EET1222/ET242 Circuit Analysis II

Sinusoidal Alternating Waveforms

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- Intro. to Sinusoidal Alternating Waveforms
- Frequency & Period
- Phase Instantaneous
- Peak & Peak-to-Peak
- Average & Effective Values
- AC Meters

Key Words: Sinusoidal Waveform, Frequency, Period, Phase, Peak, RMS, ac Meter

Sinusoidal Alternating Waveforms

Sinusoidal alternating waveform is the time-varying voltage that is commercially available in large quantities and is commonly called the ***ac voltage***. Each waveform in Fig. 13-1 is an alternating waveform available from commercial supplies. The term alternating indicates only that the waveform alternates between two prescribed levels in a set time sequence. To be absolutely correct, the term ***sinusoid***, ***square-wave***, or ***triangular*** must be applied.

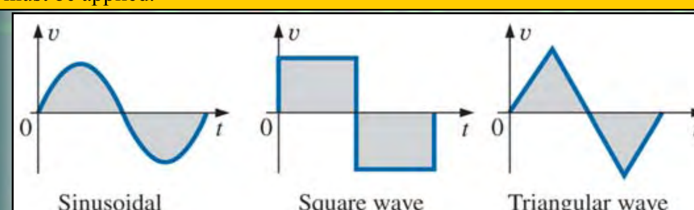


Figure 13.1 Alternating waveforms.

Sinusoidal ac Voltage Generation

Sinusoidal ac voltages are available from a variety of sources. The most common source is the typical home outlet, which provides an ac voltage that originates at a power plant. In each case, an **ac generator**, as shown in Fig. 13-2(a), is primary component in the energy-conversion process. For isolated locations where power lines have not been installed, portable ac generators [Fig. 13-2(b)] are available that run on gasoline. The turning propellers of the wind-power station [Fig. 13-2(C)] are connected directly to the shaft of ac generator to provide the ac voltage as one of natural resources. Through light energy absorbed in the form of photons, solar cells [Fig. 13-2(d)] can generate dc voltage then can be converted to one of a sinusoidal nature through an inverter. Sinusoidal ac voltages with characteristics that can be controlled by the user are available from **function generators**, such as the one in Fig.13-2(e).

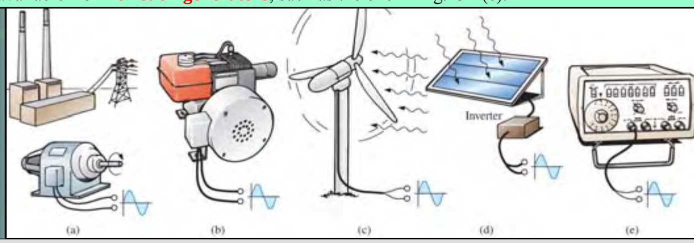


Figure 13.2 Various sources of ac power; (a) generating plant; (b) portable ac generator; (c) wind-power station; (d) solar panel; (e) function generator.

Sinusoidal ac Voltage Definitions

The ***sinusoidal waveform*** in Fig.13-3 with its additional notation will now be used as a model in defining a few basic terms. These terms, however, can be applied to any alternating waveform. It is important to remember, as you proceed through the **various definitions**, that the vertical scaling is in volts or amperes and the horizontal scaling is in units of time.

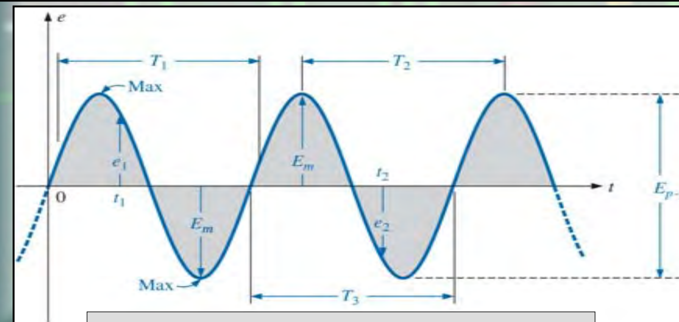


FIGURE 13.3 Important parameters for a sinusoidal voltage.

Waveform: The path traced by a quantity, such as the voltage in Fig. 13-3, plotted as a function of some variable such as time, position, degrees, radiations, temperature, and so on.

Instantaneous value: The magnitude of a waveform at any instant of time; denoted by lowercase letters (e_1, e_2 in Fig. 13-3)

Peak amplitude: The maximum value of a waveform as measured from its average, value, denoted by uppercase letters. For the waveform in Fig. 13-3, the average value is zero volts, and E_m is defined by the figure.

Peak-to-peak value: Denoted by E_{p-p} or V_{p-p} (as shown in Fig. 13-3), the full voltage between positive and negative peaks of the waveform, that is, the sum of the magnitude of the positive and negative peaks.

Periodic waveform: A waveform that continually repeats itself after the same time interval. The Fig. 13-3 is a periodic waveform.

Period (T): The time of a periodic waveform.

Cycle: The portion of a waveform contained in one period of time. The cycles within T_1, T_2, and T_3 in Fig. 13-3 may appear different in Fig. 13-3, but they are all bounded by one period of time and therefore satisfy the definition of a cycle.

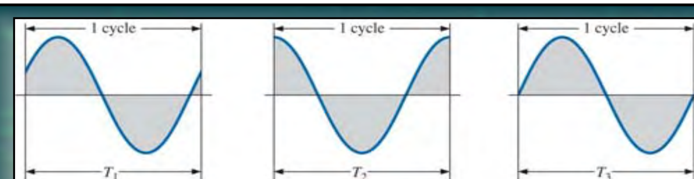


FIGURE 13.4 Defining the cycle and period of a sinusoidal waveform.

Frequency (f): The number of cycles that occur in 1 s. The frequency of the waveform in Fig. 13-5(a) is 1 cycle per second, and for Fig. 13-5(b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13-5 (c)], the frequency would be 2 cycles per second. **1 hertz (Hz) = 1 cycle per second (cps)**

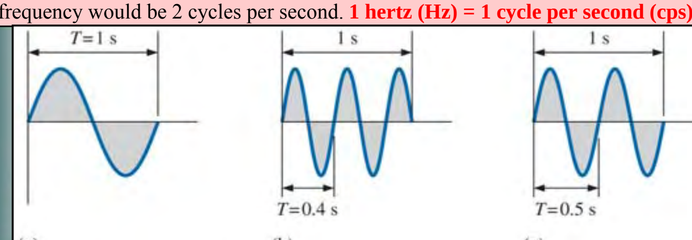


FIGURE 13.5 Demonstration of the effect of a changing frequency on the period of a sinusoidal f...

Ex. 13-1 For the sinusoidal waveform in Fig. 13-7.

a. What is the peak value?
b. What is the instantaneous value at 0.3 s and 0.6 s?
c. What is the peak-to-peak value of the waveform?
d. What is the period of the waveform?
e. How many cycles are shown?
f. What is the frequency of the waveform?

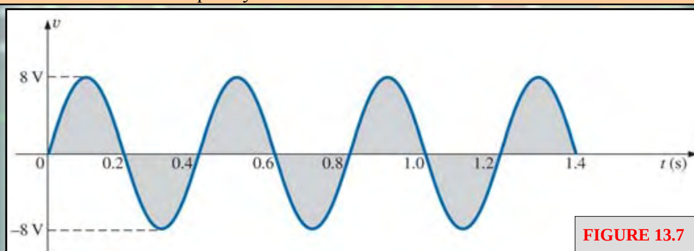


FIGURE 13.7

Frequency

Since ***the frequency is inversely related to the period–that*** is, as one increases, the other decreases by an equal amount–the two can be related by the following equation:

—

f = Hz
T = second (s)

—

Ex. 13-2 Find the periodic waveform with a frequency of
a. 60 Hz b. 1000Hz

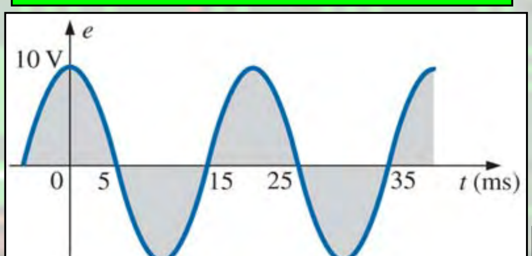
$$a.\quad T = \frac{1}{f} = \frac{1}{60\ Hz} \cong 0.01667\,s \quad or \quad 16.67\ ms$$

$$b.\quad T = \frac{1}{f} = \frac{1}{1000\ Hz} = 10^{-3}\,s = 1\,ms$$

Ex. 13-3 Determine the frequency of the waveform in Fig. 13-9.

From the figure, T = (25 ms – 5 ms) or (35 ms – 15 ms) = 20 ms, and

$$f = \frac{1}{T} = \frac{1}{20\times10^{-3}\,s} = 50\ Hz$$



The Sinusoidal Waveform

Consider the power of the following statement:

The sinusoidal waveform is the only alternating waveform whose shape is unaffected by the response characteristics of R, L, and C element.

In other word, if the voltage or current across a resistor, inductor, or capacitor is sinusoidal in nature, the resulting current or voltage for each will also have sinusoidal characteristics, as shown in Fig. 13-12.

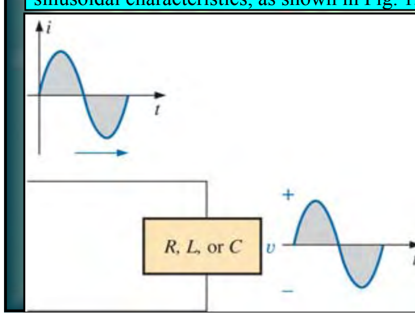


FIGURE 13.12 The sine wave is the only alternating waveform whose shape is not altered by the response characteristics of a pure resistor, indicator, or capacitor.

The unit of measurement for the horizontal axis can be **time**, **degree**, or **radians**. The term radian can be defined as follow: If we mark off a portion of the circumference of a circle by a length equal to the radius of the circle, as shown in Fig. 13-13, the angle resulting is called 1 *radian*. The result is

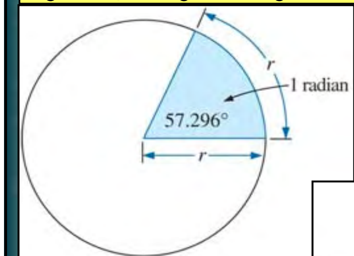


FIGURE 13.13 Defining the radian.

1 rad = 57.296° ≈ 57.3°

where 57.3° is the usual approximation applied.

One full circle has 2π radians, as shown in Fig. 13-14. That is

2π rad = 360°

$2\pi = 2(3.142) = 6.28$

$2\pi(57.3°) = 6.28(57.3°) = 359.84° \approx 360°$

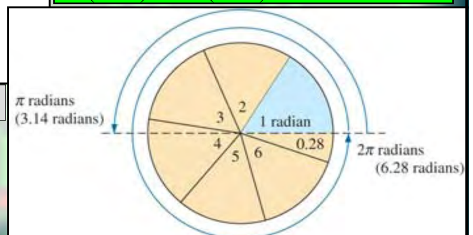


FIGURE 13.14 There are 2π radian in one full circle of 360°.

A number of electrical formulas contain a multiplier of π. For this reason, it is sometimes preferable to measure angles in radians rather than in degrees.

The quantity is the ratio of the circumference of a circle to its diameter.

$$Radians = \left(\frac{\pi}{180^o}\right) \times (degrees)$$

$$Degrees = \left(\frac{180^o}{\pi}\right) \times (radians)$$

Applying these equations, we find

$$90^o : Radians = \frac{\pi}{180^o}(90^o) = \frac{\pi}{2} rad$$

$$30^o : Radians = \frac{\pi}{180^o}(30^o) = \frac{\pi}{6} rad$$

$$\frac{\pi}{3} rad : Degrees = \frac{180^o}{\pi}(\frac{\pi}{3}) = 60^o$$

$$\frac{3\pi}{2} rad : Degrees = \frac{180^o}{\pi}(\frac{3\pi}{2}) = 270^o$$

For comparison purposes, two sinusoidal voltages are in Fig. 13-15 using degrees and radians as the units of measurement for the horizontal axis.

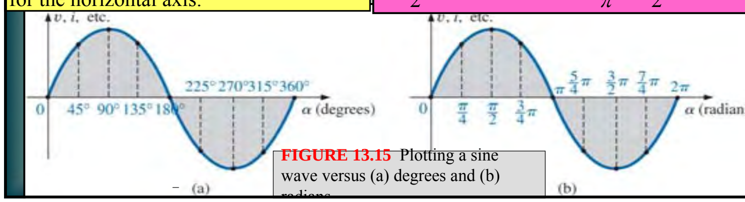


FIGURE 13.15 Plotting a sine wave versus (a) degrees and (b) radians.

