INTRODUCTION

Number systems and methods to record patterns in their surroundings were developed by every culture in history. The Mayans in Central America had one of the most sophisticated number systems in the world in the twelfth century A.D. The Chinese number system dates from around 1200 B.C.E.

The oldest evidence of a number system is from Africa near modern-day Swaziland. Archaeologists found a bone that was notched in a numerical pattern and dates from about 35,000 B.C.E.

The roots of algebra first appear in the 4,000-year-old Babylonian culture, in what is now Iraq. The Babylonians developed ways to record useful numerical relationships so that they would be easy to remember, easy to record, and helpful in solving problems. Some of the formulas developed by the Babylonians are still in use today.

You are about to embark on an exciting and useful endeavor: learning to use algebra to help you solve problems. It will take some time and effort, but do not be discouraged. Everyone can master this topic—people just like you have used it for many centuries!

CHAPTER 0 OUTLINE

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This pretest provides a preview of the types of exercises you will encounter in each section of this chapter. The answers for these exercises can be found in the back of the text. If you are working on your own, or ahead of the class, this pretest can help you identify the sections in which you should focus more of your time.

**0.1**

1. List all the factors of 42.

2. For the group of numbers 2, 3, 6, 7, 9, 18, 21, and 23, list the prime numbers and the composite numbers.

3. Find the prime factorization for each of the following numbers.
   (a) 60
   (b) 350

4. Find the greatest common factor (GCF) for each of the following groups of numbers.
   (a) 12 and 32
   (b) 24, 36, and 42

5. Find the least common multiple (LCM) for each of the following groups of numbers.
   (a) 4, 5, and 10
   (b) 36, 20, and 30

**Perform the indicated operations.**

**0.2**

6. \( \frac{3}{5} \cdot \frac{25}{12} \)

7. \( \frac{6}{7} \div \frac{12}{21} \)

8. \( \frac{5}{6} \cdot \frac{3}{4} \)

9. \( \frac{17}{18} \div \frac{5}{9} \) \(0.3\) 10. 8.123 \( \div \) 4.356

11. 7.16 \( \div \) 3.19

**0.2**

12. \( \frac{2}{5} \div \frac{1}{4} \)

13. \( \frac{5}{6} \div \frac{4}{9} \) \(0.3\) 14. 3.896 \( \div \) 1.6

**Evaluate the following expressions.**

**0.4**

15. \(21 \div 3 \div 5\)

16. \(3 \div 4 \div 2^2\)

17. \((18 \div 9) \div 2 \div 3^2\)

18. \((15 \div 12 \div 5) \div 2^2\)

**0.3**

19. Write 23% as (a) a fraction and (b) a decimal.

20. Write 0.035 as a percent.

**0.5**

21. Represent the following integers on the number line shown: 6, 8, 4, 2, 10.

22. Place the following data set in ascending order: 5, 2, 4, 0, 1, 1.

23. Determine the maximum and minimum of the following data set: 4, 1, 5, 7, 3, 2.

Evaluate the following expressions.

**0.5**

24. \(|5|\)

25. \(|6|\)

26. \(|11|\)

27. \(|11| \div |5|\)

28. \(|4| \div |5|\)

29. \(16 \div 30\)

30. 23
0.1

Prime Factorization and Least Common Multiples

< 0.1 Objectives >

1. Find the factors of a whole number
2. Determine whether a number is prime, composite, or neither
3. Find the prime factorization for a number
4. Find the GCF for two or more numbers
5. Find the LCM for two or more numbers

Overcoming Math Anxiety

Over the first few chapters, we present you with a series of class-tested techniques that are designed to improve your performance in this math class.

Hint #1 Become familiar with your textbook.
Perform each of the following tasks.

1. Use the Table of Contents to find the title of Section 5.1.
2. Use the index to find the earliest reference to the term mean. (By the way, this term has nothing to do with the personality of either your instructor or the textbook author!)
3. Find the answer to the first Check Yourself exercise in Section 0.1.
4. Find the answers to the Self-Test for Chapter 1.
5. Find the answers to the odd-numbered exercises in Section 0.1.
6. In the margin notes for Section 0.1, find the definition for the term relatively prime.

Now you know where some of the most important features of the text are. When you have a moment of confusion, think about using one of these features to help you clear up that confusion.

How would you organize the following list of objects: cow, dog, daisy, fox, lily, sunflower, cat, tulip?

Although there are many ways to organize the objects, most people would break them into two groups, the animals and the flowers. In mathematics, we call a group of things that have something in common a set.
CHAPTER 0
An Arithmetic Review

**Definition**

**Set**

A set is a collection of distinct objects that are grouped together into a single unit. Each member of a set is called an *element*.

We generally describe a set in one of two ways:

List the elements of the set.

Describe the rule(s) used to determine whether a given object is a member of the set.

We usually use braces to enclose the elements of a set when we are listing them:

\{cow, dog, fox, cat\} or \{daisy, lily, sunflower, tulip\}

Of course, in mathematics many (but not all) of the sets we are interested in are sets of numbers.

The numbers used to count things—1, 2, 3, 4, 5, and so on—are called the natural (or counting) numbers. The whole numbers consist of the natural numbers and zero—0, 1, 2, 3, 4, 5, and so on. They can be represented on a number line like the one shown. Zero (0) is considered the origin.

Any whole number can be written as a product of two whole numbers. For example, we say that 12 = 3 \times 4. We call 3 and 4 *factors* of 12.

**Definition**

**Factor**

A factor of a whole number is another whole number that divides exactly into that number. This means that the division has a remainder of 0.

**Example 1**

**Finding Factors**

*Objective 1*

List all factors of 18.

3, 6, 18. Because 3 and 6 are factors (or divisors) of 18.

2, 9, 18. 2 and 9 are also factors of 18.

1, 18, 18. 1 and 18 are factors of 18.

1, 2, 3, 6, 9, and 18 are all the factors of 18.

**Check Yourself 1**

List all factors of 24.

* Check Yourself answers appear at the end of each section in this book.
Listing factors leads us to an important classification of whole numbers. Any whole number larger than 1 is either a prime or a composite number.

**Definition**

**Prime Number**

A prime number is any whole number greater than 1 that has only 1 and itself as factors.

**NOTE**

A whole number greater than 1 always has itself and 1 as factors. Sometimes these are the only factors. For instance, 1 and 3 are the only factors of 3.

As examples, 2, 3, 5, and 7 are prime numbers. Their only factors are 1 and themselves.

To check whether a number is prime, one approach is simply to divide the smaller primes, 2, 3, 5, 7, and so on, into the given number. If no factors other than 1 and the given number are found, the number is prime.

The Sieve of Eratosthenes is an easy method for identifying prime numbers.

**NOTE**

How large can a prime number be? There is no largest prime number!

As of December 2005, the largest known prime number is $2^{30,402,457} - 1$. If you are curious, this is a number with 9,152,052 digits. Of course, a computer was used to find this number and verify that it is prime.

By the time you read this, someone may very well have found an even larger prime number.

**Step by Step**

**The Sieve of Eratosthenes**

1. Write down a sequence of counting numbers, beginning with the number 2. In the example below, we stop at 50.
2. Start at the number 2. Delete every second number after the 2. Each of the deleted numbers has 2 as a factor. This means that each deleted number is a composite number.
3. Move to the number 3. Delete every third number after 3 (some numbers will already have been deleted). Each deleted number is divisible by 3, so each deleted number is not prime.
4. Move to the next undeleted number, which is 5 (you should already have deleted 4). Delete every fifth number after 5.
5. Continue this process, deleting every seventh number after 7, and so on.
6. When you have finished, the numbers that remain are prime.

The prime numbers less than 50 are 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, and 47.
CHAPTER 0
An Arithmetic Review

Example 2

Identifying Prime Numbers

<Objective 2>
Which of the numbers 17, 29, and 33 are prime?

17 is a prime number. 1 and 17 are the only factors.
29 is a prime number. 1 and 29 are the only factors.
33 is not prime. 1, 3, 11, and 33 are all factors of 33.

Note: For two-digit numbers, if the number is not a prime, it will have one or more of the numbers 2, 3, 5, or 7 as factors.

Check Yourself 2
Which of the following numbers are prime numbers?

2, 6, 9, 11, 15, 19, 23, 35, 41

We can now define a second class of whole numbers.

Definition

Composite Number
A composite number is any whole number greater than 1 that is not prime.

Example 3

Identifying Composite Numbers

Which of the following numbers are composite: 18, 23, 25, and 38?

18 is a composite number. 1, 2, 3, 6, 9, and 18 are all factors of 18.
23 is not a composite number. 1 and 23 are the only factors. This means that 23 is a prime number.
25 is a composite number. 1, 5, and 25 are factors.
38 is a composite number. 1, 2, 19, and 38 are factors.

Check Yourself 3

Which of the following numbers are composite numbers?

2, 6, 10, 13, 16, 17, 22, 27, 31, 35

By the definitions of prime and composite numbers:

Property

0 and 1
The whole numbers 0 and 1 are neither prime nor composite.
To factor a number means to write the number as a product of whole-number factors.

**Example 4** Factoring a Composite Number

< Objective 3 >

Factor the number 10.

\[
10 = 2 \times 5
\]

The order in which you write the factors does not matter, so \( 10 = 5 \times 2 \) would also be correct.

Of course, \( 10 = 10 \times 1 \) is also a correct statement. However, in this section we are interested in products that use factors other than 1 and the given number.

Factor the number 21.

\[
21 = 3 \times 7
\]

**Check Yourself 4**

Factor 35.

In writing composite numbers as a product of factors, there may be several different possible factorizations.

**Example 5** Factoring a Composite Number

NOTE
There have to be at least two different factorizations, because a composite number has factors other than 1 and itself.

Find three ways to factor 72.

\[
72 = 8 \times 9
\]

\[
6 \times 12
\]

\[
3 \times 24
\]

**Check Yourself 5**

Find three ways to factor 42.

We now want to write composite numbers as a product of their prime factors. Look again at the first factored line of Example 5. The process of factoring can be continued until all the factors are prime numbers.

**Example 6** Factoring a Composite Number into Prime Factors

NOTE
This is often called a factor tree.

\[
72 = 2 \times 2 \times 2 \times 3 \times 3
\]

4 is still not prime, so we continue by factoring 4.

72 is now written as a product of prime factors.
NOTES
Finding the prime factorization of a number is a skill that is used when adding fractions.

Because 2 3 6 3 2, the order in which we write the factors does not matter. As a matter of convention, we usually write the factors in size order.

When we write 72 as 2 2 2 3 3, no further factorization is possible. This is called the prime factorization of 72.

Now, what if we start with the second factored line from the same example, 72 6 12?

72 6 12
Continue to factor 6 and 12.
2 3 3 4
Continue again to factor 4. Other choices for the factors of 12 are possible. The end result is always the same.

No matter which factor pair you begin with, you will always finish with the same set of prime factors. In this case, the factor 2 appears three times and the factor 3 appears twice. The order in which we write the factors does not matter.

Check Yourself 6

We could also write
72 2 36
Continue the factorization.

The Fundamental Theorem of Arithmetic
There is exactly one prime factorization for any composite number.

The method shown in Example 6 always works. However, an easier method for factoring composite numbers exists. This method is particularly useful when factoring large numbers, in which case factoring with a number tree becomes unwieldy.

Factoring by Division
To find the prime factorization of a number, divide the number by a series of primes until the final quotient is a prime number.

Example 7 Finding Prime Factors
To write 60 as a product of prime factors, divide 2 into 60 for a quotient of 30. Continue to divide by 2 again for the quotient 15. Because 2 does not divide exactly into 15, we try 3. Because the quotient 5 is prime, we are done.

\[
\begin{align*}
60 & \rightarrow 30 \rightarrow 15 \rightarrow 5 \\
2 \rightarrow 2 \rightarrow 3 \rightarrow 5
\end{align*}
\]

Our factors are the prime divisors and the final quotient. We have

\[
60 = 2 \times 2 \times 3 \times 5
\]
Check Yourself 7

Complete the process to find the prime factorization of 90.

\[
\begin{array}{c}
45 \\
2/90 \\
2/45
\end{array}
\]

Remember to continue until the final quotient is prime.

Writing composite numbers in their completely factored form can be simplified if we use a format called continued division.

Example 8 Finding Prime Factors Using Continued Division

Use the continued-division method to divide 60 by a series of prime numbers.

\[
\begin{align*}
2 & \mid 60 \\
2 & \mid 30 \\
3 & \mid 15 \\
5 & \\
\end{align*}
\]

Stop when the final quotient is prime.

To write the factorization of 60, we include each divisor used and the final prime quotient. In our example, we have

\[
60 = 2 \cdot 2 \cdot 3 \cdot 5
\]

Check Yourself 8

Find the prime factorization of 234.

We know that a factor or a divisor of a whole number divides that number exactly. The factors or divisors of 20 are

1, 2, 4, 5, 10, and 20

Each of these numbers divides 20 exactly, that is, with no remainder.

Next, we look at common factors or divisors. A common factor or divisor of two numbers is any factor that divides both the numbers exactly.

Example 9 Finding Common Factors

Look at the numbers 20 and 30. Is there a common factor for the two numbers? First, we list the factors. Then, we circle the ones that appear in both lists.

Factors

20: 1, 2, 4, 5, 10, 20
30: 1, 2, 3, 5, 6, 10, 15, 30

We see that 1, 2, 5, and 10 are common factors of 20 and 30. Each of these numbers divides both 20 and 30 exactly.
Our later work with fractions will require that we find the greatest common factor of a group of numbers.

**Definition**

**Greatest Common Factor**

The **greatest common factor (GCF)** of a group of numbers is the largest number that divides each of the given numbers exactly.

In the first part of Example 9, the common factors of the numbers 20 and 30 were listed as

\[1, 2, 5, 10\]

**Common factors of 20 and 30**

The GCF of the two numbers is 10, because 10 is the largest of the four common factors.

**Check Yourself 9**

List the factors of 30 and 36, and then find the GCF.

The method of Example 9 also works in finding the GCF of a group of more than two numbers.

**Example 10 Finding the GCF by Listing Factors**

Find the GCF of 24, 30, and 36. We list the factors of each of the three numbers.

- **24:** \[1, 2, 3, 4, 6, 8, 12, 24\]
- **30:** \[5, 6, 10, 15, 30\]
- **36:** \[4, 6, 9, 12, 18, 36\]

6 is the GCF of 24, 30, and 36.

**Check Yourself 10**

Find the GCF of 16, 24, and 32.

The process shown in Example 10 is very time-consuming when larger numbers are involved. A better approach to the problem of finding the GCF of a group of numbers uses the prime factorization of each number.

