $3\frac{2}{5}$ as an equivalent improper fraction.

Because the fraction part has a denominator of 5, we write the whole-number part as a fraction with 5 as its denominator.

NOTE

With practice, you should be able to do this mentally.

$$3\frac{2}{5} \quad 3 \quad \frac{2}{5} \quad 3 \cdot \frac{5}{5} \quad \frac{2}{5} \qquad 3 \quad \frac{3}{1}$$

$$\frac{15}{5} \quad \frac{2}{5} \quad \frac{15 \quad 2}{5}$$

$$\frac{17}{5}$$

NOTE

Multiply the denominator, 7, by the whole number, 4, and add the numerator, 5.

(b) Write $4\frac{5}{7}$ as an improper fraction.

$$4\frac{5}{7} \quad \frac{(7 \cdot 4) \quad 5}{7} \quad \frac{33}{7}$$

Check Yourself 8

Write $5\frac{3}{8}$ as an improper fraction.

When multiplying two mixed numbers, it is usually easier to change the mixed numbers to improper fractions and then perform the multiplication. Example 9 illustrates this method.

Example 9 Multiplying Two Mixed Numbers

< Objective 5 >

Multiply.

$$3\frac{2}{3} \cdot 2\frac{1}{2} \quad \frac{11}{3} \cdot \frac{5}{2}$$ Change the mixed numbers to improper fractions.

$$\frac{11 \cdot 5}{3 \cdot 2} \quad \frac{55}{6} \quad 9\frac{1}{6}$$

> CAUTION

Be careful! Students sometimes think of

$$3\frac{2}{3} \cdot 2\frac{1}{2} \quad \text{as} \quad (3 \cdot 2) \quad \frac{2}{3} \cdot \frac{1}{2}$$

This is *not* the correct multiplication pattern. You must first change the mixed numbers to improper fractions.

Check Yourself 9

Multiply.

$$2\frac{1}{3} \cdot 3\frac{1}{2}$$

When dividing mixed numbers, simply write the mixed or whole numbers as improper fractions as the first step. Then proceed with the division. Example 10 illustrates this approach.

Example 10 Dividing Two Mixed Numbers

Divide.

$2\frac{3}{8} \quad 1\frac{3}{4} \quad \frac{19}{8} \quad \frac{7}{4}$ Write the mixed numbers as improper fractions.

$\frac{19}{\cancel{8}_{2}} \quad \frac{\cancel{4}^{1}}{7}$ Invert the divisor and multiply as before.

$\frac{19}{14} \quad 1\frac{5}{14}$

Check Yourself 10

Divide $3\frac{1}{5} \quad 2\frac{2}{5}$.

When adding or subtracting mixed numbers, first write the mixed numbers as improper fractions and then proceed as you would when adding or subtracting fractions. Example 11 illustrates these concepts.

Example 11 Adding and Subtracting Mixed Numbers

< Objective 6 >

NOTE

$$\begin{array}{r} 5 \\ 24\overline{)133} \\ \underline{120} \\ 13 \end{array}$$

(a) Add, and write the result as a mixed number.

$3\frac{1}{6} \quad 2\frac{3}{8} \quad \frac{19}{6} \quad \frac{19}{8}$ The LCD of the fractions is 24. Rename them with that denominator.

$\frac{76}{24} \quad \frac{57}{24}$ Then add as before.

$\frac{133}{24}$

$5\frac{13}{24}$

(b) Subtract.

$8\frac{7}{10} \quad 3\frac{3}{8} \quad \frac{87}{10} \quad \frac{27}{8}$ Write the fractions with denominator 40.

$\frac{348}{40} \quad \frac{135}{40}$ Subtract as before.

$\frac{213}{40}$

This can be written as $5\frac{13}{40}$.

Check Yourself 11

Perform the indicated operation. Write your result as a mixed number.

(a) $5\frac{7}{10} \quad 3\frac{5}{6}$ **(b)** $7\frac{11}{12} \quad 3\frac{5}{8}$

To subtract a mixed number from a whole number, we use the same techniques.

Example 12 Subtracting Mixed Numbers

Subtract.

$$6 \quad 2\frac{3}{4}$$

$$6 \quad 2\frac{3}{4} \quad \frac{24}{4} \quad \frac{11}{4}$$

$$\frac{13}{4}$$

Write both the whole number and the mixed number as improper fractions with a common denominator.

This can be written as $3\frac{1}{4}$.

NOTE

$6 \quad \frac{6}{1} \quad \frac{24}{4}$

Multiply the numerator and denominator by 4 to form a common denominator.

Check Yourself 12

Subtract 7 $3\frac{2}{5}$.

When adding mixed numbers, some students prefer to take advantage of the fact that a mixed number is the sum of a whole number and a fraction. To do this, add the whole-number parts and add the fraction parts separately, and then combine the two. You may need to simplify the fraction. Example 13 illustrates this.

Example 13 Adding Mixed Numbers

Add $6\frac{2}{5} \quad 4\frac{4}{5}$.

$$6\frac{2}{5} \quad 4\frac{4}{5} \quad 6 \quad 4 \quad \frac{2}{5} \quad \frac{4}{5}$$

$$10 \quad \frac{6}{5}$$

Add the whole-number and fraction parts separately.

$$10 \quad 1\frac{1}{5}$$

After simplifying the fraction, you may need to add more to the whole-number part.

$$11\frac{1}{5}$$

> **CAUTION**

Do not use this method to subtract mixed numbers.

Check Yourself 13

Add $4\frac{3}{4} \quad 3\frac{2}{3}$.

In algebra, we usually use improper fractions rather than mixed numbers, so when we need to add mixed numbers, we will generally write them as improper fractions and then add them following the procedure shown in Example 11(a).

There are many applications in which fractions and mixed numbers are used. Examples 14 and 15 illustrate some of these.

Example 14 An Application of Fractions and Mixed Numbers

NOTE

The word *of* usually indicates multiplication.

Chair rail molding 136 inches (in.) long must be cut into pieces of $31\frac{1}{3}$ in. each. How many pieces can be cut from the molding?

$$136 \quad 31\frac{1}{3} \quad 136 \quad \frac{94}{3} \quad \frac{136}{1} \quad \frac{94}{3} \quad \frac{136}{1} \cdot \frac{3}{94} \quad \frac{408}{94} \quad \frac{204}{47} \quad 4\frac{16}{47}$$

So four full-length pieces can be cut from the molding.

Check Yourself 14

After a family party, $10\frac{2}{3}$ cupcakes were left. If Amanda took $\frac{3}{8}$ of these, how many did she take?

Example 15 An Application of Fractions and Mixed Numbers

José must trim $2\frac{5}{16}$ feet (ft) from a board 8 ft long. How long will the board be after it is cut?

$$8 \quad 2\frac{5}{16} \quad 7\frac{16}{16} \quad 2\frac{5}{16} \quad 5\frac{11}{16}$$

The board will be $5\frac{11}{16}$ ft long after it is cut.

Check Yourself 15

Three pieces of lumber measure $5\frac{3}{8}$ ft, $7\frac{1}{2}$ ft, and $9\frac{3}{4}$ ft. What is the total length of the lumber?

Check Yourself ANSWERS

1. Answers will vary. **2.** **(a)** $\frac{7}{11}$; **(b)** $\frac{1}{2}$; **(c)** $\frac{2}{9}$ **3.** **(a)** $\frac{6}{7}$; **(b)** 4
4. $\frac{15}{2}$ **5.** **(a)** $\frac{71}{45}$; **(b)** $\frac{11}{10}$ **6.** $\frac{7}{24}$ **7.** $6\frac{2}{5}$ **8.** $\frac{43}{8}$
9. $\frac{49}{6}$ or $8\frac{1}{6}$ **10.** $\frac{4}{3}$ or $1\frac{1}{3}$ **11.** **(a)** $\frac{143}{15}$ or $9\frac{8}{15}$; **(b)** $\frac{103}{24}$ or $4\frac{7}{24}$
12. $\frac{18}{5}$ or $3\frac{3}{5}$ **13.** $8\frac{5}{12}$ **14.** 4 **15.** $22\frac{5}{8}$ ft

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.2

(a) The numerator of a ____________ fraction is less than the denominator.

(b) Two fractions that represent the same quantity are called ____________ fractions.

(c) The ____________ of a set of fractions is the same as the LCM of their denominators.

(d) A mixed number is the sum of a whole number and a proper ____________.

0.2 exercises

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Name ____________

Section ________ Date ________

Answers

1. ____________
2. ____________
3. ____________
4. ____________
5. ____________
6. ____________
7. ____________
8. ____________

9.	10.
11.	12.
13.	14.
15.	16.
17.	18.
19.	20.
21.	22.

< Objective 1 >

Give three equivalent fractions for each given fraction.

1. $\frac{3}{7}$

2. $\frac{4}{9}$

3. $\frac{7}{8}$

4. $\frac{11}{13}$

5. $\frac{10}{17}$

6. $\frac{9}{16}$

7. $\frac{6}{11}$

8. $\frac{15}{16}$

Write each fraction in simplest form.

9. $\frac{8}{12}$

10. $\frac{12}{15}$

11. $\frac{10}{14}$

12. $\frac{15}{50}$

13. $\frac{12}{18}$

14. $\frac{28}{35}$

15. $\frac{35}{40}$

16. $\frac{21}{24}$

17. $\frac{11}{44}$

18. $\frac{10}{25}$

19. $\frac{12}{36}$

20. $\frac{18}{48}$

21. $\frac{48}{60}$

22. $\frac{48}{66}$

23. $\frac{105}{135}$ > Videos

24. $\frac{54}{126}$

25. $\frac{15}{44}$

26. $\frac{10}{63}$

< Objective 2 >

Multiply. Be sure to simplify each product.

27. $\frac{3}{4} \cdot \frac{7}{5}$

28. $\frac{2}{3} \cdot \frac{8}{5}$

29. $\frac{3}{5} \cdot \frac{5}{7}$

30. $\frac{6}{11} \cdot \frac{8}{6}$

31. $\frac{6}{13} \cdot \frac{4}{9}$

32. $\frac{5}{9} \cdot \frac{6}{11}$

33. $\frac{3}{11} \cdot \frac{7}{9}$

34. $\frac{3}{10} \cdot \frac{5}{9}$

Divide. Write each result in simplest form.

35. $\frac{5}{21} \quad \frac{25}{14}$

36. $\frac{1}{5} \quad \frac{3}{4}$

37. $\frac{2}{5} \quad \frac{1}{3}$

38. $\frac{8}{9} \quad \frac{4}{3}$

39. $\frac{8}{9} \quad \frac{11}{15}$

40. $\frac{8}{15} \quad \frac{2}{5}$

41. $\frac{5}{27} \quad \frac{15}{54}$ > Videos

42. $\frac{5}{27} \quad \frac{25}{36}$

< Objective 3 >

Add.