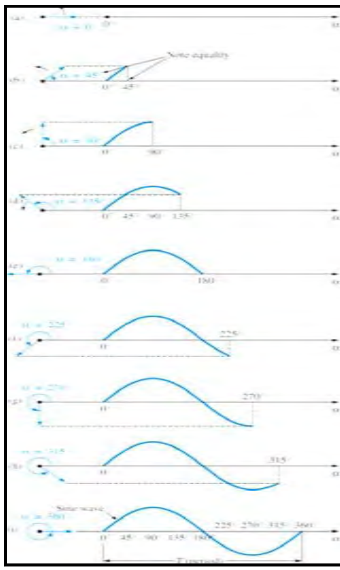


FIGURE 13.16 Generating a sinusoidal waveform through the vertical projection of a rotating vector.

In Fig. 13-16, the time required to complete one revolution is equal to the period (T) of the sinusoidal waveform. The radians subtended in this time interval are 2π. Substituting, we have

$\omega = 2\pi/T$ or $2\pi f$ **(rad/s)**

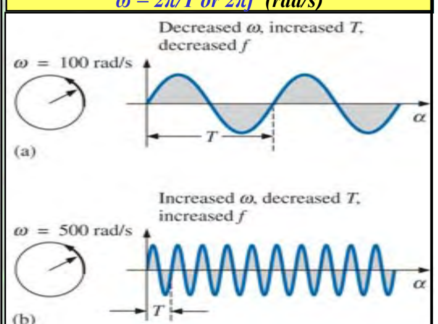


FIGURE 13.17 Demonstrating the effect of ω on the frequency and period.

Ex. 13-4 Determine the angular velocity of a sine wave having a frequency of 60 Hz.

$$\omega = 2\pi f = (2\pi)(60\ Hz) \approx 377\ rad/s$$

Ex. 13-5 Determine the frequency and period of the sine wave in Fig. 13-17 (b).

$$Since \quad \omega = 2\pi / T,$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi\ rad}{500\ rad/s} = 12.57\ ms$$

$$and \quad f = \frac{1}{T} = \frac{1}{12.57\times10^{-3} s} = 79.58\ Hz$$

Ex. 13-6 Given ω = 200 rad/s, determine how long it will take the sinusoidal waveform to pass through an angle of 90°.

General Format for the Sinusoidal Voltage or Current

The basic mathematical format for the sinusoidal waveform is

$$A_m \sin \alpha = A_m \sin \omega t$$

where A_m is the peak value of the waveform and α is the unit of measure for the horizontal axis, as shown in Fig. 13-18.

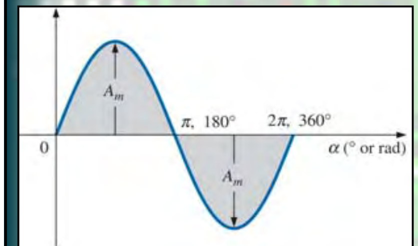


FIGURE 13.18 Basic sinusoidal function.

For electrical quantities such as current and voltage, the general format is

$$i = I_m \sin \omega t = I_m \sin \alpha$$

$$e = E_m \sin \omega t = E_m \sin \alpha$$

where the capital letters with the subscript m represent the amplitude, and the lowercase letters I and e represent the instantaneous value of current and voltage at any time t.

Ex. 13-11 Given $i = 6\times10^{-3} \sin 100t$, determine i at $t = 2\ ms$.

$$\alpha = \omega t = 1000t = (1000\ rad/s)(2\times10^{-3}\ s) = 2\ rad$$

$$\alpha(^o) = \frac{180^o}{\pi\ rad}(2\ rad) = 114.59^o$$

$$i = (6\times10^{-3})(\sin 114.59^o) = (6\ mA)(0.9093) = 5.46\ mA$$

Phase Relations

If the waveform is shifted to the right or left of 0°, the expression becomes

$$A_m \sin(\omega t \pm \theta)$$

where θ is the angle in degrees or radiations that the waveform has been shifted.

If the waveform passes through the horizontal axis with a positive going slope ***before*** 0°, as shown in Fig. 13-27, the expression is

$$A_m \sin(\omega t + \theta)$$

If the waveform passes through the horizontal axis with a positive going slope ***after*** 0°, as shown in Fig. 13-28, the expression is

$$A_m \sin(\omega t - \theta)$$

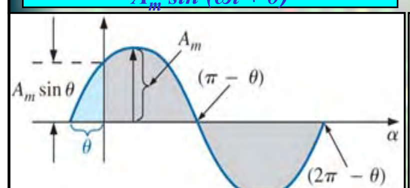


FIGURE 13.27 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope before 0°.

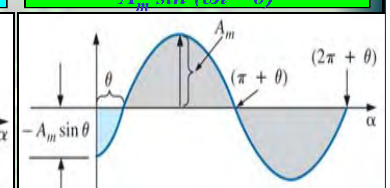
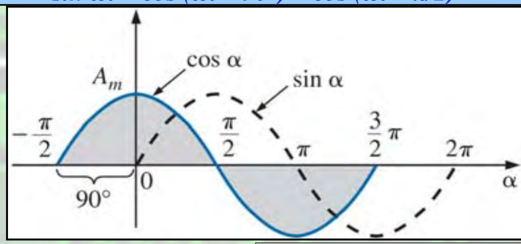


FIGURE 13.28 Defining the phase shift for a sinusoidal function that crosses the horizontal axis with a positive slope after 0°.

If the waveform crosses the horizontal axis with a positive-going slope 90° (π/2) sooner, as shown in Fig. 13-29, it is called a ***cosine wave***; that is

$$\sin(\omega t + 90°) = \sin(\omega t + \pi/2) = \cos \pi t$$

or $$\sin \omega t = \cos(\omega t - 90°) = \cos(\omega t - \pi/2)$$



The oscilloscope is an instrument that will display the sinusoidal alternating waveform in a way that permit the reviewing of all of the waveform's characteristics. ***The vertical scale is set to display voltage levels, whereas the horizontal scale is always in units of time.***

Ex. 13-13 Find the period, frequency, and peak value of the sinusoidal waveform appearing on the screen of the oscilloscope in Fig. 13-36. Note the sensitivities provided in the figure.

One cycle span 4 divisions. Therefore, the period is

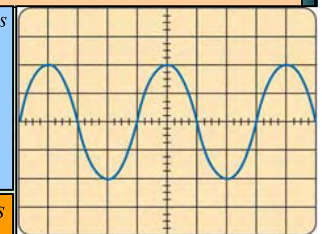
$$T = 4\,div.\left(\frac{50\ \mu s}{div}\right) = 200\ \mu s$$

and the frequency is

$$f = \frac{1}{T} = \frac{1}{200 \times 10^{-6}\,s} = 5\ kHz$$

The vertical height above the horizontal axis encompasses 2 divisions, Therefore,

$$V_m = 2\,div.\left(\frac{0.1\,V}{div.}\right) = 0.2\,V$$

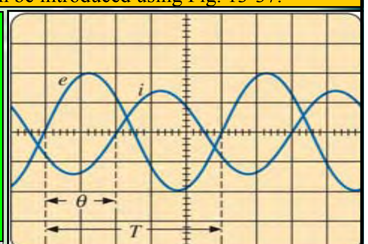


Vertical sensitivity = 0.1 V/div.
Horizontal sensitivity = 50 μs/div.

FIGURE 13.36

An oscilloscope can also be used to make phase measurements between two sinusoidal waveforms. Oscilloscopes have the dual-trace option, that is, the ability to show two waveforms at the same time. It is important that both waveforms must have the same frequency. The equation for the phase angle can be introduced using Fig. 13-37.

First, note that ***each sinusoidal function has the same frequency***, permitting the use of either waveform to determine the period. For the waveform chosen in Fig. 13-37, the period encompasses 5 divisions at 0.2 ms/div. The phase shift between the waveforms is 2 divisions. Since the full period represents a cycle of 360°, the following ratio can be formed:



Vertical sensitivity = 2 V/div.
Horizontal sensitivity = 0.2 ms/div.

Average Value

The concept of the **average value** is an important one in most technical fields. In Fig. 13-38(a), the average height of the sand may be required to determine the volume of sand available. The average height of the sand is that height obtained if the distance from one end to the other is maintained while the sand is leveled off, as shown in Fig. 13-38(b). The area under the mound in Fig. 13-38(a) then equals the area under the rectangular shape in Fig. 13-38(b) as determined by $A = b \times h$.

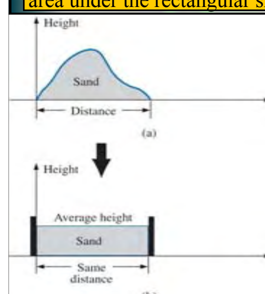


FIGURE 13.38 Defining average value.

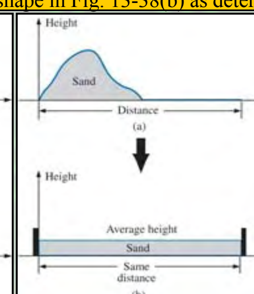


FIGURE 13.39 Effect of distance (length) on average value.

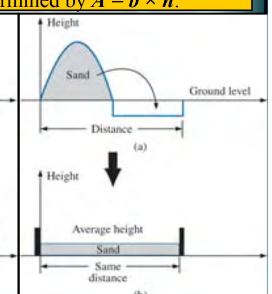


FIGURE 13.40 Effect of depressions (negative excursions) on average value.

Ex. 13-14 Determine the average value of the waveforms in Fig.13-42.

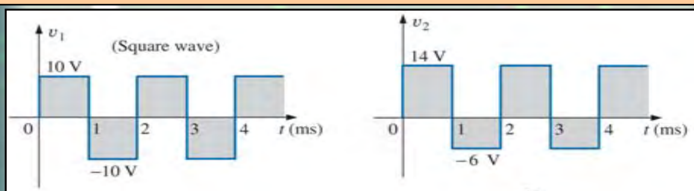


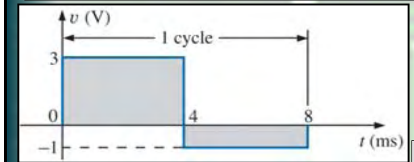
FIGURE 13.42

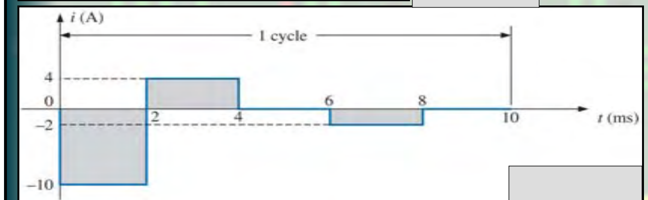
a. By inspection, the area above the axis equals the area below over one cycle, resulting in an average value of zero volts.

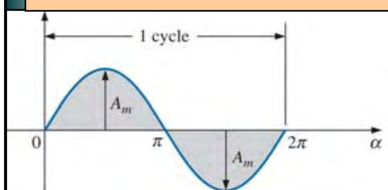
$$G\,(average\ value) = \frac{(10\,V)(1\,ms) + (-10\,V)(1\,ms)}{2\,ms} = \frac{0}{2} = 0\,V$$

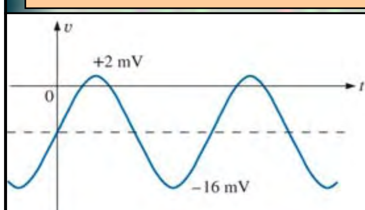
b. $$G\,(average\ value) = \frac{(14\,V)(1\,ms) + (-6\,V)(1\,ms)}{2\,ms} = \frac{8\,V}{2} = 4\,V$$

Ex. 13-15 Determine the average value of the waveforms over one full cycle:
a. Fig. 13-44.
b. Fig. 13-45









Effective (*rms*) Values

This section begins to relate dc and ac quantities with respect to the power delivered to a load. The ***average power*** delivered by the ac source is just the first term, since the average value of a cosine wave is zero even though the wave may have twice the frequency of the original input current waveform. Equation the average power delivered by the ac generator to that delivered by the dc source,

Which, in words, states that

The equivalent dc value of a sinusoidal current or voltage is 1 / √2 or 0.707 of its peak value.

The equivalent dc value is called the **rms** or **effective value** of the sinusoidal quantity

Similarly,

Ex. 13-20 Find the rms values of the sinusoidal waveform in each part of Fig. 13-58.