**Step by Step Finding the GCF**

- **Step 1:** Write the prime factorization for each of the numbers in the group.
- **Step 2:** Locate the prime factors that appear in every prime factorization.
- **Step 3:** The GCF is the **product** of all the common prime factors.
Example 11  Finding the GCF

Find the GCF of 20 and 30.

**Step 1** Write the prime factorizations of 20 and 30.

\[
20 = 2 \times 2 \times 5 \\
30 = 2 \times 3 \times 5
\]

**Step 2** Find the prime factors common to each number.

\[
20 = 2, 2, 5 \\
30 = 2, 3, 5
\]

2 and 5 are the common prime factors.

**Step 3** Form the product of the common prime factors.

\[
2 \times 5 = 10
\]

10 is the GCF.

Check Yourself 11

Find the GCF of 30 and 36.

To find the GCF of a group of more than two numbers, we use the same process.

Example 12  Finding the GCF

Find the GCF of 24, 30, and 36.

\[
24 = 2 \times 2 \times 3 \\
30 = 2 \times 3 \times 5 \\
36 = 2 \times 2 \times 3 \times 3
\]

2 and 3 are the prime factors common to all three numbers.

\[
2 \times 3 = 6
\]

6 is the GCF.

Check Yourself 12

Find the GCF of 15, 30, and 45.

Sometimes, two numbers have no common factors other than 1.
Finding the GCF

Example 13

Find the GCF of 15 and 28.

\[
\begin{array}{c|c|c}
\text{Factor} & \text{Number} & \text{Prime Factorization} \\
\hline
3 & 15 & 3 \\
5 & 15 & 5 \\
2 & 28 & 2 \\
7 & 28 & 7 \\
\end{array}
\]

There are no common prime factors listed. But remember that 1 is a factor of every whole number.

1 is the GCF.

Check Yourself 13

Find the GCF of 30 and 49.

Another idea that will be important in our work with fractions is the concept of multiples. Every whole number has an associated group of multiples.

Definition

Multiples

The multiples of a number are the product of that number with the natural numbers 1, 2, 3, 4, 5, . . . .

Example 14

Listing Multiples

Objective 5

List the multiples of 3.

The multiples of 3 are

\[3, 6, 9, 12, \ldots\]

An easy way of listing the multiples of 3 is to think of counting by threes.

Check Yourself 14

List the first seven multiples of 4.

You may see the relationship between factors and multiples. Saying “12 is a multiple of 3” is the same as saying “3 is a factor of 12.”

Sometimes we need to find common multiples of two or more numbers.

Definition

Common Multiples

If a number is a multiple of each of a group of numbers, it is called a common multiple of the numbers; that is, it is a number that is evenly divisible by all the numbers in the group.
Prime Factorization and Least Common Multiples

SECTION 0.1

Example 15

Finding Common Multiples

NOTE

Find four common multiples of 3 and 5.
Some common multiples of 3 and 5 are
15, 30, 45, 60

Check Yourself 15

List the first six multiples of 6. Then look at your list from Check Yourself 14 and list some common multiples of 4 and 6.

In our later work with fractions, we will use the least common multiple of a group of numbers.

Definition

Least Common Multiple

The least common multiple (LCM) of a group of numbers is the smallest number that is a multiple of each number in the group.

It is possible to simply list the multiples of each number and then find the LCM by inspection.

Example 16

Finding the LCM

NOTE

Find the LCM of 6 and 8.
Multiples

6: 6, 12, 18, 24, 30, 36, 42, 48, . . .
8: 8, 16, 24, 32, 40, 48, . . .

24 is also a common multiple of 6 and 8, but we are looking for the smallest such number.

We see that 24 is the smallest number common to both lists. So 24 is the LCM of 6 and 8.

Check Yourself 16

Find the LCM of 20 and 30 by listing the multiples of each number.

The technique of Example 16 will work for any group of numbers. However, it becomes tedious for larger numbers. Here is an easier approach.
Step by Step

Finding the LCM

Step 1 Write the prime factorization for each of the numbers in the group.
Step 2 List the prime factors that occur the greatest number of times in any one prime factorization.
Step 3 Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.

Some students prefer to line up the factors to help remember the process of finding the LCM of a group of numbers.

Example 17
Finding the LCM

To find the LCM of 10 and 18, we factor:

\[
\begin{array}{c}
10 \\
18
\end{array}
\begin{array}{c}
2 \\
3 3
\end{array}
\]

Bring down the factors.

2 and 5 appear, at most, one time in any one factorization. 3 appears twice in one factorization.

\[
\begin{array}{c}
2 3 3 5
\end{array}
\]

So 90 is the LCM of 10 and 18.

Check Yourself 17

Use the method of Example 17 to find the LCM of 24 and 36.

The procedure is the same for a group of more than two numbers.

Example 18
Finding the LCM

To find the LCM of 12, 18, and 20, we factor:

\[
\begin{array}{c}
12 \\
18 \\
20
\end{array}
\begin{array}{c}
2 2 3 \\
2 3 3 \\
2 2 5
\end{array}
\]

2 and 3 appear twice in one factorization; 5 appears just once.

\[
\begin{array}{c}
2 3 3 5
\end{array}
\]

So 180 is the LCM of 12, 18, and 20.

Check Yourself 18

Find the LCM of 3, 4, and 6.
Check Yourself ANSWERS

1. 1, 2, 3, 4, 6, 8, 12, and 24  
2. 2, 11, 19, 23, and 41 are prime numbers.  
3. 6, 10, 16, 22, 27, and 35 are composite numbers.  
4. 5 7  
5. 2, 21, 3, 14, 6, 7  
6. 2 2 2 3 3  
7. \( \frac{45}{2 \div 90} \)  
8. 2 3 3 13  
9. 30: 1, 2, 3, 5, 6, 10, 15, 30  
36: 1, 2, 3, 4, 6, 9, 12, 18, 36  
The GCF is 6.  
10. 16: 1, 2, 4, 8, 16  
24: 1, 2, 3, 4, 6, 8, 12, 24  
32: 1, 2, 4, 8, 16, 32  
The GCF is 8.  
11. 30  
36  
The GCF is 2 3 6.  
12. 15  
The GCF is 1; 30 and 49 are relatively prime.  
13. The first seven multiples of 4 are 4, 8, 12, 16, 20, 24, and 28.  
14. 6, 12, 18, 24, 30, 36; some common multiples of 4 and 6 are 12, 24, and 36.  
15. The multiples of 20 are 20, 40, 60, 80, 100, 120, . . . ; the multiples of 30 are 30, 60, 90, 120, 150, . . . ; the LCM of 20 and 30 is 60, the smallest number common to both lists.  
16. 24  
36  
The GCF is 2 2 3 3 72  
17. 12  
18. The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.1

(a) The centered dot in the expression 3 4 indicates _____________.

(b) A composite number is any whole number greater than 1 that is not ______________.

(c) A pair of numbers that have no common factor other than 1 are called ____________ prime.

(d) Saying “12 is a ______________ of 3” is the same as saying “3 is a factor of 12.”
0.1 exercises

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< Objective 1 >
List the factors of each of the following numbers.

1. 8
2. 6
3. 10
4. 12
5. 15
6. 21
7. 24
8. 32
9. 64
10. 66

< Objective 2 >
Use the following list of numbers for exercises 13 and 14.
0, 1, 15, 19, 23, 31, 49, 55, 59, 87, 91, 97, 103, 105

13. Which of the given numbers are prime?
14. Which of the given numbers are composite?
15. List all the prime numbers between 30 and 50.
16. List all the prime numbers between 55 and 75.

< Objective 3 >
Find the prime factorization of each number.

17. 20
18. 22
19. 30
20. 35
21. 51
22. 24
In later mathematics courses, you often will want to find factors of a number with a given sum or difference. The following problems use this technique.

37. Find two factors of 24 with a sum of 10.
38. Find two factors of 15 with a difference of 2.
39. Find two factors of 30 with a difference of 1.
40. Find two factors of 28 with a sum of 11.

< Objective 4 >
Find the GCF for each of the following groups of numbers.

41. 4 and 6
42. 6 and 9
43. 10 and 15
44. 12 and 14
45. 21 and 24
46. 22 and 33
47. 20 and 21
48. 28 and 42
49. 18 and 24
50. 35 and 36
51. 45, 60, and 75
52. 36, 54, and 180
53. 12, 36, and 60
54. 15, 45, and 90
55. 105, 140, and 175
56. 32, 80, and 112
57. 25, 75, and 150
58. 36, 72, and 144

Answers

23. 63 24. 94
25. 70 26. 90
27. 88 28. 100
29. 130 30. 66
31. 315 32. 400
33. 225 34. 132
35. 189 36. 330

37. _______ 38. _______
39. _______ 40. _______
41. _______ 42. _______
43. _______ 44. _______
45. _______ 46. _______
47. _______ 48. _______
49. _______ 50. _______
51. _______ 52. _______
53. _______ 54. _______
55. _______ 56. _______
57. _______ 58. _______
< Objective 5 >

Find the LCM for each of the following groups of numbers. Use whichever method you wish.

59. 12 and 15
60. 12 and 21
61. 18 and 36
62. 25 and 50
63. 25 and 40
64. 10 and 14
65. 3, 5, and 6
66. 2, 8, and 10
67. 18, 21, and 28
68. 8, 15, and 20
69. 20, 30, and 40
70. 12, 20, and 35

71. Prime numbers that differ by two are called twin primes. Examples are 3 and 5, 5 and 7, and so on. Find one pair of twin primes between 85 and 105.

72. The following questions refer to twin primes (see exercise 71).
   (a) Search for, and make a list of several pairs of twin primes in which the primes are greater than 3.
   (b) What do you notice about each number that lies between a pair of twin primes?
   (c) Write an explanation for your observation in part (b).

73. Obtain (or imagine that you have) a quantity of square tiles. Six tiles can be arranged in the shape of a rectangle in two different ways:

   (a) Record the dimensions of the rectangles shown.
   (b) If you use seven tiles, how many different rectangles can you form?
   (c) If you use ten tiles, how many different rectangles can you form?
   (d) What kind of number (of tiles) permits only one arrangement into a rectangle? More than one arrangement?

74. The number 10 has four factors: 1, 2, 5, and 10. We can say that 10 has an even number of factors. Investigate several numbers to determine which numbers have an even number of factors and which numbers have an odd number of factors.

75. A natural number is said to be perfect if it is equal to the sum of its divisors, except itself.
   (a) Show that 28 is a perfect number.
   (b) Identify another perfect number less than 28.

76. Find the smallest natural number that is divisible by all of the following: 2, 3, 4, 6, 8, 9.
Suppose that a school has 1,000 lockers and that they are all closed. A person passes through, opening every other locker, beginning with locker 2. Then another person passes through, changing every third locker (closing it if it is open, opening it if it is closed), starting with locker 3. Yet another person passes through, changing every fourth locker, beginning with locker 4. This process continues until 1,000 people pass through.

(a) At the end of this process, which locker numbers are closed?
(b) Write an explanation for your answer to part (a).
(Hint: It may help to attempt exercise 74 first.)

Answers

We provide the answers for the odd-numbered problems at the end of each exercise set.

1. 1, 2, 4, 8  3. 1, 2, 5, 10  5. 1, 3, 5, 15  
7. 1, 2, 3, 4, 6, 8, 12, 24  9. 1, 2, 4, 8, 16, 32, 64  
11. 1, 13  13. 19, 23, 31, 59, 97, 103  
15. 31, 37, 41, 43, 47  17. 2, 2, 5  
19. 2 3 5  21. 3 17  
23. 3 3 7  25. 2 5 7  27. 2 2 2 11 
29. 2)130 
31. 3 3 5 7  33. 3 3 5 5  
35. 3)189  
37. 4, 6  39. 5, 6  41. 2  43. 5  45. 3  
47. 1  49. 6  51. 15  53. 12  55. 35  57. 25  59. 60  
61. 36  63. 200  
65. 30  67. 252  69. 120  
71. 101, 103  73. Above and Beyond  75. Above and Beyond  
77. Above and Beyond
Fractions and Mixed Numbers

1 > Simplify a fraction
2 > Multiply and divide fractions
3 > Add and subtract fractions
4 > Write fractions as mixed numbers
5 > Multiply and divide mixed numbers
6 > Add and subtract mixed numbers

This section provides a review of the basic arithmetic operations—addition, subtraction, multiplication, and division—with fractions and mixed numbers.

In Section 0.1, we identified the set of whole numbers as the set consisting of the numbers 0, 1, 2, 3, and so on. In this section, we look at the set of positive numbers that can be written as fractions.

There are two types of fractions that we examine here: proper fractions and improper fractions. **Proper fractions** are those fractions that are less than 1, such as \( \frac{1}{2} \) and \( \frac{5}{7} \) (the numerator is less than the denominator). **Improper fractions** are those fractions that are greater than or equal to 1, such as \( \frac{7}{2} \) and \( \frac{19}{5} \) (the numerator is greater than the denominator).

Every whole number can be written in fraction form, \( \frac{a}{b} \), in which the denominator \( b \neq 0 \). In fact, there are many fraction forms for each number. This is because the fraction bar can be interpreted as division. For example, we can write \( \frac{2}{2} \) as \( \frac{2}{2} \). Of course, this is another way of writing the whole number 1. Any fraction in which the numerator and the denominator are the same is a representation of the number 1 because any nonzero number divided by itself is 1.

\[
1 \quad \frac{2}{2} \quad 1 \quad \frac{12}{12} \quad 1 \quad \frac{257}{257}
\]

These fractions are called **equivalent fractions** because they all represent the same number.

To determine whether two fractions are equivalent or to find equivalent fractions, we use the **Fundamental Principle of Fractions**. The Fundamental Principle of Fractions arises from the idea that multiplying any number by 1 does not change the number.
### Property

**The Fundamental Principle of Fractions**

\[
\frac{a}{b} \cdot \frac{c}{b} = \frac{a \cdot c}{b^2}, \text{ or } \frac{a}{b} \cdot \frac{b}{c} = \frac{a}{c}, \text{ in which neither } b \text{ nor } c \text{ is zero.}
\]

---

### Example 1

**Rewriting Fractions**

**< Objective 1 >**

**NOTE**

Each representation is a numeral, or name for the number. Each number has many names.

**NOTE**

In each case, we use the Fundamental Principle of Fractions with \(c\) equal to a different number.