43. $\frac{2}{5} \quad \frac{1}{4}$

44. $\frac{2}{3} \quad \frac{3}{10}$

45. $\frac{2}{5} \quad \frac{7}{15}$

46. $\frac{3}{10} \quad \frac{7}{12}$

Answers

23. ______ 24. ______

25. ______ 26. ______

27. ______ 28. ______

29. ______ 30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

39. ______

40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

46. ______

 The Streeter/Hutchison Series in Mathematics Beginning Algebra

Answers

47. ______ 48. ______

49. ______ 50. ______

51. ______ 52. ______

53. ______ 54. ______

55. ______ 56. ______

57. ______

58. ______

59. ______

60. ______

61. ______

62. ______

63. ______

64. ______

65. ______

66. ______

67. ______

68. ______

69. ______

70. ______

47. $\frac{3}{8} \quad \frac{5}{12}$

48. $\frac{5}{36} \quad \frac{7}{24}$

49. $\frac{2}{15} \quad \frac{9}{20}$

50. $\frac{9}{14} \quad \frac{10}{21}$

51. $\frac{7}{15} \quad \frac{13}{18}$

52. $\frac{12}{25} \quad \frac{19}{30}$

53. $\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$

54. $\frac{1}{3} \quad \frac{1}{5} \quad \frac{1}{10}$

Subtract.

55. $\frac{8}{9} \quad \frac{3}{9}$

56. $\frac{9}{10} \quad \frac{6}{10}$

57. $\frac{5}{8} \quad \frac{1}{8}$

58. $\frac{11}{12} \quad \frac{7}{12}$

59. $\frac{7}{8} \quad \frac{2}{3}$ > Videos

60. $\frac{5}{6} \quad \frac{3}{5}$

61. $\frac{11}{18} \quad \frac{2}{9}$

62. $\frac{5}{6} \quad \frac{1}{4}$

< Objective 4 >

Write the following fractions as mixed numbers.

63. $\frac{17}{4}$

64. $\frac{200}{11}$

Write the following mixed numbers as fractions.

65. $3\frac{1}{4}$

66. $6\frac{3}{4}$

< Objectives 5–6 >

Perform the indicated operations.

67. $2\frac{2}{9} \quad 3\frac{5}{9}$

68. $5\frac{2}{9} \quad 6\frac{4}{9}$

69. $1\frac{1}{3} \quad 2\frac{1}{5}$

70. $2\frac{1}{4} \quad 1\frac{1}{6}$

71. $3\frac{2}{5} - 1\frac{4}{5}$ Videos

72. $5\frac{3}{7} - 2\frac{1}{7}$

73. $3\frac{2}{3} - 2\frac{1}{4}$

74. $5\frac{4}{5} - 1\frac{1}{6}$

75. $2\frac{2}{5} \cdot 3\frac{3}{4}$

76. $2\frac{2}{7} \cdot 2\frac{1}{3}$

77. $3\frac{1}{2} \div 2\frac{4}{5}$

78. $3\frac{3}{4} \div 1\frac{3}{8}$ Videos

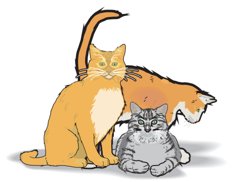
Solve the following applications.

79. **Crafts** Roseann is making shirts for her three children. One shirt requires $\frac{1}{2}$ yard (yd) of material, a second shirt requires $\frac{1}{3}$ yd of material, and the third shirt requires $\frac{1}{4}$ yd of material. How much material is required for all three shirts?

80. **Science** José rode his trail bike for 10 miles. Two-thirds of the distance was over a mountain trail. How long is the mountain trail?

81. **Business and Finance** You make $240 a day on a job. What will you receive for working $\frac{2}{3}$ of a day?

82. **Statistics** A survey has found that $\frac{3}{4}$ of the people in a city own pets. Of those who own pets, $\frac{2}{3}$ have cats. What fraction of those surveyed own cats?



83. **Social Science** The scale on a map is 1 in. = 200 miles (mi). What actual distance, in miles, does $\frac{3}{8}$ in. represent?

84. **Business and Finance** A family uses $\frac{2}{5}$ of its monthly income for housing and utilities on average. If the family's monthly income is $1,750, what is spent for housing and utilities? What amount remains?

Answers

71. ____
72. ____
73. ____
74. ____
75. ____
76. ____
77. ____
78. ____
79. ____
80. ____
81. ____
82. ____
83. ____
84. ____

Answers

85. ______

86. ______

87. ______

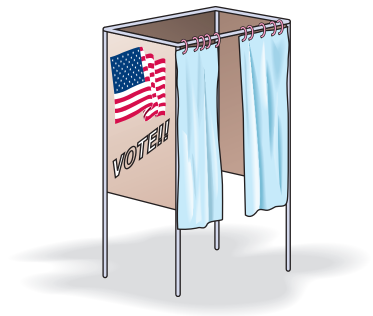
88. ______

89. ______

90. ______

91. ______

85. **Social Science** Of the eligible voters in an election, $\frac{3}{4}$ were registered. Of those registered, $\frac{5}{9}$ actually voted. What fraction of those people who were eligible voted?



86. **Statistics** A survey has found that $\frac{7}{10}$ of the people in a city own pets. Of those who own pets, $\frac{2}{3}$ have dogs. What fraction of those surveyed own dogs?

87. **Science** A jet flew at an average speed of 540 mi/h on a $4\frac{2}{3}$-h flight. What was the distance flown?

88. **Geometry** A piece of land that has $11\frac{2}{3}$ acres is being subdivided for home lots. It is estimated that $\frac{2}{7}$ of the area will be used for roads. What amount remains to be used for lots?

89. **Geometry** To find the approximate circumference or distance around a circle, we multiply its diameter by $\frac{22}{7}$. What is the circumference of a circle with a diameter of 21 in.?

90. **Geometry** The length of a rectangle is $\frac{6}{7}$ yd, and its width is $\frac{21}{26}$ yd. What is its area in square yards? (The area of a rectangle is the product of its length and its width.)

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

91. Every fraction (rational number) has a corresponding decimal form that either terminates or repeats. For example, $\frac{5}{16}$ 0.3125 (the decimal form terminates), and $\frac{4}{11}$ 0.363636 . . . (the decimal form repeats). Investigate a number of fractions to determine which ones terminate and which ones repeat. (Hint: You can focus on the denominator; study the prime factorizations of several denominators.)

92. Complete the following sums:

$$\frac{1}{2} \quad \frac{1}{4}$$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8}$$

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16}$$

Based on these, predict the sum:

$$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{16} \quad \frac{1}{32} \quad \frac{1}{64} \quad \frac{1}{128}$$

chapter 0 > Make the Connection

Answers

92. ____________

Answers

For exercises 1–7, answers will vary.

1. $\frac{6}{14}, \frac{9}{21}, \frac{12}{28}$ **3.** $\frac{14}{16}, \frac{35}{40}, \frac{70}{80}$ **5.** $\frac{20}{34}, \frac{30}{51}, \frac{100}{170}$ **7.** $\frac{12}{22}, \frac{18}{33}, \frac{24}{44}$

9. $\frac{2}{3}$ **11.** $\frac{5}{7}$ **13.** $\frac{2}{3}$ **15.** $\frac{7}{8}$ **17.** $\frac{1}{4}$ **19.** $\frac{1}{3}$ **21.** $\frac{4}{5}$

23. $\frac{7}{9}$ **25.** $\frac{15}{44}$ **27.** $\frac{21}{20}$ **29.** $\frac{3}{7}$ **31.** $\frac{8}{39}$ **33.** $\frac{7}{33}$ **35.** $\frac{2}{15}$

37. $\frac{6}{5}$ **39.** $\frac{40}{33}$ **41.** $\frac{2}{3}$ **43.** $\frac{13}{20}$ **45.** $\frac{13}{15}$ **47.** $\frac{19}{24}$ **49.** $\frac{7}{12}$

51. $\frac{107}{90}$ **53.** $\frac{7}{8}$ **55.** $\frac{5}{9}$ **57.** $\frac{1}{2}$ **59.** $\frac{5}{24}$ **61.** $\frac{7}{18}$ **63.** $4\frac{1}{4}$

65. $\frac{13}{4}$ **67.** $5\frac{7}{9}$ **69.** $3\frac{8}{15}$ **71.** $1\frac{3}{5}$ **73.** $1\frac{5}{12}$ **75.** 9 **77.** $1\frac{1}{4}$

79. $1\frac{1}{12}$ yd **81.** $160 **83.** 75 mi **85.** $\frac{5}{12}$ **87.** 2,520 mi

89. 66 in. **91.** Above and Beyond

0.3 Decimals and Percents

< **0.3 Objectives** >

1 > Write a fraction as a decimal

2 > Write a decimal as a fraction

3 > Add and subtract decimals

4 > Multiply and divide decimals

5 > Write a percent as a fraction or decimal

6 > Write a decimal or fraction as a percent

Because a fraction can be interpreted as division, we can divide the numerator of a fraction by its denominator to write the fraction as an equivalent decimal. The result is called its **decimal equivalent.**

Example 1 — Writing a Fraction as a Decimal

< Objective 1 >

RECALL

5 can be written as 5.0, 5.00, 5.000, and so on. In this case, we continue the division by adding zeros to the dividend until a 0 remainder is reached.

Write $\frac{5}{8}$ as a decimal.

$$\begin{array}{r}
0.625 \\
8\,\overline{)\,5.000} \\
\underline{4\;8} \\
20 \\
\underline{16} \\
40 \\
\underline{40} \\
0
\end{array}$$

Because $\frac{5}{8}$ means 5   8, divide 8 into 5.

We see that $\frac{5}{8}$ 0.625; 0.625 is the decimal equivalent of $\frac{5}{8}$.

Check Yourself 1

Find the decimal equivalent of $\frac{7}{8}$.

You should recall that the decimal 0.625 in Example 1 means $\frac{625}{1{,}000}$. This is equivalent to $\frac{625}{1{,}000} \cdot \frac{10}{10}$ $\frac{6{,}250}{10{,}000}$ 0.6250.

We could choose to round our answer to some specific decimal place. If we round to the nearest tenth, then we are rounding to one decimal place 0.625   0.6 (to the nearest tenth), whereas if we rounded to the nearest hundredth, we are rounding to two decimal places, 0.625   0.63 (to the nearest hundredth).

When a decimal does not terminate, we usually round it to a specific place, as in Example 2.

Example 2 Writing a Fraction as a Decimal

Write $\frac{3}{7}$ as a decimal. Round the answer to the nearest thousandth.

$$\begin{array}{r} 0.4285 \\ 7\overline{)3.0000} \\ \underline{2\ 8} \\ 20 \\ \underline{14} \\ 60 \\ \underline{56} \\ 40 \\ \underline{35} \\ 5 \end{array}$$

In this example, we are choosing to round to three decimal places, so we must add enough zeros to carry the division to four decimal places.

So $\frac{3}{7}$ 0.429 (to the nearest thousandth).

Check Yourself 2

Find the decimal equivalent of $\frac{5}{12}$ to the nearest thousandth.