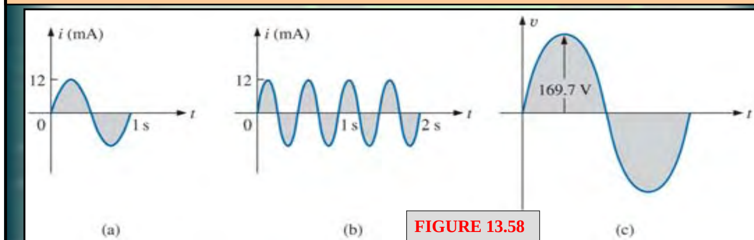


FIGURE 13.58

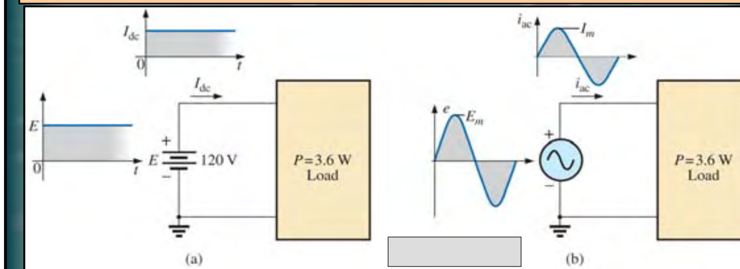
$$a.\quad I_{rms} = \frac{12\times10^{-3}\,A}{\sqrt{2}} = 8.48mA$$

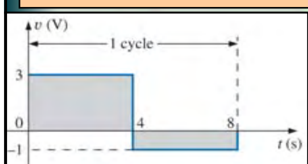
$b.\quad I_{rms} = 8.48\ mA$

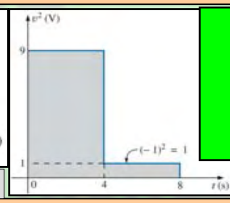
Note that frequency did not change the effective value in (b) compared to (a).

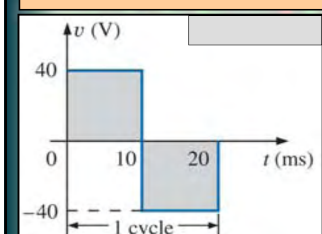
$$c.\quad V_{rms} = \frac{169.73\,V}{\sqrt{2}} = 120\,V$$

Ex. 13-21 The 120 V dc source in Fig. 13-59(a) delivers 3.6 W to the load. Determine the peak value of the applied voltage (E_m) and the current (I_m) if the ac source [Fig. 13-59(b)] is to deliver the same power to the load.





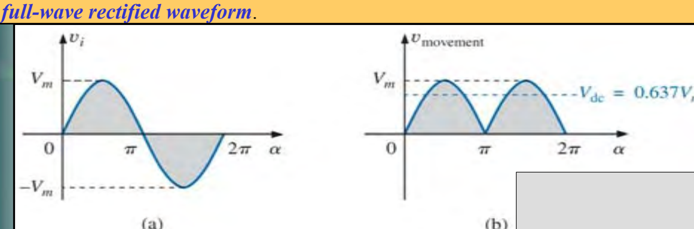




Ac Meters and Instruments

It is important to note whether the DMM in use is a true rms meter or simply meter where the average value is calculated to indicate the rms level. ***A true rms meter reads the effective value of any waveform and is not limited to only sinusoidal waveforms.***

Fundamentally, conduction is permitted through the diodes in such a manner as to convert the sinusoidal input of Fig. 13-68(a) to one having been effectively "flipped over" by the bridge configuration. The resulting waveform in Fig. 13-68(b) is called a ***full-wave rectified waveform***.



Forming the ratio between the ***rms*** and dc levels results in

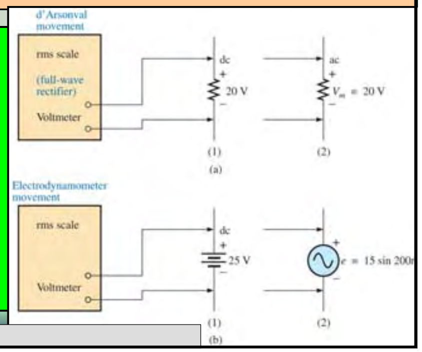
$$\frac{V_{rms}}{V_{dc}} = \frac{0.707\,V_m}{0.637\,V_m} \cong 1.11$$

For Fig. 13-71(a), situation (1):
Meter indication = 1.11(20V) = 22.2V

For Fig.13-71(a), situation (2):
$V_{rms} = 0.707V_m = 0.707(20V) = 14.14V$

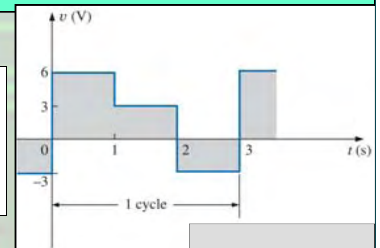
For Fig. 13-71(b), situation (1):
$V_{rms} = V_{dc} = 25\text{ V}$

For Fig.13-71(b), situation (2):
$V_{rms} = 0.707V_m = 0.707(15V) \approx 10.6V$



$$G = \frac{(6V)(1s) + (3V)(1s) - (3V)(1s)}{3s}$$

$$= \frac{6V}{3} = 2V$$



HW 13-42 Find the rms value of the following sinusoidal waveforms:

a. $v = 140\sin(377t + 60°)$

b. $i = 6\times10^{-3}\sin(2\pi1000t)$

c. $v = 40\times10^{-6}\sin(2\pi5000t + 30°)$

a. $V_{rms} = 0.7071(140\,V) = 98.99\,V$

b. $I_{rms} = 0.7071(6mA) = 4.24\,mA$

c. $V_{rms} = 0.7071(40\,\mu V) = 28.28\,\mu V$

Homework 13: 10-18, 30-32, 37, 42, 43

EET1222/ET242 Circuit Analysis II

Response of Basic Elements to AC Input

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

OUTLINES

- **Introduction**
- **Derivative**
- **Response of Basic Elements to ac Input**
- **Frequency Response of the Basic Elements**

Key Words: Sinusoidal Waveform, ac Element, ac Input, Frequency Response

INTRODUCTION

The response of the basic *R*, *L*, and *C* elements to a sinusoidal voltage and current are examined in this class, with special note of how frequency affects the "opposing" characteristic of each element. Phasor notation is then introduced to establish a method of analysis that permits a direct correspondence with a number of the methods, theorems, and concepts introduced in the dc chapter.

DERIVATIVE

The derivative *dx/dt* is defined as the rate of change of *x* with respect to time. If *x* fails to change at a particular instant, $dx = 0$, and the derivative is zero. For the sinusoidal waveform, *dx/dt* is zero only at the positive and negative peaks ($\omega t = \pi/2$ and $\frac{3}{2}\pi$ in Fig. 14-1), since *x* fails to change at these instants of time. The derivative dx/dt is actually the slope of the graph at any instant of time.

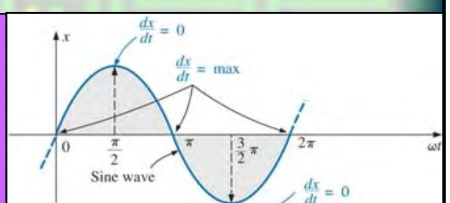
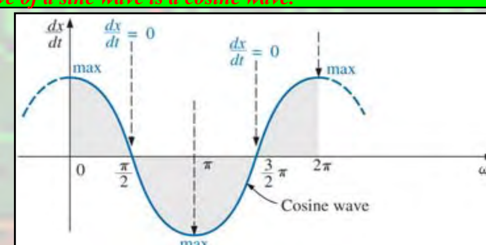


Figure 14.1 Defining those points in a sinusoidal waveform that have maximum and minimum

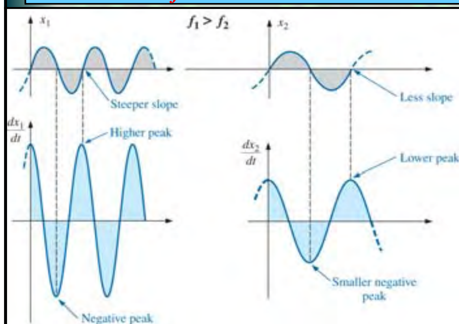
A close examination of the sinusoidal waveform will also indicate that the greatest change in *x* occurs at the instants $\omega t = 0$, π, and 2π. The derivative is therefore a maximum at these points. At *0* and 2π, *x* increases at its greatest rate, and the derivative is given positive sign since *x* increases with time. At π, *dx/dt* decreases at the same rate as it increases at *0* and 2π, but the derivative is given a negative sign since *x* decreases with time. For various values of ωt between these maxima and minima, the derivative will exist and have values from the minimum to the maximum inclusive. A plot of the derivative in Fig. 14-2 shows that

the derivative of a sine wave is a cosine wave.



The peak value of the cosine wave is directly related to the frequency of the original waveform. The higher the frequency, steeper the slope at the horizontal axis and the greater the value of *dx/dt*, as shown in Fig. 14-3 for two different frequencies. In addition, note that

the derivative of a sine wave has the same period and frequency as the original sinusoidal waveform.



For the sinusoidal voltage

$e(t) = E_m \sin(\omega t \pm \theta)$

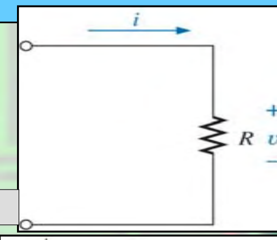
The derivative can be found directly by differentiation to produce the following:

$d\{e(t)\}/dt = \omega E_m \cos(\omega t \pm \theta)$

$= 2\pi f E_m \cos(\omega t \pm \theta)$

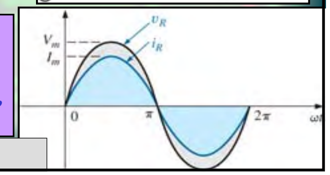
Response of Resistor to an ac Voltage or Current

For power-line frequencies, resistance is, for all practical purposes, unaffected by the frequency of the applied sinusoidal voltage or current. For this frequency region, the resistor ***R*** in Fig. 14-4 can be treated as a constant, and Ohm's law can be applied as follow. For $v = V_m \sin \omega t$,



A plot of *v* and *i* in Fig. 14-5 reveals that

For a purely resistive element, the voltage across and the current through the element are in phase, with their peak values related by Ohm's law.



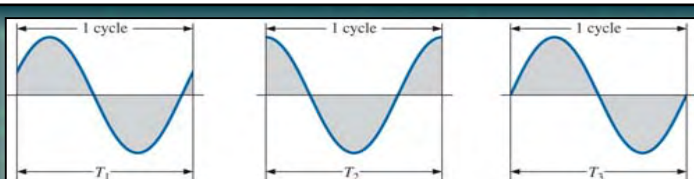


FIGURE 13.4 Defining the cycle and period of a sinusoidal waveform

Frequency (*f*): The number of cycles that occur in 1 s. The frequency of the waveform in Fig. 13-5(a) is 1 cycle per second, and for Fig. 13-5(b), 2½ cycles per second. If a waveform of similar shape had a period of 0.5 s [Fig. 13-5 (c)], the frequency would be 2 cycles per second. **1 hertz (Hz) = 1 cycle per second (cps)**

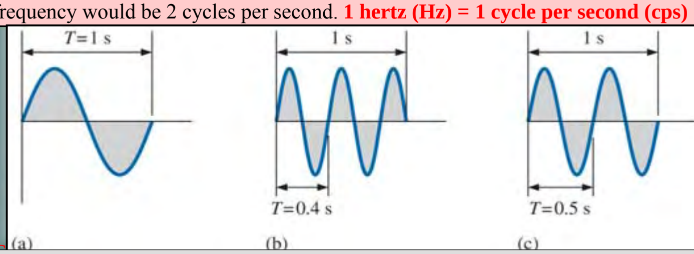
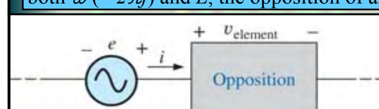


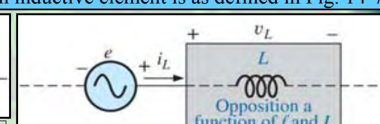
FIG... waveform

Response of Inductor to an ac Voltage or Current

For the series configuration in Fig. 14-6, the voltage $v_{element}$ of the boxed-in element opposes the source ***e*** and thereby reduces the magnitude of the current *i*. The magnitude of the voltage across the element is determined by the opposition of the element to the flow of charge, or current *i*. ***For a resistive element, we have found that the opposition is its resistance and that $v_{element}$ and i are determined by $v_{element}$ = iR.***

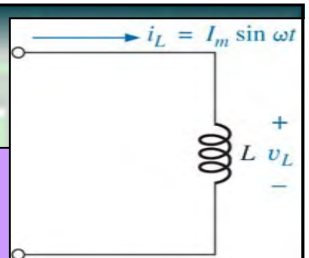
The **inductance voltage** is directly related to the frequency and the inductance of the coil. For increasing values of *f* and *L* in Fig. 14-7, the magnitude of v_L increases due the higher inductance and the greater the rate of change of the flux linkage. Using similarities between Figs. 14-6 and 14-7, we find that increasing levels of v_L are directly related to increasing levels of opposition in Fig. 14-6. Since v_L increases with both ω (= $2\pi f$) and *L*, the opposition of an inductive element is as defined in Fig. 14-7.





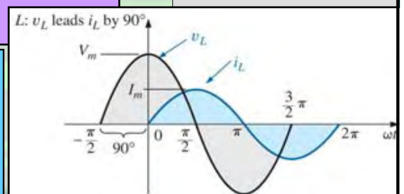
For the inductor in Fig. 14 – 8,

$$v_L = L\frac{di_L}{dt}$$



Note that the peak value of v_L is directly related to ω (= $2\pi f$) and L as predicted in the discussion previous slide. A plot of v_L and i_L in Fig. 14-9 reveals that

for an inductor, v_L leads i_L by 90°, or i_L lags v_L by 90°.