**Write three fractional representations for each number.**

(a) \(\frac{2}{3}\)

We use the Fundamental Principle of Fractions to multiply the numerator and denominator by the same number.

\[
\begin{align*}
\frac{2}{3} & \quad \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6} \\
\frac{2}{3} & \quad \frac{2 \cdot 3}{3 \cdot 3} = \frac{6}{9} \\
\frac{2}{3} & \quad \frac{2 \cdot 10}{3 \cdot 10} = \frac{20}{30}
\end{align*}
\]

(b) \(5\)

\[
\begin{align*}
5 & \quad \frac{5 \cdot 2}{1 \cdot 2} = \frac{10}{2} \\
5 & \quad \frac{5 \cdot 3}{1 \cdot 3} = \frac{15}{3} \\
5 & \quad \frac{5 \cdot 100}{1 \cdot 100} = \frac{500}{100}
\end{align*}
\]

---

**Check Yourself 1**

**Write three fractional representations for each number.**

(a) \(\frac{5}{8}\)

(b) \(\frac{4}{3}\)

(c) \(3\)

The simplest fractional representation for a number has the smallest numerator and denominator. Fractions written in this form are said to be **simplified**.
Example 2
Simplifying Fractions

Simplify each fraction.

(a) \(\frac{22}{55}\)  
(b) \(\frac{35}{45}\)  
(c) \(\frac{24}{36}\)

We first find the prime factors for the numerator and for the denominator.

(a) \(\frac{22}{55}\)  
We then use the Fundamental Principle of Fractions.

\[
\frac{22}{55} \quad \frac{2 \cdot 11}{5 \cdot 11}
\]

(b) \(\frac{35}{45}\)  
Using the fundamental principle to remove the common factor of 5 yields

\[
\frac{35}{45} \quad \frac{7 \cdot 5}{3 \cdot 3 \cdot 5} \quad \frac{7 \cdot 5}{9 \cdot 5}
\]

(c) \(\frac{24}{36}\)  
Removing the common factor \(2 \cdot 2 \cdot 3\) yields

\[
\frac{24}{36} \quad \frac{2}{2 \cdot 3 \cdot 3} \quad \frac{2}{3}
\]

Check Yourself 2

Simplify each fraction.

(a) \(\frac{21}{33}\)  
(b) \(\frac{15}{30}\)  
(c) \(\frac{12}{54}\)

The Fundamental Principle of Fractions is really based on the way in which we multiply fractions. To multiply a pair of fractions, we multiply the numerators—the result becomes the numerator of their product. Then, we multiply the denominators—the result becomes the denominator of the product.

Property

Multiplying Fractions

\[
\frac{a \cdot c}{b \cdot d} = \frac{a \cdot c}{b \cdot d}
\]

When multiplying two fractions, rewrite them in factored form, and then simplify before multiplying. To multiply a fraction by a whole number, we rewrite the whole number as a fraction in which the denominator is 1.
Example 3  Multiplying Fractions

< Objective 2 >

Find the product of the two fractions.

\[
\frac{9}{2} \cdot \frac{4}{3}
\]

\[
\frac{9}{2} \cdot \frac{4}{3} = \frac{9 \cdot 4}{2 \cdot 3} = \frac{36}{6} = 6
\]

The denominator of 1 is not necessary.

Check Yourself 3

Multiply and simplify each pair of fractions.

(a) \(\frac{3}{5} \cdot \frac{10}{7}\)  
(b) \(\frac{12}{5} \cdot \frac{10}{6}\)

Property

Dividing Fractions

\[
\frac{a}{b} \div \frac{c}{d} = \frac{a \cdot d}{b \cdot c}
\]

This rule says that to divide two fractions, invert the divisor (flip the second fraction) and multiply.

Example 4  Dividing Fractions

Find the quotient.

\[
\frac{7}{3} \div \frac{5}{6} = \frac{7 \cdot 6}{3 \cdot 5} = \frac{42}{15} = \frac{14}{5}
\]
Check Yourself 4

Find the result from dividing the two fractions.

\[
\frac{9}{2} \div \frac{3}{5}
\]

The next property tells us how to add fractions when they have the same denominator.

**Example 5 Adding Fractions**

< Objective 3 >

Find the sum of the two fractions.

\[
\frac{5}{8} + \frac{7}{12}
\]

The fractions \(\frac{5}{8}\) and \(\frac{7}{12}\) have different denominators. In order to add them, we need to find equivalent fractions that have the same denominator. To find the least common denominator (LCD) of these fractions, we find the LCM of their denominators 8 and 12.

From Section 0.1, we know that the LCM of 8 and 12 is 24, so we rewrite each fraction as an equivalent fraction with a denominator of 24.

To find equivalent fractions, multiply each fraction by 1.
Fractions and Mixed Numbers

Check Yourself 5

Find the sum for each pair of fractions.

(a) \( \frac{4}{5} + \frac{7}{9} \)  
(b) \( \frac{5}{6} + \frac{4}{15} \)

Subtracting Fractions

Subtracting fractions is treated exactly like adding them, except the numerator becomes the difference of the two numerators.

Example 6 Subtracting Fractions

Find the difference.

\[ \frac{7}{9} - \frac{1}{6} \]

The LCD is 18. We rewrite the fractions with that denominator.

\[ \frac{7}{9} = \frac{14}{18} \]
\[ \frac{1}{6} = \frac{3}{18} \]

\[ \frac{7}{9} - \frac{1}{6} = \frac{14}{18} - \frac{3}{18} = \frac{11}{18} \]

This fraction cannot be simplified.

Check Yourself 6

Find the difference \( \frac{11}{12} - \frac{5}{6} \).

Another way to write an improper fraction is as a mixed number.

Definition

Mixed Number

A mixed number is the sum of a whole number and a proper fraction.

For our later work it will be important to be able to change back and forth between improper fractions and mixed numbers. Because an improper fraction represents a
number that is greater than or equal to 1, we have the following rule:

**Property**

**Writing Improper Fractions as Mixed Numbers**

An improper fraction can always be written as either a mixed number or a whole number.

To do this, remember that you can think of a fraction as indicating division. The numerator is divided by the denominator. This leads us to the following rule:

**Step by Step**

**Writing an Improper Fraction as a Mixed Number**

1. Divide the numerator by the denominator.
2. If there is a remainder, write the remainder over the original denominator.

**Example 7**

**Writing a Fraction as a Mixed Number**

< Objective 4 >

Write \( \frac{17}{5} \) as a mixed number.

\[
\begin{align*}
5 & \quad 17 \\
\underline{15} & \quad 17 \\
\underline{15} & \quad 2
\end{align*}
\]

Divide 17 by 5.

**Check Yourself 7**

Write \( \frac{32}{5} \) as a mixed number.

In order to write a mixed number as an equivalent improper fraction, we write the whole-number part as an equivalent fraction with the same denominator as the fraction part. We then add the two fractions. This is illustrated in Example 8.

**Example 8**

**Writing a Mixed Number as an Improper Fraction**

(a) Write \( 3 \frac{2}{5} \) as an equivalent improper fraction.

Because the fraction part has a denominator of 5, we write the whole-number part as a fraction with 5 as its denominator.
Fractions and Mixed Numbers

SECTION 0.2

NOTE
With practice, you should be able to do this mentally.

NOTE
Multiply the denominator, 7, by the whole number, 4, and add the numerator, 5.

(b) Write \(4\frac{5}{7}\) as an improper fraction.

\[
\frac{5}{7} \quad \frac{(7 \cdot 4)}{7} \quad \frac{5}{7} \quad \frac{33}{7}
\]

Check Yourself 8

Write \(5\frac{3}{8}\) as an improper fraction.

When multiplying two mixed numbers, it is usually easier to change the mixed numbers to improper fractions and then perform the multiplication. Example 9 illustrates this method.

Example 9
Multiplying Two Mixed Numbers

< Objective 5 >

Multiply.

\[
\frac{2}{3} \cdot \frac{1}{2} \quad \frac{11}{3} \cdot \frac{5}{2} \quad \text{Change the mixed numbers to improper fractions.}
\]

\[
\frac{11 \cdot 5}{3 \cdot 2} \quad \frac{55}{6} \quad \frac{91}{6}
\]

Be careful! Students sometimes think of

\[
\frac{2}{3} \cdot \frac{1}{2} \quad \text{as} \quad \frac{2 \cdot 1}{3} \cdot \frac{2}{1}
\]

This is not the correct multiplication pattern. You must first change the mixed numbers to improper fractions.

Check Yourself 9

Multiply.

\[
\frac{2}{3} \cdot \frac{3}{2}
\]

When dividing mixed numbers, simply write the mixed or whole numbers as improper fractions as the first step. Then proceed with the division. Example 10 illustrates this approach.
Example 10

Dividing Two Mixed Numbers

Divide.

\[ \frac{3\frac{2}{8}}{1\frac{3}{4}} \]

Write the mixed numbers as improper fractions.

\[ \frac{19}{8} \times \frac{7}{4} \]

Invert the divisor and multiply as before.

\[ \frac{19}{8} \times \frac{7}{4} = \frac{133}{32} \]

Check Yourself 10

Divide \(3\frac{1}{5} \div 2\frac{2}{5}\).

When adding or subtracting mixed numbers, first write the mixed numbers as improper fractions and then proceed as you would when adding or subtracting fractions. Example 11 illustrates these concepts.

Example 11

Adding and Subtracting Mixed Numbers

(a) Add, and write the result as a mixed number.

\[ \frac{1}{6} + \frac{3}{8} \]

The LCD of the fractions is 24. Rename them with that denominator.

\[ \frac{19}{6} + \frac{19}{8} \]

Then add as before.

\[ \frac{76}{24} + \frac{57}{24} = \frac{133}{24} \]

\[ \frac{5}{\frac{13}{24}} \]

(b) Subtract.

\[ \frac{7}{10} - \frac{3}{8} \]

Write the fractions with denominator 40.

\[ \frac{87}{10} - \frac{27}{8} \]

Subtract as before.

\[ \frac{348}{40} - \frac{135}{40} = \frac{213}{40} \]

This can be written as \(5\frac{13}{40}\).
Fractions and Mixed Numbers

SECTION 0.2

Check Yourself 11

Perform the indicated operation. Write your result as a mixed number.

(a) \( \frac{7}{10} - \frac{5}{6} \)

(b) \( \frac{11}{12} - \frac{5}{8} \)

To subtract a mixed number from a whole number, we use the same techniques.

Example 12 Subtracting Mixed Numbers

Subtract.

\[ 6 \quad \frac{3}{4} \]

\[ 6 \quad \frac{3}{4} - \frac{24}{4} \quad \frac{11}{4} \]

Write both the whole number and the mixed number as improper fractions with a common denominator.

\[ \frac{13}{4} \]

This can be written as \( 3 \frac{1}{4} \).

Check Yourself 12

Subtract \( 7 \quad \frac{2}{5} \).

When adding mixed numbers, some students prefer to take advantage of the fact that a mixed number is the sum of a whole number and a fraction. To do this, add the whole-number parts and add the fraction parts separately, and then combine the two. You may need to simplify the fraction. Example 13 illustrates this.

Example 13 Adding Mixed Numbers

Add \( 6 \frac{2}{5} + \frac{4}{5} \).

\[ \frac{2}{5} + \frac{4}{5} \]

\[ 6 \quad 4 \quad \frac{2}{5} + \frac{4}{5} \]

\[ 10 \quad \frac{6}{5} \]

\[ 10 \quad \frac{1}{5} \]

\[ 11 \frac{1}{5} \]

Do not use this method to subtract mixed numbers.
In algebra, we usually use improper fractions rather than mixed numbers, so when we need to add mixed numbers, we will generally write them as improper fractions and then add them following the procedure shown in Example 11(a).

There are many applications in which fractions and mixed numbers are used. Examples 14 and 15 illustrate some of these.

**Check Yourself 13**

Add $\frac{3}{4}$ and $\frac{2}{3}$.

**Example 14** An Application of Fractions and Mixed Numbers

Chair rail molding 136 inches (in.) long must be cut into pieces of $3\frac{1}{3}$ in. each. How many pieces can be cut from the molding?

The word of usually indicates multiplication.

So four full-length pieces can be cut from the molding.

**Check Yourself 14**

After a family party, $10\frac{2}{3}$ cupcakes were left. If Amanda took $\frac{3}{8}$ of these, how many did she take?

**Example 15** An Application of Fractions and Mixed Numbers

José must trim $2\frac{5}{16}$ feet (ft) from a board 8 ft long. How long will the board be after it is cut?

The board will be $5\frac{11}{16}$ ft long after it is cut.

**Check Yourself 15**

Three pieces of lumber measure $5\frac{3}{8}$ ft, $7\frac{1}{2}$ ft, and $9\frac{3}{4}$ ft. What is the total length of the lumber?
Fractions and Mixed Numbers

SECTION 0.2

Check Yourself ANSWERS

1. Answers will vary.  
2. (a) \(\frac{7}{11}\); (b) \(\frac{1}{2}\); (c) \(\frac{2}{9}\)  
3. (a) \(\frac{6}{7}\); (b) \(4\)  
4. \(\frac{15}{2}\)  
5. (a) \(\frac{71}{45}\); (b) \(\frac{11}{10}\)  
6. \(\frac{7}{24}\)  
7. \(\frac{6\,\frac{2}{5}}{5}\)  
8. \(\frac{43}{8}\)  
9. \(\frac{49}{6}\) or \(\frac{1}{6}\)  
10. \(\frac{4}{3}\) or \(1\) \(\frac{1}{3}\)  
11. (a) \(\frac{143}{15}\) or \(9\) \(\frac{8}{15}\); (b) \(\frac{103}{24}\) or \(\frac{4}{7}{\frac{24}{24}}\)  
12. \(\frac{18}{5}\) or \(3\) \(\frac{3}{5}\)  
13. \(8\) \(\frac{5}{12}\)  
14. 4  
15. \(22\) \(\frac{5}{8}\) ft

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.2

(a) The numerator of a __________ fraction is less than the denominator.

(b) Two fractions that represent the same quantity are called __________ fractions.

(c) The __________ of a set of fractions is the same as the LCM of their denominators.

(d) A mixed number is the sum of a whole number and a proper __________.
Basic Skills

Objective 1

Give three equivalent fractions for each given fraction.

1. \( \frac{3}{7} \)
2. \( \frac{4}{9} \)
3. \( \frac{7}{8} \)
4. \( \frac{11}{13} \)
5. \( \frac{10}{17} \)
6. \( \frac{9}{16} \)
7. \( \frac{6}{11} \)
8. \( \frac{15}{16} \)

Write each fraction in simplest form.

9. \( \frac{8}{12} \)
10. \( \frac{12}{15} \)
11. \( \frac{10}{14} \)
12. \( \frac{15}{50} \)
13. \( \frac{12}{18} \)
14. \( \frac{28}{35} \)
15. \( \frac{35}{40} \)
16. \( \frac{21}{24} \)
17. \( \frac{11}{44} \)
18. \( \frac{10}{25} \)
19. \( \frac{12}{36} \)
20. \( \frac{18}{48} \)
21. \( \frac{48}{60} \)
22. \( \frac{48}{66} \)
< Objective 2 >

Multiply. Be sure to simplify each product.