If the decimal equivalent of a fraction does *not* terminate, it will *repeat* a sequence of digits. These decimals are called **repeating decimals.**

Example 3 Writing a Fraction as a Repeating Decimal

Write $\frac{5}{11}$ as a decimal.

$$\begin{array}{r} 0.4545 \\ 11\overline{)5.0000} \\ \underline{4\ 4} \\ 60 \\ \underline{55} \\ 5 \rightarrow 50 \\ \underline{44} \\ 60 \\ \underline{55} \\ 5 \end{array}$$

As soon as a remainder repeats itself, as 5 does here, the pattern of digits will repeat in the quotient.

$\frac{5}{11}$ 0.4545 . . .

$0.\overline{45}$

Check Yourself 3

Use the bar notation to write the decimal equivalent of $\frac{5}{7}$. (Be patient. You have to divide for a while to find the repeating pattern.)

To write a decimal as a fraction, write the decimal without the decimal point. This is the **numerator** of the fraction. The **denominator** of the fraction is a 1 followed by as many zeros as there are places in the decimal. The next two examples illustrate this process.

Example 4 Writing a Decimal as a Fraction

< Objective 2 >

$0.7 = \frac{7}{10}$ One place, One zero

$0.09 = \frac{9}{100}$ Two places, Two zeros

$0.257 = \frac{257}{1{,}000}$ Three places, Three zeros

Check Yourself 4

Write as fractions.

(a) 0.3 (b) 0.311

When a decimal is written as an equivalent fraction, the common fraction that results should be simplified.

Example 5 Converting a Decimal to a Fraction

NOTE

Divide the numerator and denominator of $\frac{395}{1{,}000}$ by 5.

Convert 0.395 to a fraction and simplify the result.

$$0.395 = \frac{395}{1{,}000} = \frac{79}{200}$$

Check Yourself 5

Write 0.275 as a fraction.

We add and subtract decimals using place value, just as we do with whole numbers. You must be sure to align the decimal points, as illustrated in Example 6.

Example 6 Adding or Subtracting Two Decimals

< Objective 3 >

Perform the indicated operation.

(a) Add 2.356 and 15.6.

Aligning the decimal points, we get

$$\begin{array}{r} 2.356 \\ 15.600 \\ \hline 17.956 \end{array}$$

Although the zeros are not necessary, they ensure proper alignment.

RECALL

We borrowed from the tenths place, just as we would with whole numbers.

$$\begin{array}{r} \overset{7}{\cancel{8}}.\overset{0\,1}{\cancel{1}}0 \\ 3.84 \\ \hline 4.26 \end{array}$$

(b) Subtract 3.84 from 8.1.

Again, we align the decimal points.

$$\begin{array}{r} 8.10 \\ 3.84 \\ \hline 4.26 \end{array}$$

When subtracting, always add zeros so that the right columns line up.

Check Yourself 6

Perform the indicated operation.

(a) 34.76 2.419 **(b)** 71.82 8.197

Example 7 illustrates the multiplication of two decimal fractions.

Example 7 Multiplying Two Decimals

< Objective 4 >

RECALL

Multiply as you would with whole numbers. The final result is given the same number of decimal places as the total number of decimal places in both factors.

Multiply 4.6 and 3.27.

$$\begin{array}{r} 4.6 \\ 3.27 \\ \hline 322 \\ 920 \\ 13800 \\ \hline 15.042 \end{array}$$

It is not necessary to align decimals being multiplied. Note that the two factors have a total of three digits to the right of the decimal point.

The decimal point of the product is moved three digits to the left.

Check Yourself 7

Multiply 5.8 and 9.62.

Dividing decimals is a bit trickier. In order to divide when the divisor is a decimal, we multiply both the dividend and divisor by a large enough power of 10 that the divisor becomes a whole number. We show you how to set this up in Example 8.

Example 8 Rewriting a Problem That Requires Dividing by a Decimal

NOTE

It is always easier to rewrite a division problem so that you're dividing by a whole number. Dividing by a whole number makes it easy to place the decimal point in the quotient.

Rewrite the division so that the divisor is a whole number.

2.57 3.4

$\dfrac{2.57}{3.4}$ Write the division as a fraction.

$\dfrac{2.57 \quad 10}{3.4 \quad 10}$ We multiply the numerator and denominator by 10 so that the divisor is a whole number. This *does not change* the value of the fraction.

$\dfrac{25.7}{34}$ Multiplying by 10, shift the decimal point in the numerator and denominator *one place to the right*.

25.7 34 Our division problem is rewritten so that the divisor is a whole number.

So

2.57   3.4   25.7   34

After we multiply the numerator and denominator by 10, we see that 2.57   3.4 is the same as 25.7   34.

Check Yourself 8

Rewrite the division problem so that the divisor is a whole number.

3.42   2.5

NOTE

Of course, multiplying by any whole-number power of 10 greater than 1 is just a matter of shifting the decimal point to the right.

Do you see the rule suggested by Example 8? We multiplied the numerator and the denominator (the dividend and the divisor) by 10. We made the divisor a whole number without altering the actual digits involved. All we did was shift the decimal point in the divisor and dividend the same number of places. This leads us to the following procedure.

Step by Step

To Divide by a Decimal

Step 1 Move the decimal point in the divisor *to the right,* making the divisor a whole number.

Step 2 Move the decimal point in the dividend to the right *the same number of places.* Add zeros if necessary.

Step 3 Place the decimal point in the quotient directly above the decimal point of the dividend.

Step 4 Divide as you would with whole numbers.

Here is an example using the division rule.

Example 9 Rounding the Result of Dividing by a Decimal

Divide 1.573 by 0.48 and give the quotient to the nearest tenth.

Write

0.48 $\overline{)1.57\,3}$

Shift the decimal points two places to the right to make the divisor a whole number.

Now divide:

```
      3.27
48 )157.30
    144
     13 3
      9 6
      3 70
      3 36
        34
```

Add a 0 to carry the division to the hundredths place. In this case, we want to find the quotient to the nearest tenth.

NOTE

Once the division statement is rewritten, place the decimal point in the quotient above that in the dividend.

Round 3.27 to 3.3. So

1.573   0.48   3.3   (to the nearest tenth)

Check Yourself 9

Divide, rounding the quotient to the nearest tenth.

3.4   1.24

We have used fractions and decimals to name parts of a whole. **Percents** can also be used to accomplish this. The word *percent* means "for each hundred." We can think of percents as fractions whose denominators are 100. So 25% can be written as $\frac{25}{100}$ or, in simplified form, $\frac{1}{4}$.

Because there are different ways of naming the parts of a whole, you need to know how to change from one of these ways to another. First, we look at writing a percent as a fraction. Because a percent is a fraction or a ratio with denominator 100, we can use the following rule.

Property

Writing a Percent as a Fraction

To write a percent as a fraction, replace the percent symbol with $\frac{1}{100}$.

We use this rule in Example 10.

Example 10 Writing a Percent as a Fraction

< Objective 5 >

Write each percent as a fraction.

(a) 7% $7 \quad \frac{1}{100} \quad \frac{7}{100}$

(b) 25% $25 \quad \frac{1}{100} \quad \frac{25}{100} \quad \frac{1}{4}$ Always simplify fractions.

Check Yourself 10

Write 12% as a fraction.

RECALL

$\frac{1}{100} \quad 0.01$

In Example 10, we wrote percents as fractions by replacing the percent sign with $\frac{1}{100}$ and multiplying. How do we convert percents when we are working with decimals? Just move the decimal point two places to the left. This gives us a second rule for rewriting percents.

Property

Writing a Percent as a Decimal

To write a percent as a decimal, replace the percent symbol with $\frac{1}{100}$. As a result of multiplying by $\frac{1}{100}$, the decimal point will move two places to the left.

Example 11 Writing a Percent as a Decimal

RECALL

Multiplying by $\frac{1}{100}$ is the same as dividing by 100.

Write each percent as a decimal.

(a) 25% $\quad 25\,\frac{1}{100} \quad 0.25$ The decimal point in 25% is understood to be after the 5.

(b) 4.5% $\quad 4.5\,\frac{1}{100} \quad 0.045$ We must add a zero to move the decimal point.

(c) 130% $\quad 130\,\frac{1}{100} \quad 1.30$

NOTE

A percent greater than 100 gives a decimal greater than 1.

Check Yourself 11

Write as decimals.

(a) 5% (b) 3.9% (c) 115%

Writing a decimal as a percent is the opposite of writing a percent as a decimal. We simply reverse the process. Here is the rule:

Property

Writing a Decimal as a Percent

To write a decimal as a percent, move the decimal point *two* places to the *right* and attach the percent symbol.

Example 12 Writing a Decimal as a Percent

< Objective 6 >

Write each decimal as a percent.

(a) 0.18 18%

(b) 0.03 3%

(c) 1.25 125%

Check Yourself 12

Write each decimal as a percent.

(a) 0.27 (b) 0.045 (c) 1.3

The following rule allows us to write fractions as percents.

Property

Writing a Fraction as a Percent

To write a fraction as a percent, write the decimal equivalent of the fraction by dividing. Then, move the decimal point two places to the right and attach the percent symbol.

Example 13 Writing a Fraction as a Percent

Write each fraction as a percent.

(a) $\frac{3}{5}$ 0.60 To find the decimal equivalent, just divide the denominator into the numerator.

Now write the percent.

$\frac{3}{5}$ 0.60 60%

(b) $\frac{1}{8}$ 0.125 12.5% or $12\frac{1}{2}$%

(c) $\frac{1}{3}$ $0.\overline{3}$ $0.33\overline{3}$ $33.\overline{3}$% or $33\frac{1}{3}$%

Check Yourself 13

Change each fraction to a percent equivalent.

(a) $\frac{3}{4}$ (b) $\frac{3}{8}$ (c) $\frac{2}{3}$

Example 14 illustrates one of the many applications using decimals.

Example 14 An Application of Decimals

NOTE

In most applications

Lucretia's car gets approximately 20 miles per gallon (mi/gal) of fuel. If 1 gal of fuel costs $1.93, how much does it cost her to drive 125 mi?

125 20 6.25 gal

6.25 · 1.93 $12.06 (rounded)

Check Yourself 14

The art department has a budget of $195.75 to purchase art supplies. After purchasing 35 paintbrushes for $1.92 each, six jars of paint remover for $0.93 each, and four cans of blue paint for $2.95 each, how much money was left in the budget?

Check Yourself ANSWERS

1. 0.875 **2.** 0.417 **3.** $0.\overline{714285}$ **4.** **(a)** $\frac{3}{10}$; **(b)** $\frac{311}{1{,}000}$ **5.** $\frac{11}{40}$
6. **(a)** 37.179; **(b)** 63.623 **7.** 55.796 **8.** 34.2 25 **9.** 2.7
10. $\frac{12}{100}$ or $\frac{3}{25}$ **11.** **(a)** 0.05; **(b)** 0.039; **(c)** 1.15 **12.** **(a)** $\frac{27}{100}$ 27%;
(b) 4.5%; **(c)** 130% **13.** **(a)** 75%; **(b)** 37.5%; **(c)** $66.\overline{6}\%$ or $66\frac{2}{3}\%$
14. $111.17

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.3

(a) To write a fraction as a ______________, divide the numerator by the denominator.