90

The opposition established by an inductor in an sinusoidal network is directly related to the product of the angular velocity and the inductance. The quantity ωL, called the reactance of an inductor, is symbolically represented by X_L and is measured in ohms;

Inductive reactance is the opposition to the flow of current, which results in the continual interchange of energy between the source and the magnetic field of inductor. In other words, ***inductive reactance, unlike resistance, does not dissipate electrical energy.***

Response of Capacitor to an ac Voltage or Current

For the capacitor, we will determine *i* for a particular voltage across the element. When this approach reaches its conclusion, we will know the relationship between the voltage and current and can determine the opposing voltage ($v_{element}$) for any sinusoidal current *i*.

For capacitive networks, the voltage across the capacitor is limited by the rate at which charge can be deposited on, or released by, the plates of the capacitor during the charging and discharging phases, respectively. In other words, an instantaneous change in voltage across a capacitor is opposed by the fact that there is an element of time required to deposit charge on the plates of a capacitor, and V = Q/C.

Since **capacitance** is a measure of the rate at which a capacitor will store charge on its plate,

for a particular change in voltage across the capacitor, the greater the value of capacitance, the greater the resulting capacitive current.

In addition, the fundamental equation relating the voltage across a capacitor to the current of a capacitor [$i = C(dv/dt)$] indicates that

for particular capacitance, the greater the rate of change of voltage across the capacitor, the greater the capacitive current.

The current of a capacitor is therefore directly to the frequency and capacitance of the capacitor. An increase in either quantity results in an increase in the current of the capacitor. For the basic configuration in Fig. 14-10, we are interested in determining the opposition of the capacitor. ***Since an increase in current corresponds to a decrease in opposition, and i_c is proportional to ω and C, the opposition of a capacitor is inversely related to ω and C.***

For the capacitor of Fig. 14 – 11,

$$i_C = C\frac{dv_C}{dt}$$

and, applying differentiation,

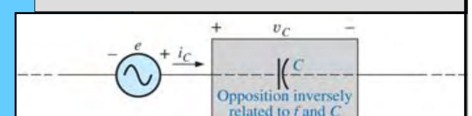
$$\frac{dv_C}{dt} = \frac{d}{dt}(V_m \sin \omega t) = \omega V_m \cos \omega t$$

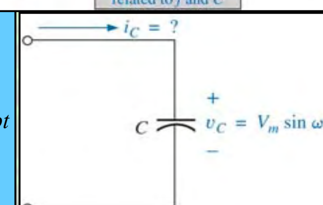
Therefore,

$$i_C = C\frac{dv_C}{dt} = C(\omega V_m \cos \omega t) = \omega C V_m \cos \omega t$$

or $i_C = I_m \sin(\omega t + 90^o)$

where $I_m = \omega C V_m$





A plot of v_C and i_C in Fig.14-12 reveals that

for a capacitor, i_C leads v_C by 90°.

If a phase angle is included in the sinusoidal expression for v_C, such as

$$v_C = V_m \sin(\omega t \pm \theta)$$

then $$i_C = \omega C V_m \sin(\omega t \pm \theta + 90°)$$

The quantity $1/\omega C$, called the reactance of a capacitor, is symbolically represented by X_C and is measured in ohms; that is,

$$X_C = \frac{1}{\omega C} \qquad (ohms, \Omega)$$

In an Ohm's law format, its magnitude can be determined from

$$X_C = \frac{V_m}{I_m} \qquad (ohms, \Omega)$$

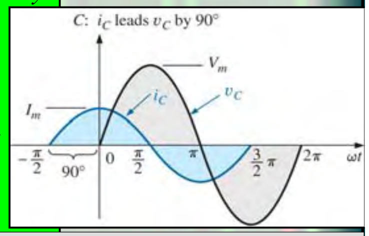


FIGURE 14.12 The current of a purely capacitive element leads the voltage across the element by 90°.

Ex. 14-1 The voltage across a resistor is indicated. Find the sinusoidal expression for the current if the resistor is 10 Ω. Sketch the curves for v and i.

a. $v = 100\sin 377t$

b. $v = 25\sin(377t + 60°)$

a. $I_m = \frac{V_m}{R} = \frac{100\ V}{10\ \Omega} = 10\ A$

(v and i are in phase), resulting in

$i = 10\sin 377t$

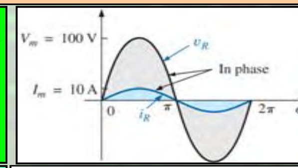
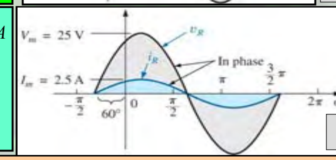


FIGURE 14.13

b. $I_m = \frac{V_m}{R} = \frac{25\ V}{10\ \Omega} = 2.5\ A$

(v and i are in phase), resulting in

$i = 2.5\sin(377t + 60°)$



Ex. 14-2 The current through a 5 Ω resistor is given. Find the sinusoidal expression for the voltage across the resistor for $i = 40\sin(377t + 30°)$.

$V_m = I_m R = (40\ A)(5\ \Omega) = 200$ (v and i are in phase), resulting in

$v = 200\sin(377t + 30°)$

Ex. 14-3 The current through a 0.1 H coil is provided. Find the sinusoidal expression for the voltage across the coil. Sketch the curves for v and i curves.

a. $i = 10\sin 377t$

b. $i = 7\sin(377t – 70°)$

a. $X_L = \omega_L = (377\ rad/s)(0.1\ H) = 37.7\ \Omega$

$V_m = I_m X_L = (10\ A)(37.7\ \Omega) = 377\ V$

and we know that for a coil v leads i by 90°. Therefore,

$v = 377\sin(377t + 90°)$

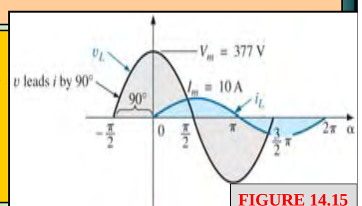


FIGURE 14.15

b. $X_L = \omega_L = (377\ rad/s)(0.1\ H) = 37.7\ \Omega$

$V_m = I_m X_L = (7\ A)(37.7\ \Omega) = 263.9\ V$

and we know that for a coil v leads i by 90°. Therefore,

$v = 263.9\sin(377t – 70° + 90°)$

and $v = 263.9\sin(377t + 20°)$

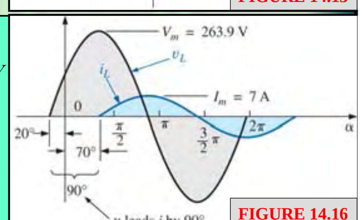
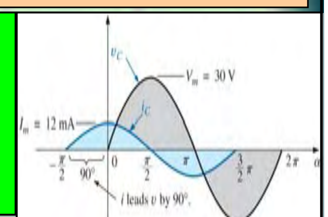


FIGURE 14.16

Ex. 14-4 The voltage across a 0.5 H coil is provided below. What is the sinusoidal expression for the current? $v = 100\sin 20t$

Ex. 14-5 The voltage across a $1\ \mu F$ capacitor is provided below. What is the sinusoidal expression for the current? Sketch the v and i curves. $v = 30\sin 400t$



Ex. 14-6 The current through a 100 μF capacitor is given. Find the sinusoidal expression for the voltage across the capacitor. ***i = 40 sin(500t + 60°)***

$$X_C = \frac{1}{\omega C} = \frac{1}{(500\,rad/s)(100\times10^{-6})} = \frac{10^6\,\Omega}{5\times10^4} = \frac{10^2\,\Omega}{5} = 20\,\Omega$$

$$V_m = I_m X_C = (40\,A)(20\,\Omega) = 800\,V$$

and we know that for a capacitor v las i by 90°. Therefore,

$$v = 800\sin(500t + 60^o - 90^o)$$

$$\text{and} \quad v = 800\sin(500t - 30^o)$$

Ex. 14-7 For the following pairs of voltage and currents, determine whether the element involved is a capacitor, an inductor, or a resistor. Determine the value of *C*, *L*, or *R* if sufficient data are provided (Fig. 14-18):.

a. $v = 100\sin(\omega t + 40°)$ $i = 20\sin(\omega t + 40°)$
b. $v = 1000\sin(377t + 10°)$ $i = 5\sin(377t - 80°)$
c. $v = 500\sin(157t + 30°)$ $i = 1\sin(157t + 120°)$
d. $v = 50\cos(\omega t + 20°)$ $i = 5\sin(\omega t + 110°)$

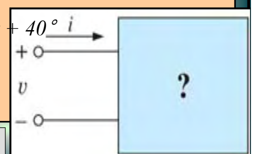


FIGURE 14.18

a. Since v and i are in phase, the element is a resistor, and

$$R = \frac{V_m}{I_m} = \frac{100\,V}{20\,A} = 5\,\Omega$$

b. Since v leads i by 90°, the element is an inductor,

$$\text{and} \quad X_L = \frac{V_m}{I_m} = \frac{1000\,V}{5\,A} = 200\,\Omega$$

$$\text{So that} \quad X_L = \omega L = 200\,\Omega \quad \text{or}$$

$$L = \frac{200\,\Omega}{\omega} = \frac{200\,\Omega}{377\,rad/s} = 0.53\,H$$

Frequency Response of the Basic Elements

Thus far, each description has been for a set frequency, resulting in a fixed level of impedance foe each of the basic elements. We must now investigate ***how a change in frequency affects the impedance level of the basic elements***. It is an important consideration because most signals other than those provided by a power plant contain a variety of frequency levels.

Ideal Response

Resistor R : For an ideal resistor, frequency will have absolutely no effect on the impedance level, as shown by the response in Fig. 14-19

Note that 5 kHz or 20 kHz, the resistance of the resistor remain at 22 Ω; there is no change whatsoever. For the rest of the analyses in this text, the resistance level remains as the nameplate value; no matter what frequency is applied.

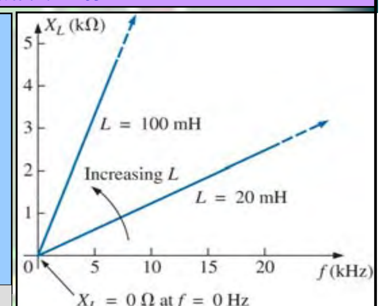
Inductor L : For the ideal inductor, the equation for the reactance can be written as follows to isolate the frequency term in the equation. The result is a constant times the frequency variable that changes as we move down the horizontal axis of a plot:

$$X_L = \omega L = 2\pi f L = (2\pi L)f = kf \quad \text{with } k = 2\pi L$$

The resulting equation can be compared directly with the equation for a straight line:

$$y = mx + b = kf + 0 = kf$$

where $b = 0$ and slope is k or $2\pi L$. X_L is the y variable, and f is the x variable, as shown in Fig. 14-20. Since the inductance determines the slope of the curve, the higher the inductance, the steeper the straight-line plot as shown in Fig. 14-20 for two levels of inductance.



In particular, note that at $f = 0\ Hz$, the reactance of each plot is zero ohms as determined by substituting $f = 0\ Hz$ into the basic equation for the reactance of an inductor:

$$X_L = 2\pi f L = 2\pi(0\,Hz)L = 0\,\Omega$$

Since a reactance of zero ohms corresponds with the characteristics of a short circuit, we can conclude that

at a frequency of 0 Hz an inductor takes on the characteristics of a short circuit, as shown in Fig. 14-21.

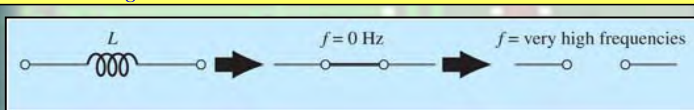


FIGURE 14.21 Effect of low and high frequencies on the circuit model of an inductor.

As shown in Fig. 14-21, as the frequency increases, the reactance increases, until it reaches an extremely high level at very high frequencies.

at very high frequencies, the characteristics of an inductor approach those of an open circuit, as shown in Fig. 14-21.

The inductor, therefore, is capable of handling impedance levels that cover the entire range, from ohms to infinite ohms, changing at a steady rate determined by the inductance level. The higher the inductance, the faster it approaches the open-circuit equivalent.

Capacitor C : For the capacitor, the equation for the reactance

$$X_C = \frac{1}{2\pi f C}$$

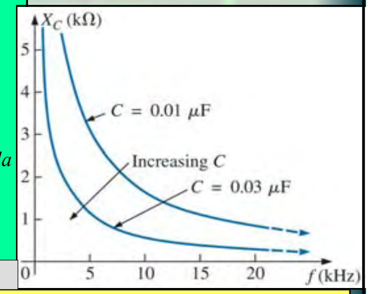
can be written as

$$X_C f = \frac{1}{2\pi C} = k \quad \textit{(a constant)}$$

which matches the basic format for a hyperbola

$$yx = k$$

where X_C is the y variable, and k a constant equal to $1/(2\pi C)$



Hyperbolas have the shape appearing in Fig. 14-22 for two levels of capacitance. Note that the higher the capacitance, the closer the curve approaches the vertical and horizontal axes at low and high frequencies. At 0 Hz, the reactance of any capacitor is extremely high, as determined by the basic equation for capacitance:

The result is that

at or near 0 Hz, the characteristics of a capacitor approach those of an open circuit, as shown in Fig. 14-23.