27. \( \frac{3}{4} \cdot \frac{7}{5} \)

28. \( \frac{2}{3} \cdot \frac{8}{5} \)

29. \( \frac{3}{5} \cdot \frac{5}{7} \)

30. \( \frac{6}{11} \cdot \frac{8}{6} \)

31. \( \frac{6}{13} \cdot \frac{4}{9} \)

32. \( \frac{5}{9} \cdot \frac{6}{11} \)

33. \( \frac{3}{11} \cdot \frac{7}{9} \)

34. \( \frac{3}{10} \cdot \frac{5}{9} \)

< Objective 3 >

Add.

43. \( \frac{2}{5} + \frac{1}{4} \)

44. \( \frac{2}{3} + \frac{3}{10} \)

45. \( \frac{2}{5} + \frac{7}{15} \)

46. \( \frac{3}{10} + \frac{7}{12} \)
SECTION 0.2

Answers

47. \(\frac{3}{8}\) \(\frac{5}{12}\)
48. \(\frac{5}{36}\) \(\frac{7}{24}\)
49. \(\frac{2}{15}\) \(\frac{9}{20}\)
50. \(\frac{9}{14}\) \(\frac{10}{21}\)
51. \(\frac{7}{15}\) \(\frac{13}{18}\)
52. \(\frac{12}{25}\) \(\frac{19}{30}\)
53. \(\frac{1}{2}\) \(\frac{1}{4}\) \(\frac{1}{8}\)
54. \(\frac{1}{3}\) \(\frac{1}{5}\) \(\frac{1}{10}\)

Subtract.

55. \(\frac{8}{9}\) \(\frac{3}{9}\)
56. \(\frac{9}{10}\) \(\frac{6}{10}\)
57. \(\frac{5}{8}\) \(\frac{1}{8}\)
58. \(\frac{11}{12}\) \(\frac{7}{12}\)
59. \(\frac{7}{8}\) \(\frac{2}{3}\)
60. \(\frac{5}{6}\) \(\frac{3}{5}\)
61. \(\frac{11}{18}\) \(\frac{2}{9}\)
62. \(\frac{5}{6}\) \(\frac{1}{4}\)

< Objective 4 >

Write the following fractions as mixed numbers.

63. \(\frac{17}{4}\)
64. \(\frac{200}{11}\)

Write the following mixed numbers as fractions.

65. \(3\frac{1}{4}\)
66. \(6\frac{3}{4}\)

< Objectives 5–6 >

Perform the indicated operations.

67. \(\frac{2}{9}\) \(\frac{3}{9}\)
68. \(\frac{2}{9}\) \(\frac{4}{9}\)
69. \(1\frac{1}{3}\) \(2\frac{1}{5}\)
70. \(2\frac{1}{4}\) \(1\frac{1}{6}\)
71. $\frac{2}{5} - \frac{4}{5}$  
72. $\frac{3}{7} - \frac{2}{7}$

73. $\frac{2}{3} - \frac{1}{4}$  
74. $\frac{4}{5} - \frac{1}{6}$

75. $\frac{2}{5} \cdot \frac{3}{4}$  
76. $\frac{2}{7} \cdot \frac{1}{3}$

77. $\frac{3}{2} + \frac{4}{5}$  
78. $\frac{3}{4} + \frac{3}{8}$

Solve the following applications.

79. **Crafts** Roseann is making shirts for her three children. One shirt requires $\frac{1}{2}$ yard (yd) of material, a second shirt requires $\frac{1}{3}$ yd of material, and the third shirt requires $\frac{1}{4}$ yd of material. How much material is required for all three shirts?

80. **Science** José rode his trail bike for 10 miles. Two-thirds of the distance was over a mountain trail. How long is the mountain trail?

81. **Business and Finance** You make $240 a day on a job. What will you receive for working $\frac{2}{3}$ of a day?

82. **Statistics** A survey has found that $\frac{3}{4}$ of the people in a city own pets. Of those who own pets, $\frac{2}{3}$ have cats. What fraction of those surveyed own cats?

83. **Social Science** The scale on a map is 1 in. = 200 miles (mi). What actual distance, in miles, does $\frac{3}{8}$ in. represent?

84. **Business and Finance** A family uses $\frac{2}{5}$ of its monthly income for housing and utilities on average. If the family’s monthly income is $1,750, what is spent for housing and utilities? What amount remains?
85. **SOCIAL SCIENCE** Of the eligible voters in an election, \( \frac{3}{4} \) were registered. Of those registered, \( \frac{5}{9} \) actually voted. What fraction of those people who were eligible voted?

86. **STATISTICS** A survey has found that \( \frac{7}{10} \) of the people in a city own pets. Of those who own pets, \( \frac{2}{3} \) have dogs. What fraction of those surveyed own dogs?

87. **SCIENCE** A jet flew at an average speed of 540 mi/h on a \( \frac{2}{3} \)-h flight. What was the distance flown?

88. **GEOMETRY** A piece of land that has \( 11\frac{2}{3} \) acres is being subdivided for home lots. It is estimated that \( \frac{2}{7} \) of the area will be used for roads. What amount remains to be used for lots?

89. **GEOMETRY** To find the approximate circumference or distance around a circle, we multiply its diameter by \( \frac{22}{7} \). What is the circumference of a circle with a diameter of 21 in.?

90. **GEOMETRY** The length of a rectangle is \( \frac{6}{7} \) yd, and its width is \( \frac{21}{26} \) yd. What is its area in square yards? (The area of a rectangle is the product of its length and its width.)

91. Every fraction (rational number) has a corresponding decimal form that either terminates or repeats. For example, \( \frac{5}{16} = 0.3125 \) (the decimal form terminates), and \( \frac{4}{11} = 0.363636 \ldots \) (the decimal form repeats). Investigate a number of fractions to determine which ones terminate and which ones repeat. (Hint: You can focus on the denominator; study the prime factorizations of several denominators.)
92. Complete the following sums:

\[
\begin{array}{cccc}
\frac{1}{2} & \frac{1}{4} \\
\frac{1}{2} & \frac{1}{4} & \frac{1}{8} \\
\frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} \\
\end{array}
\]

Based on these, predict the sum:

\[
\begin{array}{cccc}
\frac{1}{2} & \frac{1}{4} & \frac{1}{8} & \frac{1}{16} & \frac{1}{32} & \frac{1}{64} & \frac{1}{128} \\
\end{array}
\]

Answers

For exercises 1–7, answers will vary.

1. \(\frac{6}{14}, \frac{9}{21}, \frac{12}{28}\)
2. \(\frac{2}{3}, \frac{5}{7}\)
3. \(\frac{14}{16}, \frac{35}{40}, \frac{70}{80}\)
4. \(\frac{20}{34}, \frac{30}{51}, \frac{100}{170}\)
5. \(\frac{7}{12}, \frac{18}{22}, \frac{24}{33}\)
6. \(\frac{11}{53}, \frac{5}{8}\)
7. \(\frac{3}{7}, \frac{8}{15}, \frac{39}{7}\)
8. \(\frac{5}{5}, \frac{19}{7}, \frac{33}{42}\)
9. \(\frac{7}{13}, \frac{9}{22}, \frac{15}{52}\)
10. \(\frac{1}{4}, \frac{90}{4}, \frac{8}{24}\)
11. \(\frac{1}{14}, \frac{3}{7}, \frac{9}{15}\)
12. \(\frac{1}{2}, \frac{3}{5}, \frac{7}{12}\)
13. \(\frac{1}{12}, \frac{1}{12}, \frac{1}{12}\)
14. \(\frac{3}{8}, \frac{3}{15}, \frac{3}{15}\)
15. \(\frac{2}{5}, \frac{1}{5}, \frac{1}{5}\)
16. \(\frac{5}{7}, \frac{1}{5}, \frac{1}{5}\)
17. \(\frac{6}{12}, \frac{9}{18}, \frac{1}{2}\)
18. \(\frac{7}{14}, \frac{15}{30}, \frac{1}{3}\)
19. \(\frac{\text{yd}}{\text{yd}}, \frac{\$160}{\$160}, \frac{75 \text{ mi}}{75 \text{ mi}}\)
20. \(\frac{5}{12}, \frac{7520 \text{ mi}}{7520 \text{ mi}}\)
21. \(\frac{1}{4}, \frac{1}{4}, \frac{1}{4}\)

91. Above and Beyond
Decimals and Percents

1. Write a fraction as a decimal
2. Write a decimal as a fraction
3. Add and subtract decimals
4. Multiply and divide decimals
5. Write a percent as a fraction or decimal
6. Write a decimal or fraction as a percent

Because a fraction can be interpreted as division, we can divide the numerator of a fraction by its denominator to write the fraction as an equivalent decimal. The result is called its decimal equivalent.

**Example 1** Writing a Fraction as a Decimal

**Objective 1**

Write $\frac{5}{8}$ as a decimal.

RECALL

5 can be written as 5.0, 5.00, 5.000, and so on. In this case, we continue the division by adding zeros to the dividend until a 0 remainder is reached.

\[
\begin{array}{c}
8 \div 5.000 \\
8 &
\end{array}
\]

Because $\frac{5}{8}$ means $\frac{5}{8} \div 8$, divide 8 into 5.

\[
\begin{array}{c}
8 \div 5.000 \\
8 &
\end{array}
\]

0.625

We see that $\frac{5}{8} = 0.625$; 0.625 is the decimal equivalent of $\frac{5}{8}$.

**Check Yourself 1**

Find the decimal equivalent of $\frac{7}{8}$.

You should recall that the decimal 0.625 in Example 1 means $\frac{625}{1000}$. This is equivalent to $\frac{625}{1000} = \frac{6.250}{10000} = 0.6250$. 

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We could choose to round our answer to some specific decimal place. If we round to the nearest tenth, then we are rounding to one decimal place \(0.625\) \(0.6\) (to the nearest tenth), whereas if we rounded to the nearest hundredth, we are rounding to two decimal places, \(0.625\) \(0.63\) (to the nearest hundredth).

When a decimal does not terminate, we usually round it to a specific place, as in Example 2.

**Example 2** Writing a Fraction as a Decimal

Write \(\frac{3}{7}\) as a decimal. Round the answer to the nearest thousandth.

\[
\begin{array}{rll}
0.4285 & \quad \text{In this example, we are choosing to round} \\
7 & 3.0000 & \text{to three decimal places, so we must add} \\
2 & 8 & \text{enough zeros to carry the division to four} \\
20 & 14 & \text{decimal places.} \\
60 & 56 & \\
40 & 35 & \\
5 &
\end{array}
\]

So \(\frac{3}{7} = 0.429\) (to the nearest thousandth).

**Check Yourself 2**

Find the decimal equivalent of \(\frac{5}{12}\) to the nearest thousandth.

If the decimal equivalent of a fraction does not terminate, it will repeat a sequence of digits. These decimals are called *repeating decimals*.

**Example 3** Writing a Fraction as a Repeating Decimal

Write \(\frac{5}{11}\) as a decimal.

\[
\begin{array}{rll}
0.4545 & \quad \text{As soon as a remainder repeats itself, as 5 does here,} \\
11 & 5.0000 & \text{the pattern of digits will repeat in the quotient.} \\
4 & 4 & \quad \frac{5}{11} = 0.4545\ldots \\
60 & 55 & \\
5 &
\end{array}
\]

\
\]
Check Yourself 3

Use the bar notation to write the decimal equivalent of \( \frac{5}{7} \). (Be patient. You have to divide for a while to find the repeating pattern.)

To write a decimal as a fraction, write the decimal without the decimal point. This is the numerator of the fraction. The denominator of the fraction is a 1 followed by as many zeros as there are places in the decimal. The next two examples illustrate this process.

Example 4

Writing a Decimal as a Fraction

< Objective 2 >

<table>
<thead>
<tr>
<th>Decimal</th>
<th>Fraction</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>( \frac{7}{10} )</td>
</tr>
<tr>
<td>0.09</td>
<td>( \frac{9}{100} )</td>
</tr>
<tr>
<td>0.257</td>
<td>( \frac{257}{1,000} )</td>
</tr>
</tbody>
</table>

One place One zero Two places Two zeros Three places Three zeros

Check Yourself 4

Write as fractions.

(a) 0.3
(b) 0.311

When a decimal is written as an equivalent fraction, the common fraction that results should be simplified.

Example 5

Converting a Decimal to a Fraction

Convert 0.395 to a fraction and simplify the result.

\[
\frac{0.395}{1,000} = \frac{395}{1,000} = \frac{79}{200}
\]

Check Yourself 5

Write 0.275 as a fraction.

We add and subtract decimals using place value, just as we do with whole numbers. You must be sure to align the decimal points, as illustrated in Example 6.

Example 6

Adding or Subtracting Two Decimals

< Objective 3 >

Perform the indicated operation.

(a) Add 2.356 and 15.6.

Aligning the decimal points, we get

\[
\begin{array}{c}
2.356 \\
15.600 \\
\hline
17.956
\end{array}
\]

Although the zeros are not necessary, they ensure proper alignment.
Decimals and Percents

SECTION 0.3

(b) Subtract 3.84 from 8.1.

Again, we align the decimal points.

\[
\begin{align*}
8.10 & \\
-3.84 & \\
\hline
4.26 & \\
\end{align*}
\]

When subtracting, always add zeros so that the right columns line up.

Check Yourself 6

Perform the indicated operation.

\[
\begin{align*}
(a) & \quad 34.76 - 2.419 \\
(b) & \quad 71.82 - 8.197
\end{align*}
\]

Example 7 illustrates the multiplication of two decimal fractions.

Example 7

Multiplying Two Decimals

< Objective 4 >

Multiply 4.6 and 3.27.

\[
\begin{align*}
4.6 & \\
\times 3.27 & \\
\hline
322 & \\
920 & \\
\hline
15.042 & \\
\end{align*}
\]

It is not necessary to align decimals being multiplied. Note that the two factors have a total of three digits to the right of the decimal point.

The decimal point of the product is moved three digits to the left.

Check Yourself 7

Multiply 5.8 and 9.62.

Dividing decimals is a bit trickier. In order to divide when the divisor is a decimal, we multiply both the dividend and divisor by a large enough power of 10 that the divisor becomes a whole number. We show you how to set this up in Example 8.

Example 8

Rewriting a Problem That Requires Dividing by a Decimal

Rewrite the division so that the divisor is a whole number.

\[
\begin{align*}
2.57 & \quad 3.4 \\
\frac{2.57}{3.4} & \\
2.57 & \quad 10 \\
\frac{2.57}{3.4} & \quad 10 \\
25.7 & \quad 34 \\
\frac{25.7}{34} & \\
\end{align*}
\]

Write the division as a fraction.