(b) We use bar notation to indicate a ______________ decimal.

(c) Fractions, decimals, and ______________ are all ways of naming parts of a whole.

(d) To write a percent as a decimal, move the decimal two places to the ______________, and remove the percent symbol.

0.3 exercises

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Name ____________

Section ________ Date ________

< Objective 1 >

Find the decimal equivalents for each of the following fractions.

1. $\frac{3}{4}$ **2.** $\frac{4}{5}$

3. $\frac{9}{20}$ **4.** $\frac{3}{10}$

5. $\frac{1}{5}$ **6.** $\frac{1}{8}$

7. $\frac{5}{16}$ **8.** $\frac{11}{20}$

9. $\frac{7}{10}$ **10.** $\frac{7}{16}$

11. $\frac{27}{40}$ **12.** $\frac{17}{32}$

Find the decimal equivalents rounded to the indicated place.

13. $\frac{5}{6}$; thousandth **14.** $\frac{7}{12}$; hundredth

15. $\frac{4}{15}$; thousandth

Write the decimal equivalents using the bar notation.

16. $\frac{1}{18}$ **17.** $\frac{4}{9}$

18. $\frac{3}{11}$ > Videos

< Objective 2 >

Write each of the following as a fraction and simplify.

19. 0.9 **20.** 0.3

Answers

1. ________ 2. ________
3. ________ 4. ________
5. ________ 6. ________
7. ________ 8. ________
9. ________ 10. ________
11. ________ 12. ________
13. ________
14. ________
15. ________
16. ________
17. ________
18. ________
19. ________
20. ________

Answers

21. ____
22. ____
23. ____
24. ____
25. ____
26. ____
27. ____
28. ____
29. ____
30. ____
31. ____ 32. ____
33. ____ 34. ____
35. ____ 36. ____
37. ____ 38. ____
39. ____ 40. ____
41. ____ 42. ____
43. ____ 44. ____
45. ____ 46. ____
47. ____ 48. ____
49. ____ 50. ____

21. 0.8 **22.** 0.6

23. 0.37 **24.** 0.97

25. 0.587 **26.** 0.379

27. 0.48 > Videos **28.** 0.75

29. 0.58 **30.** 0.65

< Objectives 3–4 >

Perform the indicated operations.

31. 7.1562 14.78 **32.** 6.2358 3.14

33. 11.12 8.3792 **34.** 6.924 5.2

35. 9.20 2.85 **36.** 17.345 11.12

37. 18.234 13.64 **38.** 21.983 9.395

39. 3.21 2.1 **40.** 15.6 7.123

41. 6.29 9.13 > Videos **42.** 8.245 3.1

Divide.

43. 16.68 6 **44.** 43.92 8

45. 1.92 4 **46.** 5.52 6

47. 5.48 8 **48.** 2.76 8

49. 13.89 6 **50.** 21.92 5

51. 185.6   32

52. 165.6   36

53. 79.9   34

54. 179.3   55

55. $52\overline{)\,13.78}$

56. $76\overline{)\,26.22}$

57. $0.6\overline{)\,11.07}$

58. $0.8\overline{)\,10.84}$

59. $3.8\overline{)\,7.22}$

60. $2.9\overline{)\,13.34}$

61. $5.2\overline{)\,11.622}$

62. $6.4\overline{)\,3.616}$

< Objective 5 >

Write as fractions.

63. 6%

64. 17%

65. 75%

66. 20%

67. 65%

68. 48%

69. 50%

70. 52%

71. 46%

72. 35%

73. 66%

74. 4%

Write as decimals.

75. 20%

76. 70%

77. 35%

78. 75%

Answers

51. ____ 52. ____
53. ____ 54. ____
55. ____ 56. ____
57. ____ 58. ____
59. ____ 60. ____
61. ____ 62. ____
63. ____ 64. ____
65. ____
66. ____
67. ____
68. ____
69. ____
70. ____
71. ____
72. ____
73. ____
74. ____
75. ____
76. ____
77. ____
78. ____

Answers

79. ____ 80. ____
81. ____ 82. ____
83. ____ 84. ____
85. ____ 86. ____
87. ____
88. ____
89. ____
90. ____
91. ____
92. ____
93. ____
94. ____
95. ____
96. ____
97. ____
98. ____
99. ____
100. ____
101. ____
102. ____

79. 39% **80.** 27%

81. 5% **82.** 7%

83. 135% **84.** 250%

85. 240% **86.** 160%

< Objective 6 >

Write each decimal as a percent.

87. 4.40 **88.** 5.13

89. 0.065 **90.** 0.095

91. 0.025 **92.** 0.085

93. 0.002 **94.** 0.008

Write each fraction as a percent.

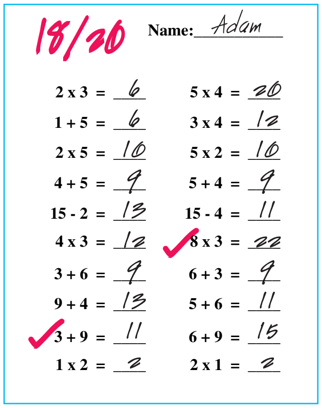
95. $\frac{1}{4}$ **96.** $\frac{4}{5}$

97. $\frac{2}{5}$ **98.** $\frac{1}{2}$

99. $\frac{1}{5}$ **100.** $\frac{3}{4}$

101. $\frac{5}{8}$ **102.** $\frac{7}{8}$ > Videos

103. **Statistics** On a math quiz, Adam answered 18 of 20 questions correctly, or $\frac{18}{20}$ of the quiz. Write the decimal equivalent of this fraction.



104. **Statistics** In a weekend baseball tournament, Joel had 4 hits in 13 times at bat. That is, he hit safely $\frac{4}{13}$ of the time. Write the decimal equivalent for Joel's hitting, rounding to three decimal places. (That number is Joel's batting average.)

105. **Business and Finance** A restaurant bought 50 glasses at a cost of \$39.90. What was the cost per glass, to the nearest cent?

106. **Business and Finance** The cost of a case of 48 items is \$28.20. What is the cost of an individual item, to the nearest cent?

107. **Business and Finance** An office bought 18 handheld calculators for \$284. What was the cost per calculator, to the nearest cent?

108. **Business and Finance** Al purchased a new refrigerator that cost \$736.12 with interest included. He paid \$100 as a down payment and agreed to pay the remainder in 18 monthly payments. What amount will he be paying per month?

109. **Business and Finance** The cost of a television set with interest is \$490.64. If you make a down payment of \$50 and agree to pay the balance in 12 monthly payments, what will be the amount of each monthly payment?

Answers

1. 0.75 **3.** 0.45 **5.** 0.2 **7.** 0.3125 **9.** 0.7 **11.** 0.675
13. 0.833 **15.** 0.267 **17.** $0.\overline{4}$ **19.** $\frac{9}{10}$ **21.** $\frac{4}{5}$ **23.** $\frac{37}{100}$

Answers

103. ____________

104. ____________

105. ____________

106. ____________

107. ____________

108. ____________

109. ____________

25. $\frac{587}{1,000}$ 27. $\frac{12}{25}$ 29. $\frac{29}{50}$ 31. 21.9362 33. 19.4992
35. 6.35 37. 4.594 39. 6.741 41. 57.4277 43. 2.78
45. 0.48 47. 0.685 49. 2.315 51. 5.8 53. 2.35
55. 0.265 57. 18.45 59. 1.9 61. 2.235 63. $\frac{3}{50}$ 65. $\frac{3}{4}$
67. $\frac{13}{20}$ 69. $\frac{1}{2}$ 71. $\frac{23}{50}$ 73. $\frac{33}{50}$ 75. 0.2 77. 0.35
79. 0.39 81. 0.05 83. 1.35 85. 2.4 87. 440% 89. 6.5%
91. 2.5% 93. 0.2% 95. 25% 97. 40% 99. 20%
101. 62.5% 103. 0.9 105. \$0.80 or 80¢ 107. \$15.78
109. \$36.72

0.4 Exponents and the Order of Operations

< 0.4 Objectives >

1 > Write a product of factors in exponential form

2 > Evaluate an expression involving several operations

NOTE

5 5 5 15
and
3 5 15

RECALL

A factor is a number or a variable that is being multiplied by another number or variable.

Often in mathematics we define symbols that allow us to write a mathematical statement in a more compact or "shorthand" form. This is an idea that you have encountered before. For example, the repeated addition

5 5 5

can be rewritten as

3 5

Thus, multiplication is shorthand for repeated addition.

In algebra, we frequently have a number or variable that is repeated as a factor in an expression several times. For instance, we might have

5 5 5

To abbreviate this product, we write

5 5 5 5^3

This is called **exponential notation** or **exponential form.** The exponent or power, here 3, indicates the number of times that the factor or base, here 5, appears in a product.

Exponent or power

5 5 5 5^3

Factor or base

This is read "5 to the third power" or "5 cubed."

> CAUTION

5^3 is *not* the same as 5 3.
5^3 5 5 5 125 and
5 3 15.

Example 1 Writing Products in Exponential Form

< Objective 1 >

Write 3 3 3 3 using exponential form.

The number 3 appears four times in the product, so

Four factors of 3

3 3 3 3 3^4

This is read "3 to the fourth power."

Check Yourself 1

Rewrite each expression using exponential form.

(a) 4 4 4 4 4 4 **(b)** 7 7 7 7

> **CAUTION**
>
> Only one of these results can be correct.

To evaluate an arithmetic expression, you need to know the order in which the operations are done. To see why, simplify the expression 5 2 3.

Method 1	or	*Method 2*
5 2 3		5 2 3
Add first.		Multiply first.
7 3		5 6
21		11

> **NOTE**
>
> Parentheses, brackets, and fraction bars are all examples of grouping symbols. You will learn other grouping symbols in later chapters.

Because we get different answers depending on how we do the problem, the language of mathematics would not be clear if there were no agreement on which method is correct. The following rules tell us the order in which operations should be done.

Step by Step: The Order of Operations

Step 1 Evaluate all expressions inside grouping symbols first.
Step 2 Evaluate all expressions involving exponents.
Step 3 Do any multiplication or division in order, working from left to right.
Step 4 Do any addition or subtraction in order, working from left to right.

Example 2 Evaluating Expressions

< Objective 2 >

Evaluate 5 2 3.

There are no parentheses or exponents, so start with step 3: First multiply and then add.

5 2 3 ← Multiply first.

5 6 ← Then add.

11

> **NOTE**
>
> Method 2 shown above is the correct one.

Check Yourself 2

Evaluate the following expressions.