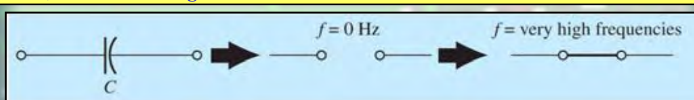


FIGURE 14.23 Effect of low and high frequencies on the circuit model of a capacitor.

As the frequency increases, the reactance approaches a value of zero ohms. The result is that

at very high frequencies, a capacitor takes on the characteristics of a short circuit, as shown in Fig. 14-23.

It is important to note in Fig. 14-22 that the reactance drops very rapidly as frequency increases. For capacitive elements, the change in reactance level can be dramatic with a relatively small change in frequency level. Finally, recognize the following:

As frequency increases, the reactance of an inductive element increases while that of a capacitor decreases, with one approaching an open-circuit equivalent as the other approaches a short-circuit equivalent.

HW 14-18 The current through a 10 Ω capacitive reactance is given. Write the sinusoidal expression for the voltages. Sketch the *v* and *i* sinusoidal waveforms on the same set of axes.

$a.\quad i = 50 \times 10^{-3} \sin \omega t$

$b.\quad i = 2 \times 10^{-6} \sin(\omega t + 60°)$

$c.\quad i = -6 \sin(\omega t - 30°)$

$d.\quad i = 3 \cos(\omega t + 10°)$

a. $V_m = I_m X_C = (50 \times 10^{-3}\ \text{A})(10\ \Omega) = 0.5\ \text{V}$
$\boldsymbol{\upsilon = 0.5 \sin(\omega t - 90°)}$

b. $V_m = I_m X_C = (2 \times 10^{-6})(10\ \Omega) = 20\ \mu\text{V}$
$\boldsymbol{\upsilon = 20 \times 10^{-6} \sin(\omega t - 30°)}$

c. $i = -6 \sin(\omega t - 30°) = 6 \sin(\omega t + 150°)$
$V_m = I_m X_C = (6\ \text{A})(10\ \Omega) = 60\ \text{V}$
$\boldsymbol{\upsilon = 60 \sin(\omega t + 60°)}$

d. $i = 3 \sin(\omega t + 100°)$
$V_m = I_m X_C = (3\ \text{A})(10\ \Omega) = 30\ \text{V}$
$\boldsymbol{\upsilon = 30 \sin(\omega t + 10°)}$

EET1222/ET242 Circuit Analysis II

Average Power and Power Factor

Electrical and Telecommunication Engineering Technology

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

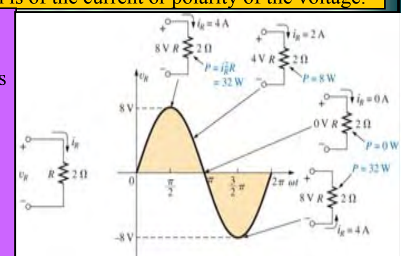
- Average Power and Power Factor
- Complex Numbers
- Rectangular Form
- Polar Form
- Conversion Between Forms

Key Words: Average Power, Power Factor, Complex Number, Rectangular, Polar

Average Power and Power Factor

A common question is, ***How can a sinusoidal voltage or current deliver power to load if it seems to be delivering power during one part of its cycle and taking it back during the negative part of the sinusoidal cycle?*** The equal oscillations above and below the axis seem to suggest that over one full cycle there is no net transfer of power or energy. However, there is a net transfer of power over one full cycle because power is delivered to the load at each instant of the applied voltage and current no matter what the direction is of the current or polarity of the voltage.

To demonstrate this, consider the relatively simple configuration in Fig. 14-29 where an 8 V peak sinusoidal voltage is applied across a 2 Ω resistor. When the voltage is at its positive peak, the power delivered at that instant is 32 W as shown in the figure. At the midpoint of 4 V, the instantaneous power delivered drops to 8 W; when the voltage crosses the axis, it drops to 0 W. Note that when the voltage crosses the its negative peak, 32 W is still being delivered to the resistor



In total, therefore,

Even though the current through and the voltage across reverse direction and polarity, respectively, power is delivered to the resistive lead at each instant time.

If we plot the power delivered over a full cycle, the curve in Fig. 14-30 results. Note that the applied voltage and resulting current are in phase and have twice the frequency of the power curve.

The fact that the power curve is always above the horizontal axis reveals that power is being delivered to the load an each instant of time of the applied sinusoidal voltage.

If we substitute the equation for the peak value in terms of the rms value as follow:

$$P_{av} = \frac{V_m I_m}{2} = \frac{(\sqrt{2}\,V_{rms})(\sqrt{2}\,I_{rms})}{2} = \frac{2V_{rms}I_{rms}}{2}$$
$$= V_{rms} I_{rms}$$

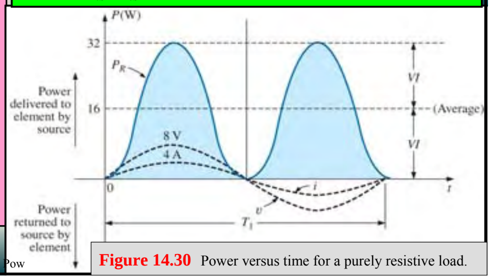


Figure 14.30 Power versus time for a purely resistive load.

In Fig. 14-31, a voltage with an initial phase angle is applied to a network with any combination of elements that results in a current with the indicated phase angle.

The power delivered at each instant of time is then defined by

$$P = vi = V_m \sin(\omega t + \theta_v)\cdot I_m \sin(\omega t + \theta_i)$$
$$= V_m I_m \sin(\omega t + \theta_v)\cdot \sin(\omega t + \theta_i)$$

Using the trigonometric identity

$$\sin A \cdot \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

the function $\sin(\omega t+\theta_v)\cdot\sin(\omega t+\theta_i)$ becomes

$$\sin(\omega t+\theta_v)\cdot\sin(\omega t+\theta_i)$$
$$= \frac{\cos[(\omega t+\theta_v)-(\omega t+\theta_i)] - \cos[(\omega t+\theta_v)+(\omega t+\theta_i)]}{2}$$
$$= \frac{\cos(\theta_v-\theta_i) - \cos(2\omega t+\theta_v+\theta_i)}{2}$$

so that

$$p = \overbrace{\left[\frac{V_m I_m}{2}\cos(\theta_v-\theta_i)\right]}^{\text{Fixed value}} - \overbrace{\left[\frac{V_m I_m}{2}\cos(2\omega t+\theta_v+\theta_i)\right]}^{\text{Time-varying (function of t)}}$$

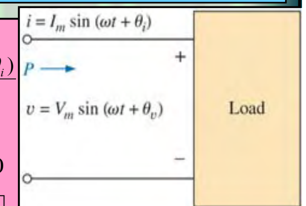


Figure14.31 Determining the power delivered in a sinusoidal ac network.

The average value of the second term is zero over one cycle, producing no net transfer of energy in any one direction. However, the first term in the preceding equation has a constant magnitude and therefore provides some net transfer of energy. This term is referred to as the **average power** or **real power** as introduced earlier. The angle ($\Theta_v - \Theta_i$) is the phase angle between v and i. Since $\cos(-\alpha) = \cos\alpha$,

the magnitude of average power delivered is independent of whether v leads i or i leads v.

Defining θ *as equal to* $|\theta_v - \theta_i|$, *where* $|\ |$ *indicates that only the magnitude is important and the sign is immaterial, we have*

$$P = \frac{V_m I_m}{2}\cos\theta \qquad (watts, W)$$

where P is the average power in watts. This equation can also be written

$$P = \left(\frac{V_m}{\sqrt{2}}\right)\left(\frac{I_m}{\sqrt{2}}\right)\cos\theta$$

or, since $V_{rms} = \frac{V_m}{\sqrt{2}}$ *and* $I_{rms} = \frac{I_m}{\sqrt{2}}$

$$P = V_{rms} I_{rms}\cos\theta$$

Resistor: In a purely resistive circuit, since v and i are in phase, $|\theta_v - \theta_i| = \theta = 0°$, and $\cos\theta = \cos 0° = 1$, so that

$$P = \frac{V_m I_m}{2} = V_{rms} I_{rms} \qquad (W)$$

Or, ***since*** $I_{rms} = \frac{V_{rms}}{R}$

then $P = \frac{V_{rms}^2}{R} = I_{rms}^2 R \quad (W)$

Inductor: In a purely inductive circuit, since v leads i by 90°, $|\theta_v - \theta_i| = \theta = 90°$, therefore

$$P = \frac{V_m I_m}{2}\cos 90^o = \frac{V_m I_m}{2}(0) = 0\,W$$

The average power or power dissipated by the ideal inductor (no associate resistor) is zero watts.

Capacitor: In a purely capacitive circuit, since i leads v by 90°, $|\theta_v - \theta_i| = \theta = |-90°| = 90°$, therefore

$$P = \frac{V_m I_m}{2}\cos 90^o = \frac{V_m I_m}{2}(0) = 0\,W$$

The average power or power dissipated by the ideal capacitor (no associate resistor) is zero watts.

Ex. 14-10 Find the average power dissipated in a network whose input current and voltage are the following:

$i = 5 \sin(\omega t + 40°)$

$v = 10 \sin(\omega t + 40°)$

Since v and i are in phase, the circuit appears to be purely resistive at the input terminals. Therefore,

$$P = \frac{V_m I_m}{2} = \frac{(10\,V)(5\,A)}{2} = 25\,W$$

$$or \quad R = \frac{V_m}{I_m} = \frac{10\,V}{5\,A} = 2\,\Omega$$

$$and \quad P = \frac{V_{rms}^2}{R} = \frac{[(0.707)(10\,V)]^2}{2} = 25\,W$$

$$or \quad P = I_{rms}^2 R = [(0.707)(5\,A)]^2(2) = 25\,W$$

Ex. 14-11 Determine the average power delivered to networks having the following input voltage and current:

a. $v = 100 \sin(\omega t + 40°)$ $\quad i = 20 \sin(\omega t + 70°)$

b. $v = 150 \sin(\omega t - 70°)$ $\quad i = 3 \sin(\omega t - 50°)$

$$a. \quad V_m = 100\,V, \quad \theta_v = 40^o \quad and \quad I_m = 20\,A, \quad \theta_i = 70^o$$

$$\theta = |\theta_v - \theta_i| = |40^o - 70^o| = |-30^o| = 30^o$$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(100\,V)(20\,A)}{2}\cos(30^o) = (1000\,W)(0.866) = 866\,W$$

$$b. \quad V_m = 150\,V, \quad \theta_v = -70^o \quad and \quad I_m = 3\,A, \quad \theta_i = -50^o$$

$$\theta = |\theta_v - \theta_i| = |-70^o - (-50^o)| = |-20^o| = 20^o$$

and

$$P = \frac{V_m I_m}{2}\cos\theta = \frac{(150\,V)(3\,A)}{2}\cos(20^o) = (225\,W)(0.9397) = 211.43\,W$$

Power Factor

In the equation $P = (V_m I_m/2)\cos\theta$, the factor that has significant control over the delivered power level is the $\cos\theta$. No matter how large the voltage or current, if $\cos\theta = 0$, the power is zero; if $\cos\theta = 1$, the power delivered is a maximum. Since it has such control, the expression was given the name **power factor** and is defined by $\quad$ *Power factor* $= F_p = \cos\theta$

For a purely resistive load such as the one shown in Fig. 14-33, the phase angle between *v* and *i* is 0° and $F_p = \cos\theta = \cos 0° = 1$. The power delivered is a maximum of $(V_m I_m/2)\cos\theta = ((100V)(5A)/2)(1) = 250W$. For purely reactive load (inductive or capacitive) such as the one shown in Fig. 14-34, the phase angle between *v* and *i* is 90° and $F_p = \cos\theta = \cos 90° = 0$. The power delivered is then the minimum value of zero watts, even though the current has the same peak value as that encounter in Fig. 14-33.

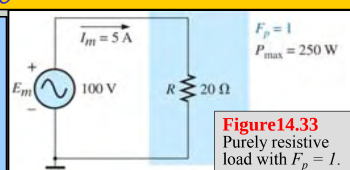


Figure14.33 Purely resistive load with $F_p = 1$.

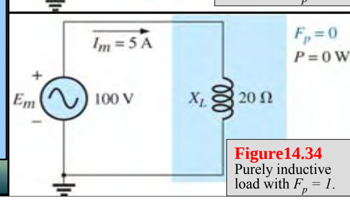


Figure14.34 Purely inductive load with $F_p = 1$.

For situations where the load is a combination of resistive and reactive elements, the *power factor varies between 0 and 1*. The more resistive the total impedance, the closer the power factor is to 1; the more reactive the total impedance, the closer power factor is to 0.