We multiply the numerator and denominator by 10 so that the divisor is a whole number. This does not change the value of the fraction.

Multiplying by 10, shift the decimal point in the numerator and denominator one place to the right.

Our division problem is rewritten so that the divisor is a whole number.
So
\[\frac{2.57}{3.4} = \frac{25.7}{34}\]
After we multiply the numerator and denominator by 10, we see that \(2.57\) \(3.4\) is the same as \(25.7\) \(34\).

**Check Yourself 8**
Rewrite the division problem so that the divisor is a whole number.

\[\frac{3.42}{2.5}\]

Do you see the rule suggested by Example 8? We multiplied the numerator and the denominator (the dividend and the divisor) by 10. We made the divisor a whole number without altering the actual digits involved. All we did was shift the decimal point in the divisor and dividend the same number of places. This leads us to the following procedure.

**NOTE**
Of course, multiplying by any whole-number power of 10 greater than 1 is just a matter of shifting the decimal point to the right.

**Step by Step**
To Divide by a Decimal

**Step 1** Move the decimal point in the divisor to the right, making the divisor a whole number.

**Step 2** Move the decimal point in the dividend to the right the same number of places. Add zeros if necessary.

**Step 3** Place the decimal point in the quotient directly above the decimal point of the dividend.

**Step 4** Divide as you would with whole numbers.

Here is an example using the division rule.

**Example 9**
Rounding the Result of Dividing by a Decimal

Divide 1.573 by 0.48 and give the quotient to the nearest tenth.

Write
\[0.48 \div 1.573\]
Shift the decimal points two places to the right to make the divisor a whole number.

Now divide:
\[\begin{array}{c}
3.27 \\
48 \\
157.30 \\
144 \\
13 & 3 & 6 \\
3 & 70 \\
3 & 36 \\
34
\end{array}\]
Add a 0 to carry the division to the hundredths place. In this case, we want to find the quotient to the nearest tenth.

Round 3.27 to 3.3. So
\[1.573 \div 0.48 = 3.3\] (to the nearest tenth)
We have used fractions and decimals to name parts of a whole. Percents can also be used to accomplish this. The word percent means “for each hundred.” We can think of percents as fractions whose denominators are 100. So 25% can be written as \( \frac{25}{100} \) or, in simplified form, \( \frac{1}{4} \).

Because there are different ways of naming the parts of a whole, you need to know how to change from one of these ways to another. First, we look at writing a percent as a fraction. Because a percent is a fraction or a ratio with denominator 100, we can use the following rule.

**Example 10 Writing a Percent as a Fraction**

< **Objective 5** >

Write each percent as a fraction.

(a) 7% \( \frac{7}{100} \) 

(b) 25% \( \frac{25}{100} \) or \( \frac{1}{4} \)

**Check Yourself 10**

Write 12% as a fraction.

In Example 10, we wrote percents as fractions by replacing the percent sign with \( \frac{1}{100} \) and multiplying. How do we convert percents when we are working with decimals? Just move the decimal point two places to the left. This gives us a second rule for rewriting percents.

**Property Writing a Percent as a Decimal**

To write a percent as a decimal, replace the percent symbol with \( \frac{1}{100} \). As a result of multiplying by \( \frac{1}{100} \), the decimal point will move two places to the left.
Example 11  Writing a Percent as a Decimal

RECALL

Write each percent as a decimal.

(a) \(25\% = \frac{25}{100} = 0.25\)  The decimal point in 25\% is understood to be after the 5.

(b) \(4.5\% = \frac{4.5}{100} = 0.045\)  We must add a zero to move the decimal point.

(c) \(130\% = \frac{130}{100} = 1.30\)

NOTE

A percent greater than 100 gives a decimal greater than 1.

Check Yourself 11

Write as decimals.

(a) 5\%  (b) 3.9\%  (c) 115\%

Writing a decimal as a percent is the opposite of writing a percent as a decimal. We simply reverse the process. Here is the rule:

Property

Writing a Decimal as a Percent

To write a decimal as a percent, move the decimal point two places to the right and attach the percent symbol.

Example 12  Writing a Decimal as a Percent

< Objective 6 >

Write each decimal as a percent.

(a) 0.18 18\%
(b) 0.03 3\%
(c) 1.25 125\%

Check Yourself 12

Write each decimal as a percent.

(a) 0.27  (b) 0.045  (c) 1.3

The following rule allows us to write fractions as percents.
Writing a Fraction as a Percent

To write a fraction as a percent, write the decimal equivalent of the fraction by dividing. Then, move the decimal point two places to the right and attach the percent symbol.

Example 13

Writing a Fraction as a Percent

Write each fraction as a percent.

(a) \( \frac{3}{5} \) 0.60 \( \text{To find the decimal equivalent, just divide the denominator into the numerator.} \)

Now write the percent.

\[ \frac{3}{5} = 0.60 = 60\% \]

(b) \( \frac{1}{8} \) 0.125 12.5% or 12 1/2%

(c) \( \frac{1}{3} \) 0.3 0.333 33.3% or 33 1/3%

Check Yourself 13

Change each fraction to a percent equivalent.

(a) \( \frac{3}{4} \) \( \text{(b) } \frac{3}{8} \text{ (c) } \frac{2}{3} \)

Example 14 illustrates one of the many applications using decimals.

Example 14

An Application of Decimals

Lucretia’s car gets approximately 20 miles per gallon (mi/gal) of fuel. If 1 gal of fuel costs $1.93, how much does it cost her to drive 125 mi?

\[
\begin{align*}
125 & \quad 20 \\
6.25 & \quad \text{6.25 gal} \\
6.25 \times 1.93 & \quad 12.06 \text{ (rounded)}
\end{align*}
\]

Check Yourself 14

The art department has a budget of $195.75 to purchase art supplies. After purchasing 35 paintbrushes for $1.92 each, six jars of paint remover for $0.93 each, and four cans of blue paint for $2.95 each, how much money was left in the budget?
Check Yourself ANSWERS

1. 0.875  
2. 0.417  
3. 0.714285  
4. (a) \(\frac{3}{10}\); (b) \(\frac{311}{1,000}\)  
5. \(\frac{11}{40}\)  
6. (a) 37.179; (b) 63.623  
7. 55.796  
8. 34.2  
9. 2.7  
10. \(\frac{12}{100}\) or \(\frac{3}{25}\)  
11. (a) 0.05; (b) 0.039; (c) 1.15  
12. (a) \(\frac{27}{100}\)  
(b) 4.5%; (c) 130%  
13. (a) 75%; (b) 37.5%; (c) 66\%\ or \(\frac{2}{3}\)  
14. $111.17

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.3

(a) To write a fraction as a ____________, divide the numerator by the denominator.

(b) We use bar notation to indicate a ____________ decimal.

(c) Fractions, decimals, and ____________ are all ways of naming parts of a whole.

(d) To write a percent as a decimal, move the decimal two places to the ____________, and remove the percent symbol.
< Objective 1 >

Find the decimal equivalents for each of the following fractions.

1. \( \frac{3}{4} \)  
2. \( \frac{4}{5} \)  

3. \( \frac{9}{20} \)  
4. \( \frac{3}{10} \)  

5. \( \frac{1}{5} \)  
6. \( \frac{1}{8} \)  

7. \( \frac{5}{16} \)  
8. \( \frac{11}{20} \)  

9. \( \frac{7}{10} \)  
10. \( \frac{7}{16} \)  

11. \( \frac{27}{40} \)  
12. \( \frac{17}{32} \)  

Find the decimal equivalents rounded to the indicated place.

13. \( \frac{5}{6} \); thousandth  
14. \( \frac{7}{12} \); hundredth

15. \( \frac{4}{15} \); thousandth

Write the decimal equivalents using the bar notation.

16. \( \frac{1}{18} \)  
17. \( \frac{4}{9} \)

18. \( \frac{3}{11} \)

< Objective 2 >

Write each of the following as a fraction and simplify.

19. 0.9  
20. 0.3
0.3 exercises

Answers

21. 0.8
22. 0.6
23. 0.37
24. 0.97
25. 0.587
26. 0.379
27. 0.48
28. 0.75
29. 0.58
30. 0.65

< Objectives 3–4 >
Perform the indicated operations.

31. 7.1562 14.78
32. 6.2358 3.14
33. 11.12 8.3792
34. 6.924 5.2
35. 9.20 2.85
36. 17.345 11.12
37. 18.234 13.64
38. 21.983 9.395
39. 3.21 2.1
40. 15.6 7.123
41. 6.29 9.13
42. 8.245 3.1

Divide.

43. 16.68 6
44. 43.92 8
45. 1.92 4
46. 5.52 6
47. 5.48 8
48. 2.76 8
49. 13.89 6
50. 21.92 5
51. 185.6 32
52. 165.6 36
53. 79.9 34
54. 179.3 55
55. 52 13.78
56. 76 26.22
57. 0.6 11.07
58. 0.8 10.84
59. 3.8 7.22
60. 2.9 13.34
61. 5.2 11.622
62. 6.4 3.616

< Objective 5 >
Write as fractions.

63. 6%
64. 17%
65. 75%
66. 20%
67. 65%
68. 48%
69. 50%
70. 52%
71. 46%
72. 35%
73. 66%
74. 4%

Write as decimals.

75. 20%
76. 70%
77. 35%
78. 75%

Answers

51. 
52. 
53. 
54. 
55. 
56. 
57. 
58. 
59. 
60. 
61. 
62. 
63. 
64. 
65. 
66. 
67. 
68. 
69. 
70. 
71. 
72. 
73. 
74. 
75. 
76. 
77. 
78. 
0.3 exercises

 Answers

79. 39% 80. 27%
81. 5% 82. 7%
83. 135% 84. 250%
85. 240% 86. 160%
87. 
88. 
89. 
90. 
91. 
92. 
93. 
94. 
95. 
96. 
97. 
98. 
99. 
100. 
101. 
102. 

< Objective 6 >
Write each decimal as a percent.

87. 4.40 88. 5.13
89. 0.065 90. 0.095
91. 0.025 92. 0.085
93. 0.002 94. 0.008

Write each fraction as a percent.

95. \( \frac{1}{4} \) 96. \( \frac{4}{5} \)
97. \( \frac{2}{5} \) 98. \( \frac{1}{2} \)
99. \( \frac{1}{5} \) 100. \( \frac{3}{4} \)
101. \( \frac{5}{8} \) 102. \( \frac{7}{8} \)
103. **Statistics** On a math quiz, Adam answered 18 of 20 questions correctly, or \( \frac{18}{20} \) of the quiz. Write the decimal equivalent of this fraction.

\[
\begin{array}{c|c}
2 \times 3 & 5 \times 4 = 20 \\
1 \times 5 & 3 \times 4 = 12 \\
2 \times 5 & 10 \\
4 \times 5 & 5 \times 4 = 20 \\
15 \div 2 & 15 \div 4 = 11.25 \\
4 \times 3 & 5 \times 3 = 15 \\
5 \div 6 & 6 \div 3 = 2 \\
9 \div 4 & 5 \div 6 = 1.5 \\
5 \div 9 & 6 \div 9 = 0.667 \\
1 \times 2 & 2 \times 1 = 2
\end{array}
\]

104. **Statistics** In a weekend baseball tournament, Joel had 4 hits in 13 times at bat. That is, he hit safely of the time. Write the decimal equivalent for Joel’s hitting, rounding to three decimal places. (That number is Joel’s batting average.)

105. **Business and Finance** A restaurant bought 50 glasses at a cost of $39.90. What was the cost per glass, to the nearest cent?

106. **Business and Finance** The cost of a case of 48 items is $28.20. What is the cost of an individual item, to the nearest cent?

107. **Business and Finance** An office bought 18 handheld calculators for $284. What was the cost per calculator, to the nearest cent?

108. **Business and Finance** Al purchased a new refrigerator that cost $736.12 with interest included. He paid $100 as a down payment and agreed to pay the remainder in 18 monthly payments. What amount will he be paying per month?

109. **Business and Finance** The cost of a television set with interest is $490.64. If you make a down payment of $50 and agree to pay the balance in 12 monthly payments, what will be the amount of each monthly payment?

**Answers**

1. 0.75  
3. 0.45  
5. 0.2  
7. 0.3125  
9. 0.7  
11. 0.675

13. 0.833  
15. 0.267  
17. 0.4  
19. \( \frac{9}{10} \)  
21. \( \frac{4}{5} \)  
23. \( \frac{37}{100} \)
0.3 exercises

25. \( \frac{587}{1000} \)  
27. \( \frac{12}{25} \)  
29. \( \frac{29}{50} \)  
31. 21.9362  
33. 19.4992

35. 6.35  
37. 4.594  
39. 6.741  
41. 57.4277  
43. 2.78

45. 0.48  
47. 0.685  
49. 2.315  
51. 5.8  
53. 2.35

55. 0.265  
57. 18.45  
59. 1.9  
61. 2.235  
63. \( \frac{3}{50} \)  
65. \( \frac{3}{4} \)

67. \( \frac{13}{20} \)  
69. \( \frac{1}{2} \)  
71. \( \frac{23}{50} \)  
73. \( \frac{33}{50} \)  
75. 0.2  
77. 0.35

79. 0.39  
81. 0.05  
83. 1.35  
85. 2.4  
87. 440%  
89. 6.5%

91. 2.5%  
93. 0.2%  
95. 25%  
97. 40%  
99. 20%

101. 62.5%  
103. 0.9  
105. $0.80 or 80¢  
107. $15.78

109. $36.72
Exponents and the Order of Operations

1. Write a product of factors in exponential form

2. Evaluate an expression involving several operations

Often in mathematics we define symbols that allow us to write a mathematical statement in a more compact or “shorthand” form. This is an idea that you have encountered before. For example, the repeated addition

\[ 5 \times 5 \times 5 \]

can be rewritten as

\[ 5^3 \]

Thus, multiplication is shorthand for repeated addition.

In algebra, we frequently have a number or variable that is repeated as a factor in an expression several times. For instance, we might have

\[ 5 \times 5 \times 5 \]

To abbreviate this product, we write

\[ 5 \times 5 \times 5 \times 5^3 \]

This is called exponential notation or exponential form. The exponent or power, here 3, indicates the number of times that the factor or base, here 5, appears in a product.

Example 1 Writing Products in Exponential Form

Write \( 3 \times 3 \times 3 \times 3 \) using exponential form. The number 3 appears four times in the product, so

\[ 3^4 \]

This is read “3 to the fourth power.”
Check Yourself 1

Rewrite each expression using exponential form.