(a) 20 3 4 **(b)** 9 6 3

When there are no parentheses, evaluate the exponents first.

Example 3 Evaluating Expressions

Evaluate $5 \quad 3^2$.

$5 \quad 3^2 \quad 5 \quad 9$

Evaluate the power first.

45

Check Yourself 3

Evaluate $4 \quad 2^4$.

Both scientific and graphing calculators correctly interpret the order of operations, as demonstrated in Example 4.

Example 4 Using a Calculator to Evaluate Expressions

Use your scientific or graphing calculator to evaluate each expression. Round the answer to the nearest tenth.

(a) $24.3 \quad 6.2 \quad 3.53$

When evaluating expressions by hand, you must consider the order of operations. In this case, the multiplication must be done first, and then the addition. With a calculator, you need only enter the expression correctly. The calculator is programmed to follow the order of operations.

> Calculator

Entering 24.3 ☐ 6.2 ☐ 3.53 ENTER

yields the evaluation 46.186. Rounding to the nearest tenth, we have 46.2.

(b) $2.45^3 \quad 49 \quad 8{,}000 \quad 12.2 \quad 1.3$

Some calculators use the caret (^) to designate powers. Others use the symbol x^y (or y^x).

Entering 2.45 ^ 3 ☐ 49 ☐ 8000 ☐ 12.2 ☐ 1.3 ENTER

or 2.45 y^x 3 ☐ 49 ☐ 8000 ☐ 12.2 ☐ 1.3 ☐

yields the evaluation 30.56. Rounding to the nearest tenth, we have 30.6.

NOTE

With most graphing calculators, the final command is ENTER. With most scientific calculators, the key is marked ☐.

Check Yourself 4

Use your scientific or graphing calculator to evaluate each expression.

(a) $67.89 \quad 4.7 \quad 12.7$ **(b)** $4.3 \quad 55.5 \quad 3.75^3 \quad 8{,}007 \quad 1{,}600$

Operations inside grouping symbols are always done first.

Example 5 Evaluating Expressions

Evaluate $(5 \quad 2) \quad 3$.

Do the operation inside the parentheses as the first step.

$(5 \quad 2) \quad 3 \quad\quad 7 \quad 3 \quad\quad 21$

Add.

Check Yourself 5

Evaluate $4 \quad (9 \quad 3)$.

The principle is the same when more than two "levels" of operations are involved.

Example 6 Using Order of Operations

(a) Evaluate $4 \quad (2 \quad 3)^3$.

Add inside the parentheses first.

$4 \quad (2 \quad 3)^3 \quad\quad 4 \quad (5)^3$

Evaluate the power.

$4 \quad 125$

Multiply.

500

(b) Evaluate $5 \quad (7 \quad 3)^2 \quad\quad 10$.

Evaluate the expression inside the parentheses.

$5 \quad (7 \quad 3)^2 \quad\quad 10 \quad\quad 5(4)^2 \quad\quad 10$

Evaluate the power.

$5 \quad 16 \quad\quad 10$

Multiply.

$80 \quad\quad 10 \quad\quad 70$

Subtract.

Check Yourself 6

Evaluate.

(a) $4 \quad 3^3 \quad\quad 8 \quad 11$ **(b)** $12 \quad\quad 4 \quad (2 \quad\quad 3)^2$

The correct order of operations must be followed within a set of grouping symbols, as shown in Example 7.

Example 7 Using Order of Operations

Evaluate $3 \cdot [(1 \quad 2)^2 \quad 5] \quad 8$.

We evaluate the expression in the parentheses within the brackets first. Next, we evaluate the exponent before proceeding to the subtraction. After evaluating everything within the brackets, we follow the correct order of operations by multiplying first, and then adding.

$$
\begin{aligned}
3 \cdot [(1 \quad 2)^2 \quad 5] \quad 8 \qquad & 3 \cdot [(3)^2 \quad 5] \quad 8 \\
& 3 \cdot [9 \quad 5] \quad 8 \\
& 3 \cdot (4) \quad 8 \\
& 12 \quad 8 \\
& 20
\end{aligned}
$$

Check Yourself 7

Evaluate $8 \quad 2 \quad [(5 \quad 3)^2 \quad 1]$.

We stated that parentheses and brackets are not the only types of grouping symbols. Example 8 demonstrates the fraction bar as a grouping symbol.

Example 8 Using the Order of Operations with Grouping Symbols

> **CAUTION**
>
> You may not "cancel" the 2's, because the numerator is being added, not multiplied.
>
> $\dfrac{\cancel{2} \quad 14}{\cancel{2}}$ is incorrect!

Evaluate $3 \quad \dfrac{2 \quad 14}{2} \cdot 5$.

$$
\begin{aligned}
3 \quad \frac{2 \quad 14}{2} \cdot 5 \qquad & 3 \quad \frac{16}{2} \cdot 5 && \text{The fraction bar acts as a grouping symbol.} \\
& 3 \quad 8 \cdot 5 && \text{We perform the division first because it precedes the multiplication.} \\
& 3 \quad 40 \\
& 43
\end{aligned}
$$

Check Yourself 8

Evaluate $4 \cdot \dfrac{3^2 \quad 2 \cdot 3}{5} \quad 6$.

Check Yourself ANSWERS

1. **(a)** 4^6; **(b)** 7^4 **2.** **(a)** 8; **(b)** 11 **3.** 64 **4.** **(a)** 8.2; **(b)** 190.92
5. 24 **6.** **(a)** 20; **(b)** 112 **7.** 2 **8.** 18

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.4

(a) Multiplication is shorthand for repeated ______________.

(b) The ______________ or power indicates the number of times the base appears in a product.

(c) Operations inside ______________ symbols are done first when evaluating an expression.

(d) ______________, brackets, and fraction bars are all examples of grouping symbols.

0.4 exercises

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Name ______________

Section ________ Date ________

< Objective 1 >

Write each expression in exponential form.

1. 7 7 7 7

2. 2 2 2 2 2 2

3. 6 6 6 6 6

4. 4 4 4 4 4 4 4

5. 8 8 8 8 8 8 8 8 8 8

6. 10 10 10

7. 15 15 15 15 15 15

8. 31 31 31 31 31 31 31 31 31 31

< Objective 2 >

Evaluate each of the following expressions.

9. 5 3 4

10. 10 4 2

11. (7 2) 6

12. (10 4) 2

13. 12 8 4

14. 20 10 2

> Videos

15. (24 12) 6

16. (10 20) 5

17. 8 7 2 2

18. 56 7 8 4

19. 7 (8 3) 3

20. 48 (8 4) 2

21. 3 5^2

22. 5 2^3

23. $(2 \quad 4)^2$

24. $(5 \quad 2)^3$

25. 4 3^2 2

> Videos

26. 3 2^4 8

Answers

1.	2.
3.	4.
5.	6.
7.	8.
9.	10.
11.	12.
13.	14.
15.	16.
17.	18.
19.	20.
21.	22.
23.	24.
25.	26.

0.4 exercises

Answers

27. ________

28. ________

29. ________

30. ________

31. ________

32. ________

33. ________

34. ________

35. ________

36. ________

37. ________

38. ________

39. ________

40. ________

41. ________

42. ________

43. ____ 44. ____

45. ____ 46. ____

47. ____ 48. ____

49. ____ 50. ____

27. $5 \quad [3 \quad (4 \quad 2)^2] \quad (3 \quad 5)$

28. $14 \quad 7 \quad [12 \quad (4 \quad 2)^2 \quad 5] \quad 3^3$

> Videos

29. $3 \quad 2^4 \quad 6 \quad 2$

30. $4 \quad 2^3 \quad 5 \quad 6$

31. $4 \quad (2 \quad 6)^2$

32. $3 \quad (8 \quad 4)^2$

33. $(4 \quad 2 \quad 6)^2$

34. $[25 \quad (2^3 \quad 3)] \quad 2$

35. $64 \quad [(16 \quad 2 \quad 4) \quad 16]$

36. $5 \quad (4 \quad 2)^3$

37. $12 \quad \dfrac{2 \quad 3}{2} \cdot 3$

38. $10 \cdot \dfrac{3^2 \quad 4}{4} \quad 6$

39. $3 \cdot \dfrac{2 \cdot 4 \quad 1 \cdot 5^2}{2 \quad 2} \quad 6$

40. $\dfrac{16 \quad (2 \quad 1)^2 \quad 1}{2^3}$

41. $(4 \quad 2 \quad 3)^2 \quad 25$

> Videos

42. $8 \quad (2 \quad 3 \quad 3)^2$

43. $2 \quad [16 \quad (1 \quad 3)^2]$

44. $[(2 \quad 3)^2 \quad 4 \quad 5] \quad 7$

45. **Social Science** Over the last 2,000 years, Earth's population has doubled approximately five times. Use exponential notation to write an expression that indicates doubling five times.

46. **Geometry** The volume of a cube with each edge of length 9 in. is given by 9 9 9. Write the volume using exponential notation.

Basic Skills | Advanced Skills | Vocational-Technical Applications | **Calculator/Computer** | Above and Beyond | Getting Ready

Use a calculator to evaluate each expression. Round your results to the nearest tenth.

47. $(1.2)^3 \quad 2.0736 \quad 2.4 \quad 1.6935 \quad 2.4896$

48. $(5.21 \quad 3.14 \quad 6.2154) \quad 5.12 \quad 0.45625$

49. $1.23 \quad 3.169 \quad 2.05194 \quad (5.128 \quad 3.15 \quad 10.1742)$

50. $4.56 \quad (2.34)^4 \quad 4.7896 \quad 6.93 \quad 27.5625 \quad 3.1269 \quad (1.56)^2$

Insert grouping symbols in the proper place(s) so that the given value of the expression is obtained.

51. 36 4 2 4; 2

52. 48 3 2 2 3; 2

53. 6 9 3 16 4 2; 29

54. 5 3 2 8 5 2; 28

Answers

1. 7^4 **3.** 6^5 **5.** 8^{10} **7.** 15^6 **9.** 17 **11.** 54 **13.** 10
15. 2 **17.** 60 **19.** 231 **21.** 75 **23.** 64 **25.** 34
27. 8 **29.** 36 **31.** 256 **33.** 196 **35.** 4 **37.** $\frac{9}{2}$
39. $\frac{75}{4}$ **41.** 96 **43.** 0 **45.** 2^5 **47.** 1.2 **49.** 7.8
51. 36 (4 2) 4 **53.** (6 9) 3 (16 4) 2

Answers

51. ____________

52. ____________

53. ____________

54. ____________

0.5 Positive and Negative Numbers

< 0.5 Objectives >

1 > Represent integers on a number line

2 > Order real numbers

3 > Find the opposite of a number

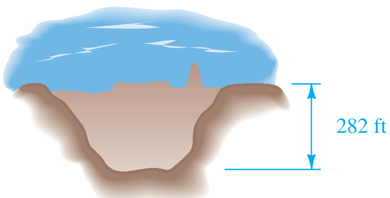
4 > Evaluate numerical expressions involving absolute value

When numbers are used to represent physical quantities (altitudes, temperatures, and money are examples), it is often necessary to distinguish between *positive* and *negative* amounts. It is convenient to represent these quantities with plus ( ) or minus ( ) signs. Some instances of this are shown here.