In terms of the average power and the terminal voltage and current,

$$F_p = \cos\theta = \frac{P}{V_{rms} I_{rms}}$$

The terms leading and lagging are often written in conjunction with the power factor. *They are defined by the current through the load.* If the current leads the voltage across a load, the load has a **leading power factor**. If the current lags the voltage across the load, the load has a **lagging power factor**. In other words,

capacitive networks have leading power factor, and inductive networks have lagging power factors.

Ex. 14-12 Determine the power factors of the following loads, and indicate whether they are leading or lagging:

a. Fig. 14-35 b. Fig. 14-36 c. Fig. 14-37

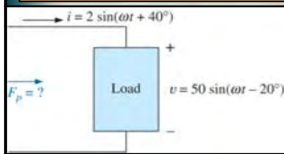


Figure 14.35

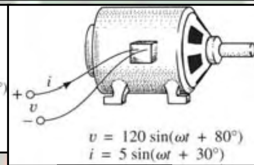


Figure 14.36

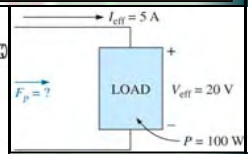


Figure 14.37

$$a.\quad F_p = \cos\theta = \cos\left|40^o - (-20^o)\right| = \cos 60^o = 0.5\ leading$$

$$b.\quad F_p = \cos\theta\left|80^o - 30^o\right| = \cos 50^o = 0.64\ lagging$$

$$c.\quad F_p = \cos\theta = \frac{P}{V_{eff} I_{eff}} = \frac{100W}{(20V)(5A)} = \frac{100W}{100W} = 1$$

The load is resistive, and F_p is neither leading nor lagging.

Complex Numbers

In our analysis of dc network, we found it necessary to determine the algebraic sum of voltages and currents. Since the same will be also be true for ac networks, the question arises, ***How do we determine the algebraic sum of two or more voltages (or current) that are varying sinusoidally?*** Although one solution would be to find the algebraic sum on a point-to-point basis, this would be a long and tedious process in which accuracy would be directly related to the scale used.

It is purpose to introduce a system of **complex numbers** that, when related to the sinusoidal ac waveforms that is quick, direct, and accurate. The technique is extended to permit the analysis of sinusoidal ac networks in a manner very similar to that applied to dc networks.

A **complex number** represents a points in a two-dimensional plane located with reference to two distinct axes. This point can also determine a radius vector drawn from the original to the point. The horizontal axis called the *real axis*, while the vertical axis called the *imaginary axis*. Both are labeled in Fig. 14-38.

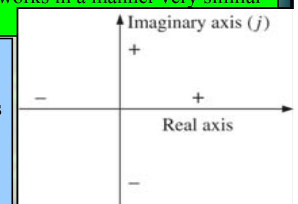


Figure 14.38 Defining the real and imaginary axes of a complex plane.

In the complex plane, the horizontal or real axis represents all positive numbers to the right of the imaginary axis and all negative numbers to the left of imaginary axis. All positive imaginary numbers are represented above the real axis, and all negative imaginary numbers, below the real axis. The symbol ***j*** (or sometimes ***i***) is used to denote the imaginary component.

Two forms are used to represent a point in the plane or a radius vector drawn from the origin to that point.

Rectangular Form

The format for the rectangular form is

$$\mathbf{C} = X + jY$$

As shown in Fig. 14-39. The letter **C** was chosen from the word "complex." The **boldface** notation is for any number with magnitude and direction. The *italic* is for magnitude only.

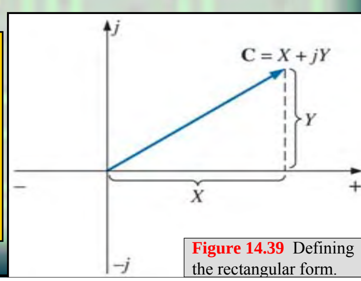


Figure 14.39 Defining the rectangular form.

Ex. 14-13 Sketch the following complex numbers in the complex plane.

a. **C** = *3 + j4* b. **C** = *0 – j6* c. **C** = *–10 –j20*

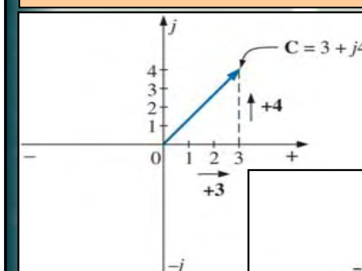


Figure 14.40 Example 14-13 (a)

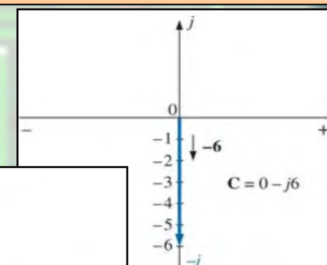


Figure 14.41 Example 14-13 (b)

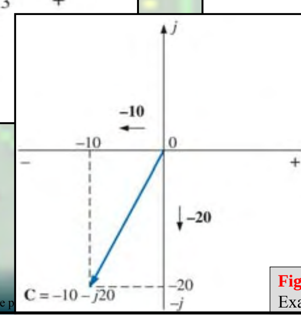


Figure 14.42 Example 14-13 (c)

Polar Form

The format for the ***polar form*** *is*

$$C = Z\angle\theta$$

with the letter Z chosen from the sequence X, Y, Z.

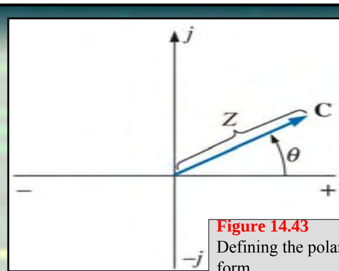


Figure 14.43 Defining the polar form

Z indicates magnitude only and θ ***is always measured counterclockwise (CCW) from the positive real axis***, as shown in Fig. 14-43. Angles measured in the clockwise direction from the positive real axis must have a negative sign associated with them. A negative sign in front of the polar form has the effect shown in Fig. 14-44. Note that it results in a complex number directly opposite the complex number with a positive sign.

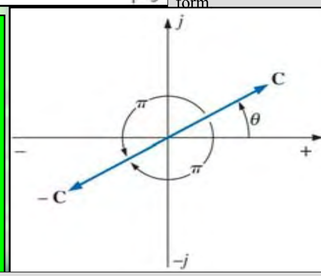


Figure 14.44 Demonstrating the effect of a negative sign on the polar form.

Ex. 14-14 Sketch the following complex numbers in the complex plane:

a. $\mathbf{C} = 5\angle 30^o$ *b.* $\mathbf{C} = 7\angle -120^o$ *c.* $\mathbf{C} = -4.2\angle 60^o$

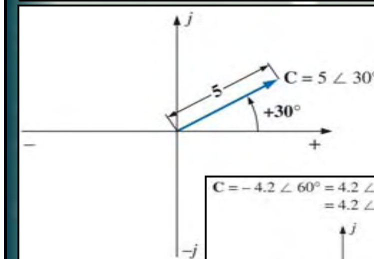


Figure 14.45 Example 14-14 (a)

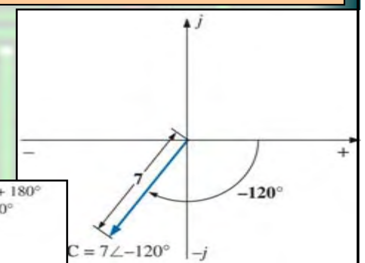


Figure 14.46 Example 14-14 (b)

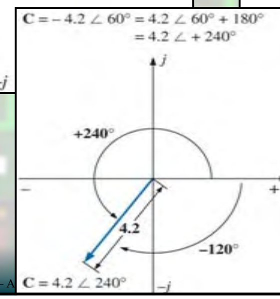


Figure 14.47 Example 14-13 (c)

Conversion Between Forms

The two forms are related by the following equations, as illustrated in Fig. 14-48.

Rectangular to Polar

$$Z = \sqrt{X^2 + Y^2}$$

$$\theta = \tan^{-1}\frac{Y}{X}$$

Polar to Rectangular

$$X = Z\cos\theta$$

$$Y = Z\sin\theta$$

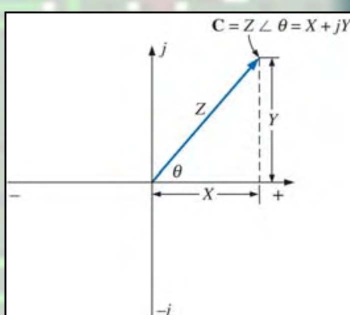


Figure 14.48 Conversion between forms.

Ex. 14-15 Convert the following from rectangular to polar form:

C = *3 + j4* (Fig. 14-49)

$$Z = \sqrt{(3)^2 + (4)^2} = \sqrt{25} = 5$$

$$\theta = \tan^{-1}\left(\frac{4}{3}\right) = 53.13^o$$

$$\mathbf{C} = 5\angle 53.13^o$$

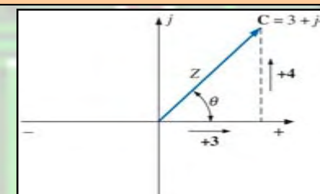


Figure 14.49

Ex. 14-16 Convert the following from polar to rectangular form:

C = *10 ∠ 45°* (Fig. 14-50)

$$X = 10\cos 45^o = (10)(0.707) = 7.07$$

$$Y = 10\sin 45^o = (10)(0.707) = 7.07$$

$$and \quad \mathbf{C} = 7.07 + j7.07$$

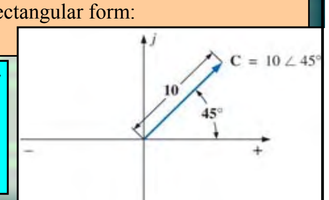


Figure 14.50

Ex. 14-17 Convert the following from rectangular to polar form:

C = *–6 + j3* (Fig. 14-51)

$$Z = \sqrt{(-6)^2 + (3)^2} = \sqrt{45} = 6.71$$

$$\beta = \tan^{-1}\left(\frac{3}{-6}\right) = -26.57^o$$

$$\theta = 180^o - 26.57^o = 153.43^o$$

$$\mathbf{C} = 5\angle 153.43^o$$

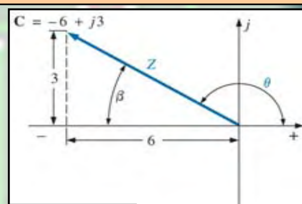


Figure 14.51

Ex. 14-18 Convert the following from polar to rectangular form:

C = *10 ∠ 230°* (Fig. 14-52)

$$X = 10\cos 230^o = -6.43$$

$$Y = 10\sin 230^o = -7.66$$

$$\textit{and} \qquad \mathbf{C} = -6.43 - j7.66$$

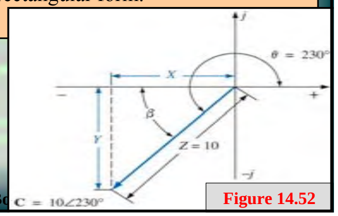


Figure 14.52

HW 14-31 If the current through and voltage across an element are $i = 8\sin(\omega t + 40^o)$ and $v = 48\sin(\omega t + 40^o)$, respectively, compute the power by I^2R, $(V_mI_m/2)\cos\theta$, and $VI\cos\theta$, and compare answers.

$$R = \frac{V_m}{I_m} = \frac{48\,V}{8\,A} = 6\,\Omega, \quad P = I^2R = \left(\frac{8A}{\sqrt{2}}\right)^2 6\Omega = 192\,W$$

$$P = \frac{V_mI_m}{2}\cos\theta = \frac{(48V)(8A)}{\sqrt{2}}\cos 0° = 192\,W$$

$$P = VI\cos\theta = \left(\frac{48\,V}{\sqrt{2}}\right)\left(\frac{8\,A}{\sqrt{2}}\right)\cos 0° = 192\,W$$

Homework 14: 28, 31, 34-36

EET1222/ET242 Circuit Analysis II

Phasors

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on “Introduction to Circuit Analysis” by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- Mathematical Operations with Complex Numbers
- Psasors – Polar and Rectangular Formats
- Conversion Between Forms

Key Words: Complex Number, Phasor, Time Domain, Phase Domain

Mathematical Operations with Complex Numbers

Complex numbers lend themselves readily to the basic mathematical operations of ***addition***, ***subtraction***, ***multiplication***, and ***division***. A few basic rules and definitions must be understood before considering these operations.