(a) 4 • 4 • 4 • 4 • 4
(b) 7 • 7 • 7 • 7

To evaluate an arithmetic expression, you need to know the order in which the operations are done. To see why, simplify the expression $5^2 \cdot 3$.

Method 1

Method 2

$5^2 \cdot 3 = 25 \cdot 3 = 75$

$5 \cdot 3^2 = 5 \cdot 9 = 45$

Because we get different answers depending on how we do the problem, the language of mathematics would not be clear if there were no agreement on which method is correct. The following rules tell us the order in which operations should be done.

Step by Step

The Order of Operations

Step 1: Evaluate all expressions inside grouping symbols first.

Step 2: Evaluate all expressions involving exponents.

Step 3: Do any multiplication or division in order, working from left to right.

Step 4: Do any addition or subtraction in order, working from left to right.

Example 2

Evaluating Expressions

< Objective 2 >

Evaluate $5^2 \cdot 3$.

There are no parentheses or exponents, so start with step 3: First multiply and then add.

$5 \cdot 3^2 = 5 \cdot 9 = 45$

Method 2 shown above is the correct one.

Check Yourself 2

Evaluate the following expressions.

(a) $20 \cdot 3 \cdot 4$
(b) $9 \cdot 6 \cdot 3$

When there are no parentheses, evaluate the exponents first.
**Exponents and the Order of Operations**

**SECTION 0.4**

### Example 3

**Evaluating Expressions**

Evaluate \(5^3\):

\[5^3 = 5 \times 5 \times 5 = 125\]

Evaluate the power first.

\[45\]

### Check Yourself 3

Evaluate \(4^2\).

Both scientific and graphing calculators correctly interpret the order of operations, as demonstrated in Example 4.

### Example 4

**Using a Calculator to Evaluate Expressions**

Use your scientific or graphing calculator to evaluate each expression. Round the answer to the nearest tenth.

(a) \(24.3 \div 6.2 \times 3.53\)

When evaluating expressions by hand, you must consider the order of operations. In this case, the multiplication must be done first, and then the addition. With a calculator, you need only enter the expression correctly. The calculator is programmed to follow the order of operations.

Entering \(24.3 \div 6.2 \times 3.53 \text{ ENTER}\) yields the evaluation 46.186. Rounding to the nearest tenth, we have 46.2.

(b) \(2.45^3 \div 49 \div 8,000 \times 12.2 \div 1.3\)

Some calculators use the caret (^) to designate powers. Others use the symbol \(x^y\) (or \(y^x\)).

Entering \(2.45^3 \text{ ENTER} \div 49 \text{ ENTER} \div 8,000 \text{ ENTER} \times 12.2 \text{ ENTER} \div 1.3 \text{ ENTER}\) yields the evaluation 30.56. Rounding to the nearest tenth, we have 30.6.

### Check Yourself 4

Use your scientific or graphing calculator to evaluate each expression.

(a) \(67.89 \div 4.7 \times 12.7\)

(b) \(4.3 \div 55.5 \times 3.75^3 \div 8,007 \times 1,600\)

Operations inside grouping symbols are always done first.
Example 5 Evaluating Expressions

Evaluate \((5 - 2)^3\).
Do the operation inside the parentheses as the first step.
\[(5 - 2)^3 = 7^3 = 343\]

Add.

Check Yourself 5

Evaluate \(4(9 - 3)\).

The principle is the same when more than two “levels” of operations are involved.

Example 6 Using Order of Operations

(a) Evaluate \(4(2 \cdot 3)^3\).
Add inside the parentheses first.
\[4(2 \cdot 3)^3 = 4(6)^3 = 4 \cdot 216 = 864\]

(b) Evaluate \(5(7 - 3)^2 - 10\).
Evaluate the expression inside the parentheses.
\[5(7 - 3)^2 = 5(4)^2 = 5 \cdot 16 = 80\]

Subtract.

Check Yourself 6

Evaluate.

(a) \(4 \cdot 3^2 + 8 - 11\)
(b) \(12 - 4(2 - 3)^2\)

The correct order of operations must be followed within a set of grouping symbols, as shown in Example 7.
Using Order of Operations

Example 7

Evaluate $3 \cdot [(1 - 2)^2 - 5] + 8$.

We evaluate the expression in the parentheses within the brackets first. Next, we evaluate the exponent before proceeding to the subtraction. After evaluating everything within the brackets, we follow the correct order of operations by multiplying first, and then adding.

$$3 \cdot [(1 - 2)^2 - 5] + 8 = 3 \cdot [(3)^2 - 5] + 8 = 3 \cdot [9 - 5] + 8 = 3 \cdot [4] + 8 = 12 + 8 = 20$$

Check Yourself 7

Evaluate $8 - 2 \cdot [(5 - 3)^2 - 1]$.

We stated that parentheses and brackets are not the only types of grouping symbols. Example 8 demonstrates the fraction bar as a grouping symbol.

Example 8

Using the Order of Operations with Grouping Symbols

Evaluate $\frac{2 \cdot 14}{2} \cdot \frac{5}{3}$.

$$\frac{2 \cdot 14}{2} \cdot \frac{5}{3} = \frac{28}{2} \cdot \frac{5}{3} = 14 \cdot \frac{5}{3} = \frac{70}{3}$$

CAUTION

You may not “cancel” the $2$'s, because the numerator is being added, not multiplied.

The fraction bar acts as a grouping symbol.

We perform the division first because it precedes the multiplication.

$$\frac{2}{2} = 1$$

$$\frac{14}{2} = 7$$

$$\frac{5}{3} = \frac{5}{3}$$

$$\frac{70}{3} = \frac{70}{3}$$

Check Yourself 8

Evaluate $4 \cdot \frac{3^2}{5} \cdot 2 \cdot 3$.

Check Yourself ANSWERS

1. (a) $4^2$; (b) $7^4$
2. (a) 8; (b) 11
3. 64
4. (a) 8.2; (b) 190.92
5. 24
6. (a) 20; (b) 112
7. 2
8. 18
The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.4

(a) Multiplication is shorthand for repeated ______________.

(b) The ___________ or power indicates the number of times the base appears in a product.

(c) Operations inside ___________ symbols are done first when evaluating an expression.

(d) ______________, brackets, and fraction bars are all examples of grouping symbols.
< Objective 1 >

Write each expression in exponential form.

1. $7 \times 7 \times 7 \times 7$
2. $2 \times 2 \times 2 \times 2 \times 2$
3. $6 \times 6 \times 6 \times 6$
4. $4 \times 4 \times 4 \times 4 \times 4 \times 4$
5. $8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8 \times 8$
6. $10 \times 10 \times 10$
7. $15 \times 15 \times 15 \times 15 \times 15$
8. $31 \times 31 \times 31 \times 31 \times 31 \times 31 \times 31 \times 31$

< Objective 2 >

Evaluate each of the following expressions.

9. $5 \times 3 \times 4$
10. $10 \times 4 \times 2$
11. $(7 \times 2) \div 6$
12. $(10 \times 4) \div 2$
13. $12 \times 8 \times 4$
14. $20 \times 10 \times 2$
15. $(24 \times 12) \div 6$
16. $(10 \times 20) \div 5$
17. $8 \times 7 \times 2 \times 2$
18. $56 \times 7 \times 8 \times 4$
19. $7 \times (8 \times 3) \div 3$
20. $48 \times (8 \times 4) \div 2$
21. $3 \times 5^2$
22. $5 \times 2^3$
23. $(2 \times 4)^2$
24. $(5 \times 2)^3$
25. $4 \times 3^2 \div 2$
26. $3 \times 2^4 \times 8$
### Answers

27. \[ \frac{3}{2} (4 - 2)^3 \cdot 3 \]

28. \[ 5 \cdot \frac{2}{3} \cdot 3 \]

29. \[ 4 (2 - 6)^2 \]

30. \[ 64 \cdot \frac{2}{3} \cdot 4 \]

31. \[ \frac{3}{2} (16 - 2)^4 \]

32. \[ 12 \cdot \frac{2}{3} \cdot 3 \]

33. \[ 5 (4 - 2)^3 \]

34. \[ 7 \cdot \frac{2}{3} (4 - 5)^2 \]

35. \[ 4 (2 - 3)^2 \]

36. \[ 2 \cdot \frac{16}{3} \cdot 6 \]

37. \[ 2 \cdot \frac{1}{2} \cdot 2 \]

38. \[ 16 \cdot \frac{2}{3} (1 - 2)^2 \]

39. \[ 8 (2 - 3)^2 \]

40. \[ 2 \cdot \frac{1}{2} (1 - 3)^2 \]

41. \[ 2 \cdot \frac{16}{3} \cdot 4 \]

42. \[ 5 (2 - 3)^2 \]

43. \[ 2 \cdot \frac{1}{2} (1 - 3)^2 \]

44. \[ 5 (2 - 3)^2 \cdot 4 \]

45. **Social Science** Over the last 2,000 years, Earth’s population has doubled approximately five times. Use exponential notation to write an expression that indicates doubling five times.

46. **Geometry** The volume of a cube with each edge of length 9 in. is given by \[ 9 \cdot 9 \cdot 9. \] Write the volume using exponential notation.

*Use a calculator to evaluate each expression. Round your results to the nearest tenth.*

47. \[ (1.2)^3 \]

48. \[ (5.21 - 3.14 - 6.2154) \]

49. \[ 1.23 \cdot 3.169 \]

50. \[ 4.56 \cdot (2.34)^4 \]
Insert grouping symbols in the proper place(s) so that the given value of the expression is obtained.

51. \(36 \div 4 \div 2 \div 4; 2\)

52. \(48 \div 3 \div 2 \div 3; 2\)

53. \(6 \div 9 \div 3 \div 16 \div 4 \div 2; 29\)

54. \(5 \div 3 \div 2 \div 8 \div 5 \div 2; 28\)

Answers

1. \(7^4\)

2. \(3 \cdot 6^3\)

3. \(8^{10}\)

4. \(7 \cdot 15^6\)

5. \(9 \cdot 17\)

6. \(11 \cdot 54\)

7. \(13 \cdot 10\)

8. \(2 \cdot 17 \cdot 60\)

9. \(19 \cdot 231\)

10. \(21 \cdot 75\)

11. \(23 \cdot 64\)

12. \(25 \cdot 34\)

13. \(27 \cdot 8\)

14. \(29 \cdot 36\)

15. \(31 \cdot 256\)

16. \(33 \cdot 196\)

17. \(35 \cdot 4\)

18. \(37 \cdot \frac{9}{2}\)

19. \(39 \cdot \frac{75}{4}\)

20. \(41 \cdot 96\)

21. \(43 \cdot 0\)

22. \(45 \cdot 2^5\)

23. \(47 \cdot 1.2\)

24. \(49 \cdot 7.8\)

25. \(51 \cdot 36 \div (4 \div 2) \div 4\)

26. \((53 \cdot (6 \div 9) \div 3 \div (16 \div 4)\) \div 2\)
Positive and Negative Numbers

1. Represent integers on a number line
2. Order real numbers
3. Find the opposite of a number
4. Evaluate numerical expressions involving absolute value

When numbers are used to represent physical quantities (altitudes, temperatures, and money are examples), it is often necessary to distinguish between positive and negative amounts. It is convenient to represent these quantities with plus (+) or minus (−) signs. Some instances of this are shown here.

The altitude of Mount Whitney is 14,495 ft above sea level (14,495).

The altitude of Death Valley is 282 ft below sea level (−282).

The temperature in Chicago is 10° below zero (−10).
Positive and Negative Numbers

SECTION 0.5

An account could show a gain of $100 (100) or a loss of $100 (-100).

These numbers suggest the need to extend the number system to include both positive numbers (like 100) and negative numbers (like -282).

To represent the negative numbers, we extend the number line to the left of zero and name equally spaced points.

Numbers used to name points to the right of zero are positive numbers. They can be written with a positive (+) sign, but are usually written with no sign at all.

+6 and 9 are positive numbers.

Numbers used to name points to the left of zero are negative numbers. They are always written with a negative (-) sign.

3 and -20 are negative numbers.

Read “negative 3.”

Positive and negative numbers are both examples of integers.

Here, the number line is extended to include both positive and negative numbers.

Zero is neither positive nor negative. It is the origin.

Definition

**Integers**

The integers consist of the natural numbers, their negatives, and zero. We can represent the set of integers by

\[
\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots
\]

The sets of three dots are called ellipses and indicate that a pattern continues.
Example 1  
**Representing Integers on the Number Line**

< Objective 1 >

Represent the following integers on the number line shown.

\[3, 12, 8, 15, 7\]

![Number line with marked integers](image)

**Check Yourself 1**

Represent the following integers on a number line.

\[1, 9, 4, 11, 8, 20\]

![Number line with marked integers](image)

The set of numbers on the number line is *ordered*. The numbers get smaller moving to the left on the number line and larger moving to the right.

When a set of numbers is written from smallest to largest, the numbers are said to be in *ascending order*.

Example 2  
**Ordering Integers**

< Objective 2 >

Place each set of numbers in ascending order.

(a) \[9, 5, 8, 3, 7\]

From smallest to largest, the numbers are

\[8, 5, 3, 7, 9\]

Note that this is the order in which the numbers appear on a number line as we move from left to right.

(b) \[3, 2, 18, 20, 13\]

From smallest to largest, the numbers are

\[20, 13, 2, 3, 18\]

**Check Yourself 2**

Place each set of numbers in ascending order.

(a) \[12, 13, 15, 2, 8, 3\]  
(b) \[3, 6, 9, 3, 8\]

The least and greatest numbers in a set are called the *extreme values*. The least element is called the *minimum*, and the greatest element is called the *maximum*. 
Example 3

Labeling Extreme Values

For each set of numbers, determine the minimum and maximum values.

(a) 9, 5, 8, 3, 7

From our previous ordering of these numbers, we see that 8, the least element, is the minimum, and 9, the greatest element, is the maximum.

(b) 3, 2, 18, 20, 13

20 is the minimum, and 18 is the maximum.

Check Yourself 3

For each set of numbers, determine the minimum and maximum values.

(a) 12, 13, 15, 2, 8, 3  
(b) 3, 6, 9, 3, 8

Integers are not the only kind of signed numbers. Decimals and fractions can also be thought of as signed numbers.

Example 4

Identifying Numbers That Are Integers

Which of the numbers 145, 28, 0.35, and \( \frac{2}{3} \) are integers?

(a) 145 is an integer.

(b) 28 is an integer.

(c) 0.35 is not an integer.

(d) \( \frac{2}{3} \) is not an integer.