The altitude of Mount Whitney is 14,495 ft *above* sea level (14,495).

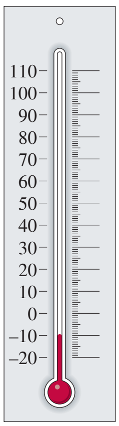
Mount Whitney

The altitude of Death Valley is 282 ft *below* sea level (282).



Death Valley

The temperature in Chicago is 10° *below* zero (10).



An account could show a *gain* of $100 (100) or a *loss* of $100 (100).

These numbers suggest the need to extend the number system to include both positive numbers (like 100) and negative numbers (like 282).

To represent the negative numbers, we extend the number line to the *left* of zero and name equally spaced points.

Numbers used to name points to the right of zero are positive numbers. They can be written with a positive ( ) sign, but are usually written with no sign at all.

+6 and 9 are positive numbers

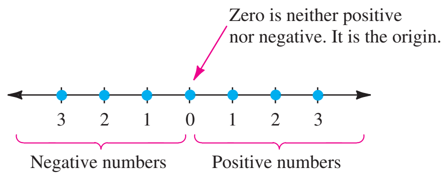
Numbers used to name points to the left of zero are negative numbers. They are always written with a negative ( ) sign.

 3 and 20 are negative numbers.

Read "negative 3."

Positive and negative numbers are both examples of **integers.**

Here, the number line is extended to include both positive and negative numbers.



Definition

Integers

The **integers** consist of the natural numbers, their negatives, and zero. We can represent the set of integers by

. . . , 3, 2, 1, 0, 1, 2, 3, ...

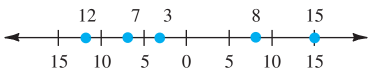
The sets of three dots are called *ellipses* and indicate that a pattern continues.

Example 1 Representing Integers on the Number Line

< Objective 1 >

Represent the following integers on the number line shown.

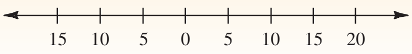
3, 12, 8, 15, 7



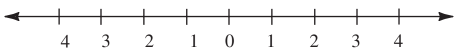
Check Yourself 1

Represent the following integers on a number line.

1, 9, 4, 11, 8, 20



The set of numbers on the number line is *ordered.* The numbers get smaller moving to the left on the number line and larger moving to the right.



When a set of numbers is written from smallest to largest, the numbers are said to be in *ascending order.*

Example 2 Ordering Integers

< Objective 2 >

Place each set of numbers in ascending order.

(a) 9, 5, 8, 3, 7

From smallest to largest, the numbers are

8, 5, 3, 7, 9

Note that this is the order in which the numbers appear on a number line as we move from left to right.

(b) 3, 2, 18, 20, 13

From smallest to largest, the numbers are

20, 13, 2, 3, 18

Check Yourself 2

Place each set of numbers in ascending order.

(a) 12, 13, 15, 2, 8, 3 **(b)** 3, 6, 9, 3, 8

The least and greatest numbers in a set are called the **extreme values.** The least element is called the **minimum,** and the greatest element is called the **maximum.**

Example 3 Labeling Extreme Values

For each set of numbers, determine the minimum and maximum values.

(a) 9, 5, 8, 3, 7

From our previous ordering of these numbers, we see that 8, the least element, is the minimum, and 9, the greatest element, is the maximum.

(b) 3, 2, 18, 20, 13

20 is the minimum, and 18 is the maximum.

Check Yourself 3

For each set of numbers, determine the minimum and maximum values.

(a) 12, 13, 15, 2, 8, 3 **(b)** 3, 6, 9, 3, 8

Integers are not the only kind of signed numbers. Decimals and fractions can also be thought of as signed numbers.

Example 4 Identifying Numbers That Are Integers

Which of the numbers 145, 28, 0.35, and $\frac{2}{3}$ are integers?

(a) 145 is an integer.

(b) 28 is an integer.

(c) 0.35 is not an integer.

(d) $\frac{2}{3}$ is not an integer.

Check Yourself 4

Which of the following numbers are integers?

23 1,054 0.23 0 500 $\frac{4}{5}$

NOTE

0 is the opposite of 0.

We refer to the negative of a number as its *opposite*. But what is the opposite of the opposite of a number? It is the number itself. Example 5 illustrates this concept.

Example 5 Finding Opposites

< Objective 3 >

Find the opposite for each number.

(a) 5 The opposite of 5 is 5.

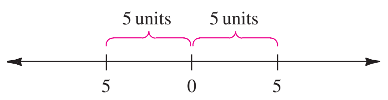
(b) 9 The opposite of 9 is 9.

Check Yourself 5

Find the opposite for each number.

(a) 17 **(b)** 12

An important idea for our work in this chapter is the **absolute value** of a number. This represents the distance of the point named by the number from the origin on the number line.



The absolute value of 5 is 5. The absolute value of 5 is also 5.

The absolute value of a positive number or zero is itself. The absolute value of a negative number is its opposite.

In symbols we write

5 5 and 5 5

Read "the absolute value of 5."

Read "the absolute value of negative 5."

The absolute value of a number does *not* depend on whether the number is to the right or to the left of the origin, but on its *distance* from the origin.

Example 6 Simplifying Absolute Value Expressions

< Objective 4 >

(a) $|7|$ 7

(b) $|\ 7|$ 7

(c) $|\ 7|$ 7

This is the *negative,* or opposite, of the absolute value of negative 7.

(d) $|\ 10|$ $|10|$ 10 10 20

Absolute value bars serve as another set of grouping symbols, so do the operation *inside* first.

(e) $|8\ \ 3|$ $|5|$ 5

(f) $|8|$ $|3|$ 8 3 5

Here, evaluate the absolute values and then subtract.

Check Yourself 6

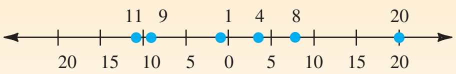
Evaluate.

(a) |8| (b) | 8| (c) | 8|

(d) | 9| |4| (e) |9 4| (f) |9| |4|

Check Yourself ANSWERS

1.



2. **(a)** 13, 8, 3, 2, 12, 15; **(b)** 9, 3, 3, 6, 8

3. **(a)** Minimum is 13, maximum is 15; **(b)** Minimum is 9, maximum is 8

4. 23, 1,054, 0, and 500 **5.** **(a)** 17; **(b)** 12

6. **(a)** 8; **(b)** 8; **(c)** 8; **(d)** 13; **(e)** 5; **(f)** 5

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.5

(a) When numbers are used to represent physical quantities, it is often necessary to distinguish between positive and ______________ quantities.

(b) It is convenient to represent negative quantities with a ___________ sign.

(c) The ______________ consist of the natural numbers, their negatives, and zero.

(d) When a set of numbers is written from smallest to largest, the numbers are said to be in ______________ order.

0.5 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

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Name ____________

Section ________ Date ________

Answers

1. ______ 2. ______

3. ______ 4. ______

5. ______

6. ______

7. ______

8. ______

9. ______

10. ______

11. ______

12. ______

13. ______

14. ______

15. ______

16. ______

< Objective 1 >

Represent each quantity with an integer.

1. An altitude of 400 ft above sea level

2. An altitude of 80 ft below sea level

3. A loss of \$200

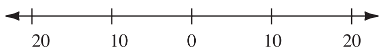
4. A profit of \$400

5. A decrease in population of 25,000

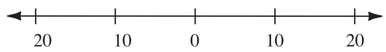
6. An increase in population of 12,500

Represent the integers on the number lines shown.

7. 5, 15, 18, 8, 3



8. 18, 4, 5, 13, 9



Which numbers in the following sets are integers?

9. 5, $\frac{2}{9}$, 175, 234, 0.64

> Videos

10. 45, 0.35, $\frac{3}{5}$, 700, 26

< Objective 2 >

Place each of the following sets of numbers in ascending order.

11. 3, 5, 2, 0, 7, 1, 8

12. 2, 7, 1, 8, 6, 1, 0

13. 9, 2, 11, 4, 6, 1, 5

> Videos

14. 23, 18, 5, 11, 15, 14, 20

15. 6, 7, 7, 6, 3, 3

16. 12, 13, 14, 14, 15, 15

Beginning Algebra

The Streeter/Hutchison Series in Mathematics

For each set, determine the maximum and minimum values.

17. 5,   6, 0, 10,   3, 15, 1, 8

18. 9,   1, 3, 11,   4, 2, 5,   2

19. 21,   15, 0, 7,   9, 16,   3, 11

20.   22, 0, 22,   31, 18,   5, 3

21. 3, 0, 1,   2, 5, 4,   1

22. 2, 7,   3, 5,   10,   5

< Objective 3 >

Find the opposite of each number.

23. 15

24. 18

25. 15

26. 34

27.   19

28.   6

29.   7

30.   54

< Objective 4 >

Evaluate.

31. $|17|$

32. $|28|$

33. $|\;\;19|$

34. $|\;\;7|$

35. $|21|$

36. $|3|$

37. $|\;\;8|$

38. $|\;\;13|$

39. $|\;\;2|\;\;|3|$

40. $|4|\;\;|\;\;3|$

41. $|\;\;9|\;\;|9|$

42. $|11|\;\;|\;\;11|$

Answers

17.

18.

19.

20.

21.

22.

23.

24.

25.

26.

27.

28.

29.

30.

31. 32.

33. 34.

35. 36.

37. 38.

39. 40.

41. 42.

0.5 exercises

Answers

43. ______

44. ______

45. ______

46. ______

47. ______

48. ______

49. ______

50. ______

51. ______

52. ______

53. ______

54. ______

55. ______

56. ______

57. ______

58. ______

59. ______

60. ______

43. $| \quad 6| \quad |6|$

44. $|5| \quad | \quad 5|$

45. $|15| \quad |8|$

46. $|11| \quad |3|$

47. $|15 \quad 8|$

48. $|11 \quad 3|$

49. $| \quad 9| \quad |2|$

50. $| \quad 7| \quad |4|$

51. $|7| \quad | \quad 6|$

52. $| \quad 9| \quad | \quad 4|$

Represent each quantity with a real number.