Let us first examine the symbol j associated with imaginary numbers, By definition,

$$j = \sqrt{-1} \quad Thus, \quad j^2 = -1$$

$$and \quad j^3 = j^2 j = -1 \cdot j = -j$$

$$with \quad j^4 = j^2 \cdot j^2 = (-1)(-1) = +1$$

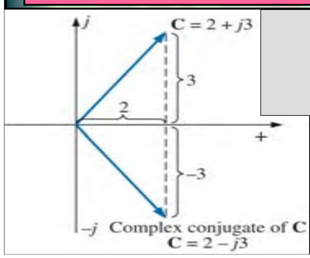
$$j^5 = j$$

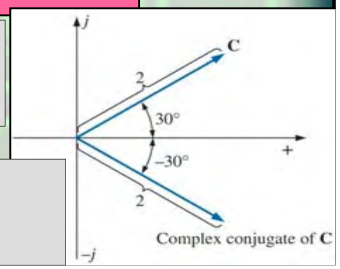
and so on. Further,

$$\frac{1}{j} = (1)\left(\frac{1}{j}\right) = \left(\frac{j}{j}\right)\left(\frac{1}{j}\right) = \left(\frac{j}{j^2}\right) = \frac{j}{-1} = -j$$

Complex Conjugate: The conjugate or complex conjugate of a complex number can be found by simply changing the sign of imaginary part in rectangular form or by using the negative of the angle of the polar form. For example, the conjugate of

$$\mathbf{C} = 2 + j3 \quad \text{is} \quad 2 - j3$$





Reciprocal: The reciprocal of a complex number is 1 devided by the complex number. For example, the reciprocal of

$$\mathbf{C} = X + jY \quad is \quad \frac{1}{X + jY}$$

$$and \ of \ Z\angle\theta,$$

$$\frac{1}{Z\angle\theta}$$

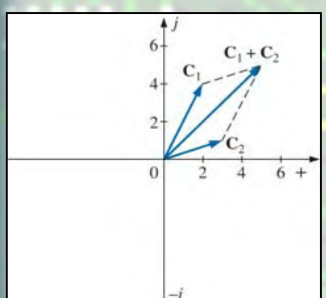
We are now prepared to consider the four basic operations of ***addition***, ***subtraction***, ***multiplication***, and ***division*** with complex numbers.

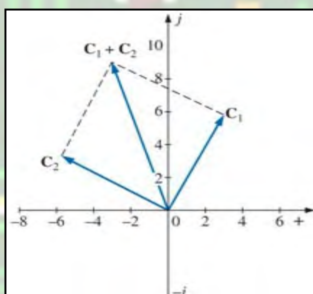
Addition: To add two or more complex numbers, add the real and imaginary parts separately. For example, if

$$\mathbf{C_1} = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathrm{C}_2 = \pm X_2 \pm jY_2$$

$$then \qquad \mathbf{C_1} + \mathbf{C_2} = (\pm X_1 \pm X_2) + j(\pm Y_1 \pm Y_2)$$

There is really no need to memorize the equation. Simply set one above the other and consider the real and imaginary parts separately, as shown in Example 14-19.



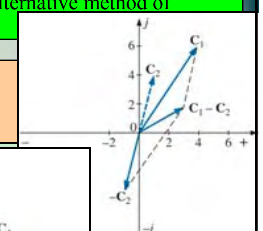


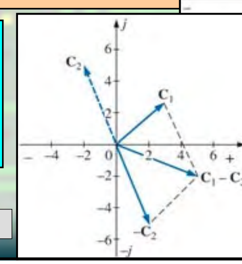
Subtraction: In subtraction, the real and imaginary parts are again considered separately. For example, if

$$\mathbf{C_1} = \pm X_1 \pm jY_1 \quad \text{and} \quad \mathbf{C_2} = \pm X_2 \pm jY_2$$

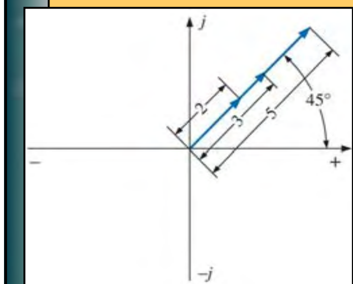
$$then \qquad \mathbf{C_1} - \mathbf{C_2} = [(\pm X_1 - (\pm X_2)] + j[(\pm Y_1 - \pm Y_2)]$$

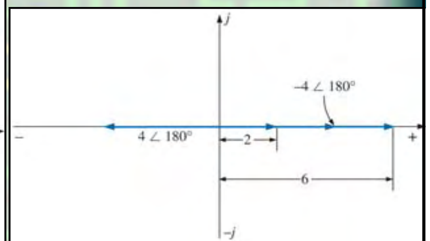
Again, there is no need to memorize the equation if the alternative method of Example 14-20 is used.





Addition or subtraction cannot be performed in polar form unless the complex numbers have the same angle θ or unless they differ only by multiples of 180°.





Multiplication: To multiply two complex numbers in rectangular form, multiply the real and imaginary parts of one in turn by the real and imaginary parts of the other. For example, if

$$\mathbf{C_1} = X_1 + jY_1 \quad \text{and} \quad \mathbf{C_2} = X_2 + jY_2$$

then $\mathbf{C_1 \cdot C_2}$:

$$\begin{array}{r} X_1 + jY_1 \\ \underline{X_2 + jY_2} \\ X_1X_2 + jY_1X_2 \\ \underline{+ jX_1Y_2 + j^2Y_1Y_2} \\ X_1X_2 + j(Y_1X_2 + X_1Y_2) + Y_1Y_2(-1) \end{array}$$

and $\boxed{\mathbf{C_1 \cdot C_2} = (X_1X_2 - Y_1Y_2) + j(Y_1X_2 + X_1Y_2)}$

In Example 14-22(b), we obtain a solution without resorting to memorizing equation above. Simply carry along the *j* factor when multiplying each part of one vector with the real and imaginary parts of the other.

Ex. 14-22

a. Find $\mathbf{C_1 \cdot C_2}$ if $\mathbf{C_1} = 2 + j3$ and $\mathbf{C_2} = 5 + j10$

b. Find $\mathbf{C_1 \cdot C_2}$ if $\mathbf{C_1} = -2 - j3$ and $\mathbf{C_2} = +4 - j6$

a. Using the format above, we have

$$\mathbf{C_1 \cdot C_2} = [(2)(3) - (3)(10)] + j[(3)(5) + (2)(10)]$$
$$= -20 + j35$$

b. Without using the format, we obtain

$$\begin{array}{l} -2 - j3 \\ \underline{+4 - j6} \\ -8 - j12 \\ \underline{\quad + j12 + j^2 18} \\ -8 + j(-12 + 12) - 18 \end{array}$$

and $\mathbf{C_1 \cdot C_2} = -26 = 26\angle 180°$

In ***polar*** form, the magnitudes are multiplied and the angles added algebraically. For example, for $C_1 = Z_1\angle\theta_1$ and $C_2 = Z_2\angle\theta_2$

we write

$$C_1 \cdot C_2 = Z_1 Z_2 \angle(\theta_1 + \theta_2)$$

To multiply a complex number in rectangular form by a real number requires that both the real part and the imaginary part be multiplied by the real number. For example,

$$(10)(2 + j3) = 20 + j30$$

and

$$50\angle 0^o (0 + j6) = j300 = 300\angle 90^o$$

Division: To divide two complex numbers in rectangular form, multiply the numerator and denominator by the conjugate of the denominator and the resulting real and imaginary parts collected. That is, if

$$C_1 = X_1 + jY_1 \quad and \quad C_2 = X_2 + jY_2$$

then
$$\frac{C_1}{C_2} = \frac{(X_1 + jY_1)(X_2 - jY_2)}{(X_2 + jY_2)(X_2 - jY_2)}$$

$$= \frac{(X_1X_2 + Y_1Y_2) + j(X_2Y_1 - X_1Y_2)}{X_2^2 + Y_2^2}$$

and
$$\boxed{\frac{C_1}{C_2} = \frac{X_1X_2 + Y_1Y_2}{X_2^2 + Y_2^2} + j\frac{X_2Y_1 - X_1Y_2}{X_2^2 + Y_2^2}}$$

The equation does not have to be memorized if the steps above used to obtain it are employed. That is, first multiply the numerator by the complex conjugate of the denominator and separate the real and imaginary terms. Then divide each term by the sum of each term of the denominator square.

Ex. 14-24
a. Find $\mathbf{C_1 / C_2}$ if $\mathbf{C_1} = 1 + j4$ and $\mathbf{C_2} = 4 + j5$
b. Find $\mathbf{C_1 / C_2}$ if $\mathbf{C_1} = -4 - j8$ and $\mathbf{C_2} = +6 - j1$

a. By preceding equation,

$$\frac{C_1}{C_2} = \frac{(1)(4) + (4)(5)}{4^2 + 5^2} + j\frac{(4)(4)}{4^2 + 5^2}$$

$$= \frac{24}{41} + j\frac{11}{41} \cong 0.59 + j0.27$$

To divide a complex number in rectangular form by a real number, both the real part and the imaginary part must be divided by the real number. For example,

$$\frac{8 + j10}{2} = 4 + j5$$

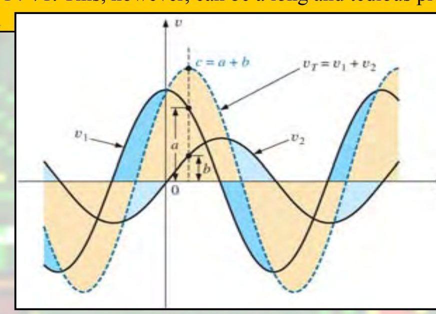
and
$$\frac{6.8 - j0}{2} = 3.4 - j0 = 3.4\angle 0^o$$

In polar form, division is accomplished by dividing the magnitude of the numerator by the magnitude of the denominator and subtracting the angle of the denominator from that of the numerator. That is, for $C_1 = Z_1\angle\theta_1 \quad and \quad C_2 = Z_2\angle\theta_2$

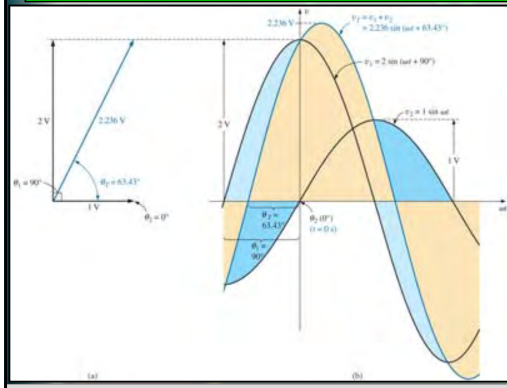
we write
$$\boxed{\frac{C_1}{C_2} = \frac{Z_1}{Z_2}\angle(\theta_1 - \theta_2)}$$

Phasors

The addition of sinusoidal voltages and current is frequently required in the analysis of ac circuits. One lengthy but valid method of performing this operation is to place both sinusoidal waveforms on the same set of axis and add a algebraically the magnitudes of each at every point along the abscissa, as shown for *c* = *a* + *b* in Fig. 14-71. This, however, can be a long and tedious process with limited accuracy.



A shorter method uses the rotating *radius vector*. This ***radius vector***, having a ***constant magnitude (length) with one end fixed at the origin***, is called a phasor when applied to electric circuits. During its rotational development of the sine wave, the phasor will, at the instant = 0, have the positions shown in Fig. 14-72(a) for each waveform in Fig. 14-72(b).



Note in Fig. 14-72(b) that v_2 passes through the horizontal axis at t = 0 s, requiring that the radius vector in Fig. 14-72(a) is equal to the peak value of the sinusoid as required by the radius vector. The other sinusoid has passed through 90° of its rotation by the time t = 0 s is reached and therefore has its maximum vertical projection as shown in Fig. 14-72(a). Since the vertical projection is a maximum, the peak value of the sinusoid that it generates is also attained at t = 0 s as shown in Fig. 14-72(b).

It can be shown [see Fig. 14-72(a)] using the vector algebra described that

$$1V\angle 0^o = 2V\angle 90^o = 2.236V\angle 63.43^o$$

In other words, if we convert v_1 and v_2 to the phasor form using

$$v = V_m \sin(\omega t \pm \theta) \Rightarrow V_m\angle \pm \theta$$

And add then using complex number algebra, we can find the phasor form for v_T with very little difficulty. It can then be converted to the time-domain and plotted on the same set of axes, as shown in Fig. 14-72(b). Fig. 14-72(a), showing the magnitudes and relative positions of the various phasors, is called a **phasor diagram**.

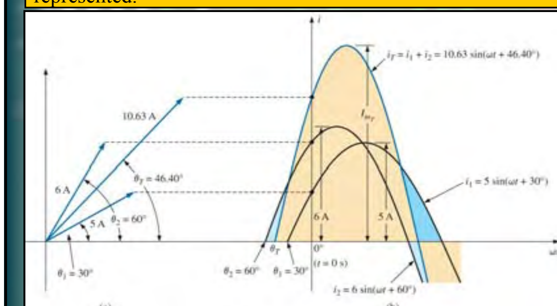
In the future, therefore, if the addition of two sinusoids is required, you should first convert them to phasor domain and find the sum using complex algebra. You can then convert the result to the time domain.

The case of two sinusoidal functions having phase angles different from 0° and 90° appears in Fig. 14-73. Note again that the vertical height of the functions in Fig. 14-73(b) at t = 0 s is determined by the rotational positions of the radius vectors in Fig. 14-73(a).