Check Yourself 4

Which of the following numbers are integers?

\[
\begin{align*}
23 & \quad 1,054 & \quad 0.23 & \quad 0 & \quad 500 & \quad \frac{4}{5}
\end{align*}
\]

NOTE

0 is the opposite of 0.

We refer to the negative of a number as its opposite. But what is the opposite of the opposite of a number? It is the number itself. Example 5 illustrates this concept.
Example 5  Finding Opposites

< Objective 3 >

Find the opposite for each number.

(a) \(5\)  
   The opposite of \(5\) is \(-5\).

(b) \(9\)  
   The opposite of \(9\) is \(-9\).

Check Yourself 5

Find the opposite for each number.

(a) \(17\)  
(b) \(12\)

An important idea for our work in this chapter is the **absolute value** of a number. This represents the distance of the point named by the number from the origin on the number line.

\[
\begin{array}{c}
\text{5 units} \\
\text{-5} \\
\text{5 units} \\
\end{array}
\]

The absolute value of \(5\) is \(5\). The absolute value of \(-5\) is also \(5\).

The absolute value of a positive number or zero is itself. The absolute value of a negative number is its opposite.

In symbols we write

\[
\begin{array}{c}
5 \\
\text{Read "the absolute value of 5."} \\
\end{array}
\quad
\begin{array}{c}
5 \\
\text{Read "the absolute value of negative 5."} \\
\end{array}
\]

The absolute value of a number does not depend on whether the number is to the right or to the left of the origin, but on its distance from the origin.

Example 6  Simplifying Absolute Value Expressions

< Objective 4 >

(a) \(|7|\)  
(b) \(|7|\)  
(c) \(|7|\)  

This is the negative, or opposite, of the absolute value of negative \(7\).

(d) \(|10|\)  
   \(|10|\)  
   \(|10|\)  
   \(|20|\)

Absolute value bars serve as another set of grouping symbols, so do the operation inside first.

(e) \(|8-3|\)  
   \(|5|\)  
   \(|5|\)

(f) \(|8-3|\)  
   \(|3|\)  
   \(|8-3|\)  
   \(|5|\)

Here, evaluate the absolute values and then subtract.
Check Yourself

Evaluate.

(a) | 8 |
(b) | 8 |
(c) | 8 |
(d) | 9 | 4 |
(e) | 9 | 4 |
(f) | 9 | 4 |

Check Yourself ANSWERS

1. 20 15 10 5 0 5 10 15 20

2. (a) 13, 8, 3, 2, 12, 15; (b) 9, 3, 3, 6, 8
3. (a) Minimum is 13, maximum is 15; (b) Minimum is 9, maximum is 8
4. 23, 1,054, 0, and 500
5. (a) 17; (b) 12
6. (a) 8; (b) 8; (c) 8; (d) 13; (e) 5; (f) 5

Reading Your Text

The following fill-in-the-blank exercises are designed to ensure that you understand some of the key vocabulary used in this section.

SECTION 0.5

(a) When numbers are used to represent physical quantities, it is often necessary to distinguish between positive and __________ quantities.

(b) It is convenient to represent negative quantities with a __________ sign.

(c) The __________ consist of the natural numbers, their negatives, and zero.

(d) When a set of numbers is written from smallest to largest, the numbers are said to be in __________ order.
0.5 exercises

Basic Skills | Advanced Skills | Vocational-Technical Applications | Calculator/Computer | Above and Beyond | Getting Ready

< Objective 1 >
Represent each quantity with an integer.

1. An altitude of 400 ft above sea level
2. An altitude of 80 ft below sea level
3. A loss of $200
4. A profit of $400
5. A decrease in population of 25,000
6. An increase in population of 12,500

Represent the integers on the number lines shown.

7. 5, 15, 18, 8, 3
8. 18, 4, 5, 13, 9

Which numbers in the following sets are integers?

9. 5, \( \frac{2}{9} \), 175, 234, 0.64
10. 45, 0.35, \( \frac{3}{5} \), 700, 26

< Objective 2 >
Place each of the following sets of numbers in ascending order.

11. 3, 5, 2, 0, 7, 1, 8
12. 2, 7, 1, 8, 6, 1, 0
13. 9, 2, 11, 4, 6, 1, 5
14. 23, 18, 5, 11, 15, 14, 20
15. 6, 7, 7, 6, 3, 3
16. 12, 13, 14, 14, 15, 15
For each set, determine the maximum and minimum values.

17. 5, 6, 0, 10, 3, 15, 1, 8
18. 9, 1, 3, 11, 4, 2, 5, 2
19. 21, 15, 0, 7, 9, 16, 3, 11
20. 22, 0, 22, 31, 18, 5, 3
21. 3, 0, 1, 2, 5, 4, 1
22. 2, 7, 3, 5, 10, 5

< Objective 3 >
Find the opposite of each number.

23. 15
24. 18
25. 15
26. 34
27. 19
28. 6
29. 7
30. 54

< Objective 4 >
Evaluate.

31. |17|
32. |28|
33. |19|
34. |7|
35. |21|
36. |3|
37. |8|
38. |13|
39. |2| |3|
40. |4| |3|
41. |9| |9|
42. |11| |11|

Answers

17. 
18. 
19. 
20. 
21. 
22. 
23. 
24. 
25. 
26. 
27. 
28. 
29. 
30. 
31. 
32. 
33. 
34. 
35. 
36. 
37. 
38. 
39. 
40. 
41. 
42. 

SECTION 0.5
Answers

43. 
44. 
45. 
46. 
47. 
48. 
49. 
50. 
51. 
52. 

Represent each quantity with a real number.

53. **SCIENCE AND TECHNOLOGY**  The erosion of 5 centimeters (cm) of topsoil from an Iowa corn field.

54. **SCIENCE AND TECHNOLOGY**  The formation of 2.5 cm of new topsoil on the African savanna.

55. **BUSINESS AND FINANCE**  The withdrawal of $50 from a checking account.

56. **BUSINESS AND FINANCE**  The deposit of $200 into a savings account.

57. **SCIENCE AND TECHNOLOGY**  The temperature change pictured.

58. **BUSINESS AND FINANCE**  An increase of 75 points in the Dow-Jones average.

59. **STATISTICS**  An eight-game losing streak by the local baseball team.

60. **SOCIAL SCIENCE**  An increase of 25,000 in the population of the city.
61. **Business and Finance** A country exported $90,000,000 more than it imported, creating a positive trade balance.

62. **Business and Finance** A country exported $60,000,000 less than it imported, creating a negative trade balance.

Determine whether each of the following statements is **true** or **false**.

63. All natural numbers are integers.

64. Zero is an integer.

65. All integers are whole numbers.

66. All real numbers are integers.

67. All negative integers are whole numbers.

68. Zero is neither positive nor negative.

For each collection of numbers given in exercises 69 to 72, answer the following:

(a) Which number is smallest?
(b) Which number lies farthest from the origin?
(c) Which number has the largest absolute value?
(d) Which number has the smallest absolute value?

69. 6, 3, 8, 7, 2

70. 8, 3, 5, 4, 9

71. 2, 6, 1, 0, 2, 5

72. 9, 0, 2, 3, 6

Place absolute value bars in the proper location(s) on the left side of the expression so that the equation is true.

73. 6 ( 2) 4

74. 8 ( 3) 5

75. 6 ( 2) 8

76. 8 ( 3) 11
77. Simplify each of the following:

\[(\ 7)\ \ (\ 7)\ \ (\ 7)\]

Based on your answers, generalize your results.

Answers

1. 400 or (400)
3. 200
5. 25,000
7. 20
9. 5, 175, 234
11. 7, 5, 1, 0, 2, 3, 8
13. 11, 6, 2, 1, 4, 5, 9
15. 7, 6, 3, 3, 6, 7
17. Max: 15; min: 6
19. Max: 21, min: 15
21. Max: 5; min: 2
23. 15
25. 15
27. 19
29. 7
31. 17
33. 19
35. 21
37. 8
39. 5
41. 18
43. 0
45. 7
47. 7
49. 11
51. 1
53. 5
55. 50
57. 10°F
59. 8
61. 60,000,000
63. True
65. False
67. False
69. (a) 6; (b) 8; (c) 8; (d) 2
71. (a) 2; (b) 6; (c) 6; (d) 0
73. |6| (2)| 4
75. |6| |2| 8
77. Above and Beyond
<table>
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<td></td>
<td>Section 0.1</td>
</tr>
<tr>
<td><strong>Factor</strong></td>
<td>The factors of 12 are 1, 2, 3, 4, 6, and 12.</td>
<td>p. 4</td>
</tr>
<tr>
<td><strong>Prime Number</strong></td>
<td>7, 13, 29, and 73 are prime numbers.</td>
<td>p. 5</td>
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<tr>
<td><strong>Composite Number</strong></td>
<td>8, 15, 42, and 65 are composite numbers.</td>
<td>p. 6</td>
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<tr>
<td><strong>Zero and One</strong></td>
<td>0 and 1 are classified as neither prime nor composite numbers.</td>
<td>p. 6</td>
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<tr>
<td><strong>Greatest Common Factor (GCF)</strong></td>
<td>The GCF of 21 and 24 is 3.</td>
<td>p. 10</td>
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<tr>
<td><strong>To Find the GCF</strong></td>
<td>To find the GCF of 24, 30, and 36:</td>
<td>p. 10</td>
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<tr>
<td>1. Write the prime factorization for each of the numbers in the group.</td>
<td>24 (2^3) (3)</td>
<td></td>
</tr>
<tr>
<td>2. Locate the prime factors that appear in every prime factorization.</td>
<td>30 (2) (3) (5)</td>
<td></td>
</tr>
<tr>
<td>3. The GCF is the product of all the common prime factors. If there are no common prime factors, the GCF is 1.</td>
<td>36 (2^2) (3) (3)</td>
<td></td>
</tr>
<tr>
<td>The GCF is 2 (\cdot) 3</td>
<td>The GCF is 2 (\cdot) 3 (\cdot) 6.</td>
<td></td>
</tr>
<tr>
<td><strong>Least Common Multiple (LCM)</strong></td>
<td>The LCM of 21 and 24 is 168.</td>
<td>p. 13</td>
</tr>
<tr>
<td><strong>To Find the LCM</strong></td>
<td>To find the LCM of 12, 15, and 18:</td>
<td></td>
</tr>
<tr>
<td>1. Write the prime factorization for each of the numbers in the group.</td>
<td>12 (2^2) (3)</td>
<td></td>
</tr>
<tr>
<td>2. Find all the prime factors that appear in any one of the prime factorizations.</td>
<td>15 (3) (5)</td>
<td></td>
</tr>
<tr>
<td>3. Form the product of those prime factors, using each factor the greatest number of times it occurs in any one factorization.</td>
<td>18 (2) (3) (3)</td>
<td></td>
</tr>
<tr>
<td>The LCM is 2 (\cdot) 3 (\cdot) 3 (\cdot) 5, or 180.</td>
<td></td>
<td></td>
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<tr>
<td><strong>Fractions and Mixed Numbers</strong></td>
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<tr>
<td><strong>The Fundamental Principle of Fractions</strong></td>
<td>(\frac{a}{b} = \frac{a \cdot c}{b \cdot c}) in which neither (b) nor (c) is zero.</td>
<td>p. 21</td>
</tr>
<tr>
<td></td>
<td>(\frac{2}{3} = \frac{2 \cdot 3}{3 \cdot 3} = \frac{6}{9})</td>
<td></td>
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### Definition/Procedure

<table>
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<tbody>
<tr>
<td>1. Multiply numerator by numerator. This gives the numerator of the product.</td>
<td>$\frac{5 \times 3}{8 \times 7} = \frac{15}{56}$</td>
<td>p. 22</td>
</tr>
<tr>
<td>2. Multiply denominator by denominator. This gives the denominator of the product.</td>
<td>$\frac{9 \times 10}{3 \times 2} = \frac{90}{6}$</td>
<td></td>
</tr>
<tr>
<td>3. Simplify the resulting fraction if possible.</td>
<td></td>
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</table>

In multiplying fractions, it is usually easiest to factor and simplify the numerator and denominator before multiplying.

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<tr>
<th><strong>Dividing Fractions</strong></th>
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</thead>
<tbody>
<tr>
<td>Invert the divisor and multiply.</td>
<td>$\frac{3}{7} \div \frac{5}{4} = \frac{3}{7} \times \frac{4}{5}$</td>
<td>p. 23</td>
</tr>
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</table>

<table>
<thead>
<tr>
<th><strong>To Add or Subtract Fractions with Different Denominators</strong></th>
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<tbody>
<tr>
<td>1. Find the LCD of the fractions.</td>
<td></td>
<td>p. 24</td>
</tr>
<tr>
<td>2. Change each fraction to an equivalent fraction with the LCD as a common denominator.</td>
<td>$\frac{3}{4} - \frac{3}{10} = \frac{15}{40} - \frac{12}{40} = \frac{3}{40}$</td>
<td></td>
</tr>
<tr>
<td>3. Add or subtract the resulting like fractions as before.</td>
<td></td>
<td></td>
</tr>
</tbody>
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<tr>
<th><strong>Mixed Number</strong></th>
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<tbody>
<tr>
<td>The sum of a whole number and a proper fraction.</td>
<td>$2\frac{1}{3} + 5\frac{7}{8}$</td>
<td>p. 25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>To Write an Improper Fraction as a Mixed Number</strong></th>
<th><strong>Example</strong></th>
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</thead>
<tbody>
<tr>
<td>1. Divide the numerator by the denominator. The quotient is the whole-number portion of the mixed number.</td>
<td>To write $\frac{22}{5}$ as a mixed number:</td>
<td>p. 26</td>
</tr>
<tr>
<td>2. If there is a remainder, write the remainder over the original denominator. This gives the fractional portion of the mixed number.</td>
<td>$\frac{22}{5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Quotient: $\frac{22}{5} = 4\frac{2}{5}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Remainder: $\frac{2}{5}$</td>
<td></td>
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</table>