53. Science and Technology The erosion of 5 centimeters (cm) of topsoil from an Iowa corn field.

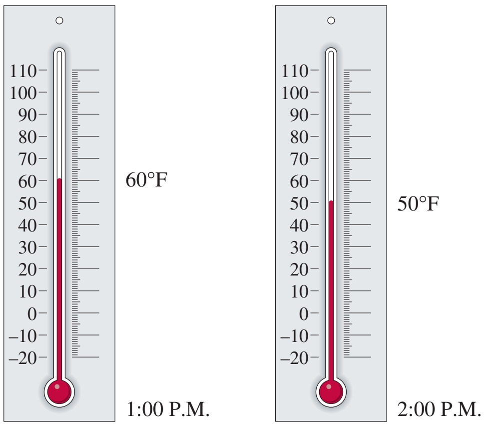
> Videos

54. Science and Technology The formation of 2.5 cm of new topsoil on the African savanna.

55. Business and Finance The withdrawal of $50 from a checking account.

56. Business and Finance The deposit of $200 into a savings account.

57. Science and Technology The temperature change pictured.



58. Business and Finance An increase of 75 points in the Dow-Jones average.

59. Statistics An eight-game losing streak by the local baseball team.

60. Social Science An increase of 25,000 in the population of the city.

61. **Business and Finance** A country exported $90,000,000 more than it imported, creating a positive trade balance.

62. **Business and Finance** A country exported $60,000,000 less than it imported, creating a negative trade balance.

Determine whether each of the following statements is **true** *or* **false.**

63. All natural numbers are integers.

64. Zero is an integer.

65. All integers are whole numbers.

66. All real numbers are integers.

67. All negative integers are whole numbers.

68. Zero is neither positive nor negative.

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | **Above and Beyond** | Getting Ready

For each collection of numbers given in exercises 69 to 72, answer the following:

(a) Which number is smallest?
(b) Which number lies farthest from the origin?
(c) Which number has the largest absolute value?
(d) Which number has the smallest absolute value?

69. 6, 3, 8, 7, 2

70. 8, 3, 5, 4, 9

71. 2, 6, 1, 0, 2, 5

72. 9, 0, 2, 3, 6

Place absolute value bars in the proper location(s) on the left side of the expression so that the equation is true.

73. 6 (2) 4

74. 8 (3) 5

75. 6 (2) 8

> Videos

76. 8 (3) 11

Answers

61. ______
62. ______
63. ______
64. ______
65. ______
66. ______
67. ______
68. ______
69. ______
70. ______
71. ______
72. ______
73. ______
74. ______
75. ______
76. ______

Answers

77. ____________________

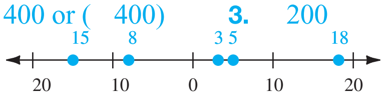
77. Simplify each of the following:

(7)   ((7))   (((7)))

Based on your answers, generalize your results.

Answers

1. 400 or (400) **3.** 200 **5.** 25,000

7. 

9. 5, 175, 234

11. 7, 5, 1, 0, 2, 3, 8 **13.** 11, 6, 2, 1, 4, 5, 9
15. 7, 6, 3, 3, 6, 7 **17.** Max: 15; min: 6 **19.** Max: 21, min: 15
21. Max: 5; min: 2 **23.** 15 **25.** 15 **27.** 19 **29.** 7
31. 17 **33.** 19 **35.** 21 **37.** 8 **39.** 5 **41.** 18 **43.** 0
45. 7 **47.** 7 **49.** 11 **51.** 1 **53.** 5 **55.** 50
57. 10°F **59.** 8 **61.** 90,000,000 **63.** True **65.** False
67. False **69.** **(a)** 6; **(b)** 8; **(c)** 8; **(d)** 2
71. **(a)** 2; **(b)** 6; **(c)** 6; **(d)** 0 **73.** |6 (2)| 4 **75.** |6| | 2| 8
77. Above and Beyond

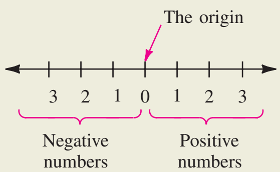
Definition/Procedure	Example	Reference
Prime Factorization and Least Common Multiples		**Section 0.1**
Factor A **factor** of a whole number is another whole number that divides exactly into that number, leaving a remainder of zero.	The factors of 12 are 1, 2, 3, 4, 6, and 12.	*p.* 4
Prime Number Any whole number greater than 1 that has only 1 and itself as factors.	7, 13, 29, and 73 are prime numbers.	*p.* 5
Composite Number Any whole number greater than 1 that is not prime.	8, 15, 42, and 65 are composite numbers.	*p.* 6
Zero and One 0 and 1 are classified as neither prime nor composite numbers.		*p.* 6
Greatest Common Factor (GCF) The GCF is the *largest* number that is a factor of each of a group of numbers.	The GCF of 21 and 24 is 3.	*p.* 10
To Find the GCF **1.** Write the prime factorization for each of the numbers in the group. **2.** Locate the prime factors that appear in every prime factorization. **3.** The GCF is the product of all the common prime factors. If there are no common prime factors, the GCF is 1.	To find the GCF of 24, 30, and 36: 24 2 2 2 3 30 2 3 5 36 2 2 3 3 The GCF is 2 3 6.	*p.* 10
Least Common Multiple (LCM) The LCM is the *smallest* number that is a multiple of each of a group of numbers.	The LCM of 21 and 24 is 168.	*p.* 13
To Find the LCM **1.** Write the prime factorization for each of the numbers in the group. **2.** Find all the prime factors that appear in any one of the prime factorizations. **3.** Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.	To find the LCM of 12, 15, and 18: 12 2 2 3 15 3 5 18 2 3 3 2 2 3 3 5 The LCM is 2 2 3 3 5, or 180.	
Fractions and Mixed Numbers		**Section 0.2**
The Fundamental Principle of Fractions $\frac{a}{b} \quad \frac{a \cdot c}{b \cdot c}$ in which neither b nor c is zero.	$\frac{2}{3} \quad \frac{2 \cdot 3}{3 \cdot 3} \quad \frac{6}{9}$	*p.* 21

Continued

Definition/Procedure	Example	Reference
Multiplying Fractions **1.** Multiply numerator by numerator. This gives the numerator of the product. **2.** Multiply denominator by denominator. This gives the denominator of the product. **3.** Simplify the resulting fraction if possible. In multiplying fractions, it is usually easiest to factor and simplify the numerator and denominator *before* multiplying.	$\frac{5}{8} \cdot \frac{3}{7} \quad \frac{5 \cdot 3}{8 \cdot 7} \quad \frac{15}{56}$ $\frac{5}{9} \cdot \frac{3}{10} \quad \frac{\overset{1}{\cancel{5}} \cdot \overset{1}{\cancel{3}}}{\underset{3}{\cancel{9}} \cdot \underset{2}{\cancel{10}}} \quad \frac{1}{6}$	*p.* 22
Dividing Fractions Invert the divisor and multiply.	$\frac{3}{7} \quad \frac{4}{5} \quad \frac{3}{7} \quad \frac{5}{4} \quad \frac{15}{28}$	*p.* 23
To Add or Subtract Fractions with Different Denominators **1.** Find the LCD of the fractions. **2.** Change each fraction to an equivalent fraction with the LCD as a common denominator. **3.** Add or subtract the resulting like fractions as before.	$\frac{3}{4} \quad \frac{7}{10} \quad \frac{15}{20} \quad \frac{14}{20} \quad \frac{29}{20}$ $\frac{8}{9} \quad \frac{5}{6} \quad \frac{16}{18} \quad \frac{15}{18} \quad \frac{1}{18}$	*p.* 24
Mixed Number The sum of a whole number and a proper fraction.	$2\frac{1}{3}$ and $5\frac{7}{8}$ are mixed numbers.	*p.* 25
To Write an Improper Fraction as a Mixed Number **1.** Divide the numerator by the denominator. The quotient is the whole-number portion of the mixed number. **2.** If there is a remainder, write the remainder over the original denominator. This gives the fractional portion of the mixed number.	To write $\frac{22}{5}$ as a mixed number: $\begin{array}{r} 4 \\ 5\overline{)22} \\ \underline{20} \\ 2 \end{array}$ Quotient, Remainder $\frac{22}{5} \quad 4\frac{2}{5}$	*p.* 26
To Write a Mixed Number as an Improper Fraction **1.** Multiply the denominator of the fraction by the whole-number portion of the mixed number. **2.** Add the numerator of the fraction to that product. **3.** Write that sum over the original denominator to form the improper fraction.	Denominator, Whole number, Numerator $5\frac{3}{4} \quad \frac{(4 \cdot 5) \quad 3}{4} \quad \frac{23}{4}$ Denominator	*p.* 26
To Add or Subtract Mixed Numbers **1.** Rewrite as improper fractions. **2.** Add or subtract the fractions. **3.** Rewrite the results as a mixed number if required.	$5\frac{1}{2} \quad 3\frac{3}{4} \quad \frac{11}{2} \quad \frac{15}{4} \quad \frac{22}{4} \quad \frac{15}{4}$ $\frac{7}{4} \quad 1\frac{3}{4}$	*p.* 28
Decimals and Percents		**Section 0.3**
To Write a Fraction as a Decimal **1.** Divide the numerator of the fraction by its denominator. **2.** The quotient is the decimal equivalent of the common fraction.	To write $\frac{1}{2}$ as a decimal: $\begin{array}{r} 0.5 \\ 2\overline{)1.0} \\ \underline{1\,0} \\ 0 \end{array}$	*p.* 38

Definition/Procedure	Example	Reference
To Write a Terminating Decimal Less Than 1 as a Fraction **1.** Write the digits of the decimal without the decimal point. This is the numerator of the fraction. **2.** The denominator of the fraction is a 1 followed by as many zeros as there are places in the decimal.	To write 0.275 as a fraction: $0.275 \quad \frac{275}{1,000} \quad \frac{11}{40}$	*p.* 40
To Add Decimals **1.** Write the numbers being added in column form with their decimal points in a vertical line. **2.** Add just as you would with whole numbers. **3.** Place the decimal point of the sum in line with the decimal points of the addends.	To add 2.7, 3.15, and 0.48: $\begin{array}{r} 2.7 \\ 3.15 \\ \underline{0.48} \\ 6.33 \end{array}$	*p.* 40
To Subtract Decimals **1.** Write the numbers being subtracted in column form with their decimal points in a vertical line. You may have to place zeros to the right of the existing digits. **2.** Subtract just as you would with whole numbers. **3.** Place the decimal point of the difference in line with the decimal points of the numbers being subtracted.	To subtract 5.875 from 8.5: $\begin{array}{r} 8.500 \\ \underline{5.875} \\ 2.625 \end{array}$	*p.* 41
To Multiply Decimals **1.** Multiply the decimals as though they were whole numbers. **2.** Add the number of decimal places in the factors. **3.** Place the decimal point in the product so that the number of decimal places in the product is the sum of the number of decimal places in the factors.	To multiply 2.85 0.045: $\begin{array}{rl} 2.85 & \longleftarrow \text{Two places} \\ \underline{0.045} & \longleftarrow \text{Three places} \\ 1425 & \\ \underline{11400} & \\ 0.12825 & \longleftarrow \text{Five places} \end{array}$	*p.* 41
To Divide by a Decimal **1.** Move the decimal point in the divisor to the right, making the divisor a whole number. **2.** Move the decimal point in the dividend to the right the same number of places. Add zeros if necessary. **3.** Place the decimal point in the quotient directly above the decimal point of the dividend. **4.** Divide as you would with whole numbers.	To divide 16.5 by 5.5, move the decimal points: $\begin{array}{r} 3 \\ 5.5\overline{)16.5} \\ \underline{16\,5} \\ 0 \end{array}$	*p.* 42
Percent Along with fractions and decimals another way of naming parts of a whole. Percent means per hundred.	$21\% \quad 21\,\frac{1}{100} \quad \frac{21}{100} \quad 0.21$	*p.* 43
To Write a Percent as a Fraction or Decimal **1.** *To write a percent as a fraction,* replace the percent symbol with $\frac{1}{100}$ and multiply. **2.** *To write a percent as a decimal,* remove the percent symbol, and move the decimal point two places to the left.	$37\% \quad 37\,\frac{1}{100} \quad \frac{37}{100}$ $37\% \quad 0.37$	*p.* 43

Continued

Definition/Procedure	Example	Reference
To Write a Decimal or Fraction as a Percent		
1. *To write a decimal as a percent,* move the decimal point two places to the right, and attach the percent symbol. **2.** *To write a fraction as a percent,* write the decimal equivalent of the fraction, and then change that decimal to a percent.	0.58 58% $\frac{3}{5}$ 0.60 60%	*p.* 44
Exponents and the Order of Operations		**Section 0.4**
Using Exponents **Base** The number that is raised to a power. **Exponent** The exponent is written to the right and above the base. The exponent tells the number of times the base is to be used as a factor.	Exponent 5^3 5 5 5 125 Base Three factors	*p.* 53
The Order of Operations *Mixed operations* in an expression should be done in the following order: **1.** Do any operations inside grouping symbols. **2.** Evaluate any powers. **3.** Do all multiplication and division in order, from left to right. **4.** Do all addition and subtraction in order, from left to right.	4 $(2 \quad 3)^2$ 7 4 5^2 7 4 25 7 100 7 93	*p.* 54
Positive and Negative Numbers		**Section 0.5**
Positive Numbers Numbers used to name points to the right of the origin on the number line. ***Negative Numbers*** Numbers used to name points to the left of the origin on the number line. ***Integers*** The natural (or counting) numbers, their negatives, and zero. The integers are {. . . , 3, 2, 1, 0, 1, 2, 3, . . .}		*p.* 63
Opposite of a Number The opposite of a number is the negative of that number.	The opposite of 2 is 2. The opposite of 9 is 9.	*p.* 65
Absolute Value The distance (on the number line) between the point named by an integer and the origin.	$\|7\|$ 7 $\|\ \ 10\|$ 10	*p.* 66

This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the *Instructor's Solutions Manual.* Your instructor will give you guidelines on how best to use these exercises in your instructional setting.

0.1 *List all the factors of the given numbers.*

1. 52

2. 41

3. 76

4. 315

Use the group of numbers 2, 5, 7, 11, 14, 17, 21, 23, 27, 39, and 43.

5. List the prime numbers; then list the composite numbers.

Find the prime factorization for the given numbers.

6. 48

7. 420

8. 60

9. 180

Find the greatest common factor (GCF).

10. 15 and 20

11. 30 and 31

12. 72 and 180

13. 240 and 900

Find the least common multiple (LCM).

14. 4 and 12

15. 8 and 16

16. 18 and 24

17. 12 and 18

0.2 *Write three fractional representations for each number.*

18. $\frac{5}{7}$

19. $\frac{3}{11}$

20. $\frac{4}{9}$

21. Write the fraction $\frac{24}{64}$ in simplest form.

22. The Pennsylvania Turnpike, from the Ohio border to the New Jersey border is 360 miles long. Miranda and Carl agree to hike along the turnpike in order to raise money for their favorite charity. On the first day, they hike $23\frac{3}{4}$ miles. The second day, they hike $24\frac{2}{3}$ miles, and on the third day they hike another $17\frac{7}{10}$ miles. How many miles did they walk over the first three days? How much farther do they have to hike in order to complete the entire distance?

0.3 *Perform the indicated operations.*

23. $\frac{7}{15} \cdot \frac{5}{21}$

24. $\frac{10}{27} \cdot \frac{9}{20}$

25. $\frac{5}{12} \quad \frac{5}{8}$

26. $\frac{7}{15} \quad \frac{14}{25}$

27. $\frac{5}{6} \quad \frac{11}{18}$

28. $\frac{5}{18} \quad \frac{7}{12}$

29. $\frac{11}{18} \quad \frac{2}{9}$

30. $\frac{11}{27} \quad \frac{5}{18}$

31. 5.123   6.4

32. 10.127   5.49

33. 5.26   3.796

34. $6\frac{5}{7} \quad 3\frac{4}{7}$

35. $5\frac{7}{10} \quad 3\frac{11}{12}$

36. $7\frac{7}{9} \quad 3\frac{4}{9}$

37. $6\frac{5}{12} \quad 3\frac{5}{8}$

38. $5\frac{1}{3} \cdot 1\frac{4}{5}$

39. $3\frac{2}{5} \cdot \frac{5}{8}$

40. $3\frac{3}{8} \quad 2\frac{1}{4}$

Divide and round the quotient to the nearest thousandth.

41. 3.042   0.37

42. 0.2549   2.87

Write the percent as a fraction or a mixed number.

43. 2%

44. 20%

45. 37.5%

46. 150%

47. $233\frac{1}{3}\%$

48. 300%

Write the percents as decimals.

49. 75%

50. 4%

51. 6.25%

52. 13.5%

53. 0.6%

54. 225%

Write as percents.

55. 0.06

56. 2.4

57. 7

58. 0.035

59. 0.005

60. $\frac{7}{10}$

61. $\frac{2}{5}$

62. $1\frac{1}{4}$

63. $2\frac{2}{3}$

64. Pierce's monthly electric bill comes to $84.52 under his equal-payment program. How much will he pay for electricity over a full year?

0.4 *Evaluate each of the following expressions.*

65. $18 \quad 3 \quad 5$

66. $(18 \quad 3) \quad 5$

67. $5 \quad 4^2$

68. $(5 \quad 4)^2$

69. $5 \quad 3^2 \quad 4$

70. $5 \quad (3^2 \quad 4)$

71. $5 \quad (4 \quad 2)^2$

72. $5 \quad 4 \quad 2^2$

73. $(5 \quad 4 \quad 2)^2$

74. $3 \quad (5 \quad 2)^2$

75. $3 \quad 5 \quad 2^2$

76. $(3 \quad 5 \quad 2)^2$

77. $8 \quad 4 \quad 2$

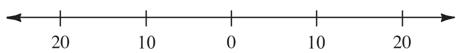
78. $36 \quad 4 \quad 2 \quad 7 \quad 6$

79. $4^2 \quad 2 \cdot \frac{10 \quad 3^2}{4 \quad (1 \quad 1)}$

80. $3 \cdot 2^2 \quad \frac{18 \quad (12 \quad 2^2)}{2^2}$

0.5

81. Represent the following integers on the number line shown: 6, 18, 3, 2, 15, 9.



Place each of the following sets in ascending order.

82. 4, 3, 6, 7, 0, 1, 2

83. 7, 8, 8, 1, 2, 3, 3, 0, 7

For each data set determine the maximum and minimum.

84. 4, 2, 5, 1, 6, 3, 4

85. 4, 2, 5, 9, 8, 1, 6

Find the opposite of each number.

86. 17

87. $\quad 63$

Evaluate.

88. $|9|$

89. $|\quad 9|$

90. $\quad |9|$

91. $\quad |\quad 9|$

92. $|12 \quad 8|$

93. $|8| + |\quad 12|$

94. $\quad |8 + 12|$

95. $|\quad 18| \quad |\quad 12|$

96. $|\quad 7| \quad |\quad 3|$

97. $|\quad 9| + |\quad 5|$

98. At the beginning of the month, Tyler had \$33.15 in his checking account. He deposited his \$425.87 paycheck and paid his \$314.89 student loan bill. What is the balance in his checking account?

self-test 0

Name ____________________

Section ________ Date ________

Answers

1. ____________________
2. ____________________
3. ________ 4. ________
5. ________ 6. ________
7. ____________________
8. ____________________
9. ____________________
10. ____________________
11. ____________________
12. ____________________
13. ____________________
14. ____________________
15. ____________________
16. ____________________
17. ____________________
18. ____________________
19. ____________________
20. ____________________

This purpose of this chapter test is to help you check your progress so that you can find sections and concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you missed any, note the section reference that accompanies the answer. Go back to that section and reread the examples until you have mastered that particular concept.

1. Which of the numbers 5, 9, 13, 17, 22, 27, 31, and 45 are prime numbers? Which are composite numbers?

2. Find the prime factorization for 264.

Find the greatest common factor (GCF) for the given numbers.

3. 36 and 84

4. 16, 24, and 72

Find the least common multiple (LCM) for the given numbers.

5. 12 and 27

6. 3, 4, and 18

Perform the indicated operations.

7. $\frac{8}{21} \cdot \frac{3}{4}$

8. $\frac{7}{12} \quad \frac{28}{36}$

9. $\frac{3}{4} \quad \frac{5}{6}$

10. $\frac{8}{21} \quad \frac{2}{7}$

11. 3.25 4.125

12. 16.234 12.35

13. 7.29 3.15

14. 6.10 13.1

15. $2\frac{2}{3} \cdot 1\frac{2}{7}$

16. $4\frac{1}{6} \quad 3\frac{3}{4}$

17. $3\frac{5}{6} \quad 2\frac{2}{9}$

18. 3.969 0.54

Write as fractions.

19. 7%

20. 72%

Answers

21. ______ 22. ______

23. ______ 24. ______

25. ______ 26. ______

27. ______ 28. ______

29. ______ 30. ______

31. ______

32. ______

33. ______

34. ______

35. ______

36. ______

37. ______

38. ______

39. ______

40. ______

41. ______

42. ______

43. ______

44. ______

45. ______

Write as decimals.

21. 42% **22.** 6% **23.** 160%

Write as percents.

24. 0.03 **25.** 0.042

26. $\frac{2}{5}$ **27.** $\frac{5}{8}$

Write using exponents.

28. 4 4 4 4 **29.** 9 9 9 9 9

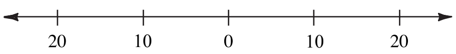
Evaluate the following expressions.

30. 23 4 5 **31.** 4 5^2 35

32. 4 (2 $4)^2$ **33.** 16 2 5^2

34. (3 2 $4)^3$ **35.** 8 3 2 5

36. Represent the following integers on the number line shown: 5, 12, 4, 7, 18, 17.



37. Place the following data set in ascending order: 4, 3, 6, 5, 0, 2, 2.

38. Determine the maximum and minimum of the following data set: 3, 2, 5, 6, 1, 2.

Evaluate.

39. $|7|$ **40.** $| \quad 7|$

41. $|18 \quad 7|$ **42.** $|18| \quad | \quad 7|$

43. $|24 \quad 5|$

Find the opposite of each number.

44. 40 **45.** 19