<table>
<thead>
<tr>
<th><strong>To Write a Mixed Number as an Improper Fraction</strong></th>
<th><strong>Example</strong></th>
<th><strong>Reference</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Multiply the denominator of the fraction by the whole-number portion of the mixed number.</td>
<td>Denominator: $\frac{3}{4}$</td>
<td>p. 26</td>
</tr>
<tr>
<td>2. Add the numerator of the fraction to that product.</td>
<td>Whole number: $4$</td>
<td></td>
</tr>
<tr>
<td>3. Write that sum over the original denominator to form the improper fraction.</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Numerator: $3$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Denominator: $4$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>To Add or Subtract Mixed Numbers</strong></th>
<th><strong>Example</strong></th>
<th><strong>Reference</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Rewrite as improper fractions.</td>
<td>$\frac{5}{2} + \frac{3}{4} + \frac{11}{2} + \frac{15}{4} + \frac{22}{4} + \frac{15}{4}$</td>
<td>p. 28</td>
</tr>
<tr>
<td>2. Add or subtract the fractions.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3. Rewrite the results as a mixed number if required.</td>
<td>$\frac{7}{4} + \frac{3}{4}$</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Decimals and Percents</strong></th>
<th><strong>Example</strong></th>
<th><strong>Reference</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>To Write a Fraction as a Decimal</strong></td>
<td><strong>Example</strong></td>
<td><strong>Reference</strong></td>
</tr>
<tr>
<td>1. Divide the numerator of the fraction by its denominator.</td>
<td>To write $\frac{1}{2}$ as a decimal:</td>
<td>p. 38</td>
</tr>
<tr>
<td>2. The quotient is the decimal equivalent of the common fraction.</td>
<td>$0.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$2 \div 0$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0.5$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>$0$</td>
<td></td>
</tr>
</tbody>
</table>

Section 0.3
### Definition/Procedure | Example | Reference
---|---|---
**To Write a Terminating Decimal Less Than 1 as a Fraction**  
1. Write the digits of the decimal without the decimal point.  
   This is the numerator of the fraction.  
2. The denominator of the fraction is a 1 followed by as many zeros as there are places in the decimal.  
   To write 0.275 as a fraction:  
   \[
   \frac{275}{1,000} = \frac{11}{40}
   \] | p. 40

**To Add Decimals**  
1. Write the numbers being added in column form with their decimal points in a vertical line.  
2. Add just as you would with whole numbers.  
3. Place the decimal point of the sum in line with the decimal points of the addends.  
   To add 2.7, 3.15, and 0.48:  
   \[
   \begin{align*}
   2.7 & \\
   3.15 & \\
   0.48 & \\
   \hline
   6.33 & 
   \end{align*}
   \] | p. 40

**To Subtract Decimals**  
1. Write the numbers being subtracted in column form with their decimal points in a vertical line.  
   You may have to place zeros to the right of the existing digits.  
2. Subtract just as you would with whole numbers.  
3. Place the decimal point of the difference in line with the decimal points of the numbers being subtracted.  
   To subtract 5.875 from 8.5:  
   \[
   \begin{align*}
   8.500 & \\
   \hline
   5.875 & \\
   2.625 & 
   \end{align*}
   \] | p. 41

**To Multiply Decimals**  
1. Multiply the decimals as though they were whole numbers.  
2. Add the number of decimal places in the factors.  
3. Place the decimal point in the product so that the number of decimal places in the product is the sum of the number of decimal places in the factors.  
   To multiply 2.85 and 0.045:  
   \[
   \begin{align*}
   2.85 \times 0.045 & \\
   1425 & \\
   0.12825 & 
   \end{align*}
   \] | p. 41

**To Divide by a Decimal**  
1. Move the decimal point in the divisor to the right, making the divisor a whole number.  
2. Move the decimal point in the dividend to the right the same number of places. Add zeros if necessary.  
3. Place the decimal point in the quotient directly above the decimal point of the dividend.  
4. Divide as you would with whole numbers.  
   To divide 16.5 by 5.5, move the decimal points:  
   \[
   \begin{align*}
   3 \div 16.5 & \\
   5.5 \vert 16.5 & \\
   16.5 & \\
   0 & 
   \end{align*}
   \] | p. 42

**Percent**  
Along with fractions and decimals another way of naming parts of a whole. Percent means per hundred.  
\[
21\% = \frac{21}{100} = \frac{21}{100} = 0.21
\] | p. 43

**To Write a Percent as a Fraction or Decimal**  
1. To write a percent as a fraction, replace the percent symbol with \(\frac{1}{100}\) and multiply.  
2. To write a percent as a decimal, remove the percent symbol, and move the decimal point two places to the left.  
   \[
   \begin{align*}
   37\% & \\
   37 \times \frac{1}{100} & \\
   \frac{37}{100} & \\
   \frac{37}{100} & \\
   0.37 & 
   \end{align*}
   \] | p. 43
### Definition/Procedure

<table>
<thead>
<tr>
<th><strong>To Write a Decimal or Fraction as a Percent</strong></th>
<th><strong>Example</strong></th>
<th><strong>Reference</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>1. <em>To write a decimal as a percent,</em> move the decimal point two places to the right, and attach the percent symbol.</td>
<td>0.58 58%</td>
<td>p. 44</td>
</tr>
<tr>
<td>2. <em>To write a fraction as a percent,</em> write the decimal equivalent of the fraction, and then change that decimal to a percent.</td>
<td>( \frac{3}{5} 0.60 60% )</td>
<td></td>
</tr>
</tbody>
</table>

### Exponents and the Order of Operations

<table>
<thead>
<tr>
<th><strong>Using Exponents</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Base</strong> The number that is raised to a power.</td>
<td><strong>Exponent</strong> The exponent is written to the right and above the base. The exponent tells the number of times the base is to be used as a factor.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>The Order of Operations</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixed operations in an expression should be done in the following order:</td>
<td>4 ((2 \cdot 3)^2) 7</td>
</tr>
<tr>
<td>1. Do any operations inside grouping symbols.</td>
<td>4 5^2 7</td>
</tr>
<tr>
<td>2. Evaluate any powers.</td>
<td>4 25 7</td>
</tr>
<tr>
<td>3. Do all multiplication and division in order, from left to right.</td>
<td>100 7</td>
</tr>
<tr>
<td>4. Do all addition and subtraction in order, from left to right.</td>
<td>93</td>
</tr>
</tbody>
</table>

### Positive and Negative Numbers

<table>
<thead>
<tr>
<th><strong>Positive Numbers</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers used to name points to the right of the origin on the number line.</td>
<td>The opposite of 2 is 2. The opposite of 9 is 9.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Negative Numbers</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Numbers used to name points to the left of the origin on the number line.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Integers</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The natural (or counting) numbers, their negatives, and zero. The integers are ( \ldots, -3, -2, -1, 0, 1, 2, 3, \ldots )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Opposite of a Number</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The opposite of a number is the negative of that number.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Absolute Value</strong></th>
<th><strong>Example</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>The distance (on the number line) between the point named by an integer and the origin.</td>
<td></td>
</tr>
</tbody>
</table>
This summary exercise set is provided to give you practice with each of the objectives of this chapter. Each exercise is keyed to the appropriate chapter section. When you are finished, you can check your answers to the odd-numbered exercises in the back of the text. If you have difficulty with any of these questions, go back and reread the examples from that section. The answers to the even-numbered exercises appear in the Instructor’s Solutions Manual. Your instructor will give you guidelines on how best to use these exercises in your instructional setting.

**0.1 List all the factors of the given numbers.**

1. 52  
2. 41  
3. 76  
4. 315

Use the group of numbers 2, 5, 7, 11, 14, 17, 21, 23, 27, 39, and 43.

5. List the prime numbers; then list the composite numbers.

**Find the prime factorization for the given numbers.**

6. 48  
7. 420  
8. 60  
9. 180

**Find the greatest common factor (GCF).**

10. 15 and 20  
11. 30 and 31  
12. 72 and 180  
13. 240 and 900

**Find the least common multiple (LCM).**

14. 4 and 12  
15. 8 and 16  
16. 18 and 24  
17. 12 and 18

**0.2 Write three fractional representations for each number.**

18. \(\frac{5}{7}\)  
19. \(\frac{3}{11}\)  
20. \(\frac{4}{9}\)

21. Write the fraction \(\frac{24}{64}\) in simplest form.
22. The Pennsylvania Turnpike, from the Ohio border to the New Jersey border is 360 miles long. Miranda and Carl agree to hike along the turnpike in order to raise money for their favorite charity. On the first day, they hike 23 miles. The second day, they hike 24\(\frac{2}{3}\) miles, and on the third day they hike another 17\(\frac{7}{10}\) miles. How many miles did they walk over the first three days? How much farther do they have to hike in order to complete the entire distance?

0.3 Perform the indicated operations.

\[
\begin{align*}
23. & \quad \frac{7}{15} \div \frac{5}{21} \\
24. & \quad \frac{10}{27} \div \frac{9}{20} \\
25. & \quad \frac{5}{12} \div \frac{5}{8} \\
26. & \quad \frac{7}{15} \div \frac{14}{25} \\
27. & \quad \frac{5}{6} \div \frac{11}{18} \\
28. & \quad \frac{5}{18} \div \frac{7}{12} \\
29. & \quad \frac{11}{18} \div \frac{2}{9} \\
30. & \quad \frac{11}{27} \div \frac{5}{18} \\
31. & \quad 5.123 \div 6.4 \\
32. & \quad 10.127 \div 5.49 \\
33. & \quad 5.26 \div 3.796 \\
34. & \quad 6\frac{5}{7} \div 3\frac{4}{7} \\
35. & \quad \frac{5}{10} \div \frac{7}{12} \\
36. & \quad \frac{7}{9} \div \frac{4}{9} \\
37. & \quad 6\frac{5}{12} \div \frac{5}{8} \\
38. & \quad \frac{5}{3} \div \frac{4}{5} \\
39. & \quad \frac{2}{5} \div \frac{5}{8} \\
40. & \quad \frac{3}{8} \div \frac{1}{4} \\
\end{align*}
\]

Divide and round the quotient to the nearest thousandth.

\[
\begin{align*}
41. & \quad 3.042 \div 0.37 \\
42. & \quad 0.2549 \div 2.87 \\
\end{align*}
\]

Write the percent as a fraction or a mixed number.

\[
\begin{align*}
43. & \quad 2\% \\
44. & \quad 20\% \\
45. & \quad 37.5\% \\
46. & \quad 150\% \\
47. & \quad 233\frac{1}{3}\% \\
48. & \quad 300\% \\
\end{align*}
\]

Write the percents as decimals.

\[
\begin{align*}
49. & \quad 75\% \\
50. & \quad 4\% \\
51. & \quad 6.25\% \\
52. & \quad 13.5\% \\
53. & \quad 0.6\% \\
54. & \quad 225\% \\
\end{align*}
\]
Write as percents.

55. 0.06  
56. 2.4  
57. 7

58. 0.035  
59. 0.005  
60. $\frac{7}{10}$

61. $\frac{2}{5}$  
62. $\frac{1}{4}$  
63. $\frac{2}{3}$

64. Pierce’s monthly electric bill comes to $84.52 under his equal-payment program. How much will he pay for electricity over a full year?

0.4 Evaluate each of the following expressions.

65. $18 \div 3 \cdot 5$  
66. $(18 \div 3) \cdot 5$  
67. $5 \cdot 4^2$

68. $(5 \div 4)^2$  
69. $5 \cdot 3^2 \div 4$  
70. $5 \cdot (3^2 \div 4)$

71. $5 \div (4 \div 2)^2$  
72. $5 \div 4 \div 2^2$  
73. $(5 \div 4)^2$

74. $3 \div (5 \div 2)^2$  
75. $3 \div 5 \div 2^2$  
76. $(3 \div 5)^2$

77. $8 \div 4 \div 2$  
78. $36 \div 4 \div 2 \div 7 \div 6$

79. $4^2 \div \frac{2 \cdot \frac{10}{4}}{\frac{3^2}{1}}$  
80. $3 \cdot 2^2 \div \frac{18}{2^2}$

0.5

81. Represent the following integers on the number line shown: 6, 18, 3, 2, 15, 9.

Place each of the following sets in ascending order.

82. 4, 3, 6, 7, 0, 1, 2  
83. 7, 8, 8, 1, 2, 3, 3, 0, 7

For each data set determine the maximum and minimum.

84. 4, 2, 5, 1, 6, 3, 4  
85. 4, 2, 5, 9, 8, 1, 6
Find the opposite of each number.

86. 17 87. 63

Evaluate.

88. |9| 89. |9|
90. |9| 91. |9|
92. |12 8| 93. |8| + |12|
94. |8 + 12| 95. |18| |12|
96. |7| |3| 97. |9| + |5|

98. At the beginning of the month, Tyler had $33.15 in his checking account. He deposited his $425.87 paycheck and paid his $314.89 student loan bill. What is the balance in his checking account?
This purpose of this chapter test is to help you check your progress so that you can find sections and concepts that you need to review before the next exam. Allow yourself about an hour to take this test. At the end of that hour, check your answers against those given in the back of this text. If you missed any, note the section reference that accompanies the answer. Go back to that section and reread the examples until you have mastered that particular concept.

1. Which of the numbers 5, 9, 13, 17, 22, 27, 31, and 45 are prime numbers? Which are composite numbers?

2. Find the prime factorization for 264.

Find the greatest common factor (GCF) for the given numbers.

3. 36 and 84

4. 16, 24, and 72

Find the least common multiple (LCM) for the given numbers.

5. 12 and 27

6. 3, 4, and 18

Perform the indicated operations.

7. \[ \frac{8}{21} \cdot \frac{3}{4} \]

8. \[ \frac{7}{12} \cdot \frac{28}{36} \]

9. \[ \frac{3}{4} \cdot \frac{5}{6} \]

10. \[ \frac{8}{21} \cdot \frac{2}{7} \]

11. 3.25 \( \cdot \) 4.125

12. 16.234 \( \cdot \) 12.35

13. 7.29 \( \cdot \) 3.15

14. 6.10 \( \cdot \) 13.1

15. \[ \frac{2}{3} \cdot \frac{2}{7} \]

16. \[ \frac{1}{6} \cdot \frac{3}{4} \]

17. \[ \frac{5}{6} \cdot \frac{2}{9} \]

18. 3.969 \( \cdot \) 0.54

Write as fractions.

19. 7%

20. 72%
Answers

21. 42%  
22. 6%  
23. 160%

Write as percents.

24. 0.03  
25. 0.042  
26. \( \frac{2}{5} \)  
27. \( \frac{5}{8} \)

Write using exponents.

28. 4 4 4 4  
29. 9 9 9 9 9

Evaluate the following expressions.

30. 23 4 5  
31. 4 5^2 35  
32. 4 (2 4)^2  
33. 16 2 5^2  
34. (3 2 4)^3  
35. 8 3 2 5

36. Represent the following integers on the number line shown: 5, 12, 4, 7, 18, 17.

37. Place the following data set in ascending order: 4, 3, 6, 5, 0, 2, 2.

38. Determine the maximum and minimum of the following data set: 3, 2, 5, 6, 1, 2.

Evaluate.

39. \( \left| 7 \right| \)  
40. \( \left| 7 \right| \)  
41. \( 18 \left| 7 \right| \)  
42. 18 \( \left| 7 \right| \)  
43. \( 24 \left| 5 \right| \)

Find the opposite of each number.

44. 40  
45. 19