In general, for all of the analysis to follow, the phasor form of a sinusoidal voltage or current will be

$$\mathbf{V} = V\angle\theta \quad and \quad \mathbf{I} = I\angle\theta$$

where V and I are rms value and θ is the phase angle. It should be pointed out that in phasor notation, the sine wave is always the reference, and the frequency is not represented.



Ex. 14-27 Convert the following from the time to the phasor domain:

Time Domain	Phasor Domain
a. $\sqrt{2}(50)\sin\omega t$	$50\angle 0°$
b. $69.9\sin(\omega$	$(0.707)(69.6)\angle 72° = 49.21\angle$

Ex. 14-28 Write the sinusoidal expression for the following phasors if the frequency is 60 Hz:

	Phasor Domain

Ex. 14-29 Find the input voltage of the circuit in Fig. 14-75 if

$$\left.\begin{array}{l} v_a = 50\sin(377t + 30^\circ) \\ v_b = 30\sin(377t + 60^\circ) \end{array}\right\} f = 60\ Hz$$

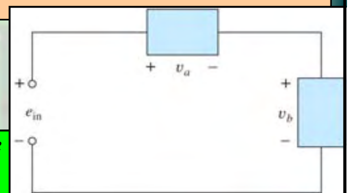


Figure 14.75

Applying Kirchhoff's voltage law, we have

$$e_m = v_a + v_b$$

Converting from the time to the phasor domain yields

$$v_a = 50\sin(377t + 30^o) \Rightarrow V_a = 35.35V\angle 30^\circ$$

$$v_b = 50\sin(377t + 60^o) \Rightarrow V_b = 21.21V\angle 60^\circ$$

Converting from polar to rectangular form for addition yields

$$V_a = 35.35V\angle 30^\circ = 30.61V + j17.68V$$

$$V_b = 21.21V\angle 30^\circ = 10.61V + j18.37V$$

Then

$$E_m = V_a + V_b = (30.61V + j17.68) + (10.61V + j18.37)$$
$$= 41.22V + j36.05V$$

Converting from rectangular to polar form, we have

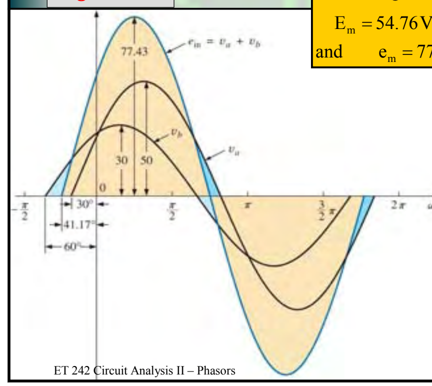
$$E_m = 41.22V + j36.05V = 54.76V\angle 41.17^\circ$$

Converting from the phasor to the time domain, we obtain

$$E_m = 54.76V\angle 41.17^\circ \Rightarrow e_m = \sqrt{2}(54.76)\sin(377t + 41.17^\circ)$$

and $e_m = 77.43\sin(377t + 41.17^\circ)$

Figure 14.76



A plot of the three waveforms is shown in Fig. 14-76. Note that at each instant of time, the sum of the two waveform does in fact add up to e_m. At $t = 0$ ($\omega t = 0$), e_m is the sum of the two positive values, while at a value of ωt, almost midway between π/2 and π, the sum of the positive value of v_a and the negative value of v_b results in $e_m = 0$.

Ex. 14-30 Determine the current i_2 for the network in Fig. 14-77.

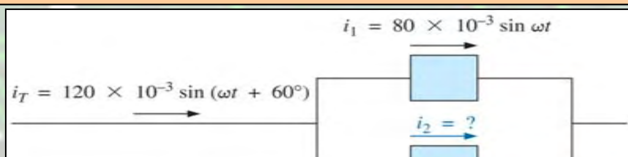


Figure 14.77

Applying Kirchhoff's current law, we have

$$i_T = i_1 + i_2 \quad or \quad i_2 = i_T - i_1$$

Converting from the time to the phasor domain yields

$$i_T = 120\times 10^{-3}\sin(\omega t + 60^o) \Rightarrow 84.84\,mA\angle 60^\circ$$

$$i_1 = 80\times 10^{-3}\sin\omega t \Rightarrow 56.56\,mA\angle 0^\circ$$

Converting from polar to rectangular form for subtracting yields

$$I_T = 84.84\,mA\angle 60^\circ = 42.42\,mA + j73.47V$$

$$I_1 = 56.56\,mA\angle 0^\circ = 56.56\,mA + j0$$

Then $I_2 = I_T - I_1 = (42.42\,mA + j73.47\,mA) - (56.56\,mA + j0)$

$$= -14.14\,mA + j73.47\,mA$$

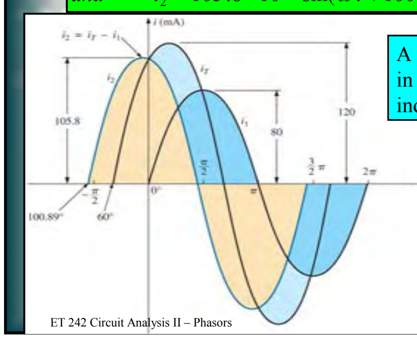
Converting from rectangular to polar form, we have

$$I_2 = 74.82\,mA\angle 100.89^\circ$$

Converting from the phasor to the time domain, we have

$$I_2 = 74.82\,mA\angle 100.89^\circ \Rightarrow i_2 = \sqrt{2}(74.82\times 10^{-3})\sin(\omega t + 100.89^\circ)$$

and $i_2 = 105.8\times 10^{-3}\sin(\omega t + 100.89^\circ)$



A plot of the three waveforms appears in Fig. 14-78. The waveforms clearly indicate that $i_T = i_1 + i_2$.

Figure 14.78

HW 14-50 For the system in Fig. 14.87, find the sinusoidal expression for the unknown voltage v_a if

$$e_m = 60\sin(377t + 20°)$$
$$v_b = 20\sin(377t - 20°)$$

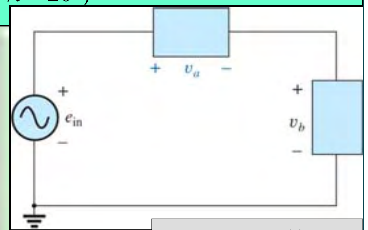


Figure 14.87 Problem 50.

(Using peak values)

$$e_{in} = v_a + v_b \Rightarrow v_a = e_{in} - v_b$$
$$= (60\,V\angle 20°) - 20\,V\angle -20°)$$
$$= 48.49\,V\angle 36.05°$$

and $e_m = 46.49\,sin(377t + 36.05°)$

HW 14-51 For the system in Fig. 14.88, find the sinusoidal expression for the unknown voltage i_1 if

$$i_s = 20 \times 10^{-6}\sin(\omega t + 60°)$$
$$i_2 = 6 \times 10^{-6}\sin(\omega t - 30°)$$

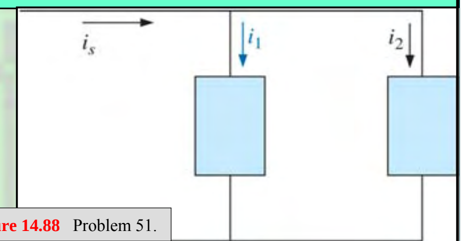


Figure 14.88 Problem 51.

$i_2 = i_1 + i_2 \Rightarrow i_1 = i_s - i_2$

(Using peak values) $= (20 \times 10^{-6}\,A\angle 60°) - (6 \times 10^{-6}\,A\angle -30°)$

$$= 20.88 \times 10^{-6}\,A\angle 76.70°$$

$i_1 = 20.88 \times 10^{-6}\,sin(\omega i + 76.70°)$

Homework 14: 39, 40, 43-45, 48, 50, 51

EET1222/ET242 Circuit Analysis II

Series AC Circuits Analysis

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Series ac Circuits Analysis
- Impedance and Phase Diagram
- Series Configuration
- Voltage Divider Rule
- Frequency Response for Series ac Circuits

Key Words: Impedance, Phase, Series Configuration, Voltage Divider Rule

Series & Parallel ac Circuits

Phasor algebra is used to develop a quick, direct method for solving both ***series and parallel ac circuits***. The close relationship that exists between this method for solving for unknown quantities and the approach used for dc circuits will become apparent after a few simple examples are considered. Once this association is established, many of the rules (current divider rule, voltage divider rule, and so on) for dc circuits can be applied to ac circuits.

Series ac Circuits

Impedance & the Phasor Diagram – Resistive Elements

From previous lesson we found, for the purely resistive circuit in Fig. 15-1, that ***v*** and ***i*** were in phase, and the magnitude

$$I_m = \frac{V_m}{R} \quad or \quad V_m = I_m R$$

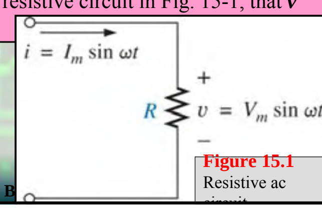

Figure 15.1 Resistive ac circuit.

$$\text{In Phasor form}, \quad v = V_m \sin \omega t \Rightarrow \mathbf{V} = V\angle 0^\circ$$

where $V = 0.707\, V_m$,

Applying Ohm's law and using phasor algebra, we have

$$\mathbf{I} = \frac{V\angle 0^\circ}{R\angle 0^\circ} = \frac{V\angle 0^\circ}{R\angle \theta^\circ}\angle(0^\circ - \theta_R)$$

Since i and v are in phase, the angle associated with i also must be 0°. To satisfy this condition, θ_R must equal 0°. Substituting $\theta_R = 0^\circ$, we found

$$\mathbf{I} = \frac{V\angle 0^\circ}{R\angle 0^\circ} = \frac{V}{R}\angle(0^\circ - 0^\circ) = \frac{V}{R}\angle 0^\circ$$

so that in the time domain, $i = \sqrt{2}\left(\frac{V}{R}\right)\sin \omega t$

We use the fact that $\theta_R = 0^\circ$ in the following polar format to ensure the proper phase relationship between the voltage and current of a resistor:

$$\mathbf{Z_R} = R\angle 0^\circ$$

Ex. 15-1 Using complex algebra, find the current i for the circuit in Fig. 15-2. Sketch the waveforms of v and i.

Note Fig. 15 – 3:

$$v = 100 \sin \omega t \Rightarrow \text{phasor form } \mathbf{V} = 70.71\, V\angle 0^\circ$$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_R}} = \frac{V\angle\theta}{R\angle 0^\circ} = \frac{70.71\, V\angle 0^\circ}{5\Omega} = 14.14\ A\angle 0^\circ$$

$$\text{and} \quad i = \sqrt{2}(14.14)\sin \omega t = 20 \sin \omega t$$

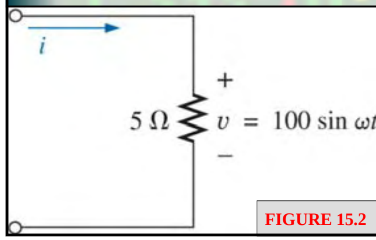

FIGURE 15.2

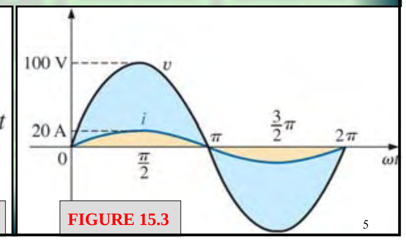

FIGURE 15.3

Ex. 15-2 Using complex algebra, find the voltage v for the circuit in Fig. 15- 4. Sketch the waveforms of v and i.

Note Fig. 15 – 5:

$$i = 4\sin(\omega t + 30^\circ) \Rightarrow \text{phasor form } \mathbf{I} = 2.828\, A\angle 30^\circ$$

$$\mathbf{V} = IZ_R = (I\angle\theta)(R\angle 0^\circ) = (2.282\, A\angle 30^\circ)(2\Omega\angle 0^\circ) = 5.656\, V\angle 30^\circ$$

$$\text{and} \quad v = \sqrt{2}(6.656)\sin(\omega t + 30^\circ) = 8.0 \sin(\omega t + 30^\circ)$$

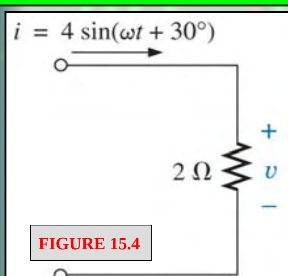

FIGURE 15.4

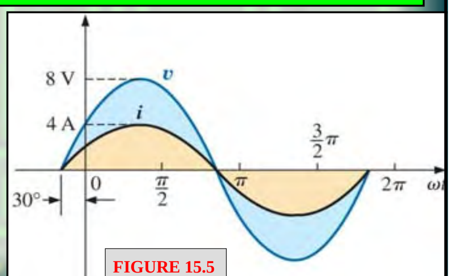

FIGURE 15.5

Series ac Circuits

Impedance & the Phasor Diagram – Inductive Elements

From previous lesson we found that the purely inductive circuit in Fig. 15-7, voltage leads the current by 90° and that the reactance of the coil X_L is determined by ωL.

$$v = V_m \sin \omega t \Rightarrow \text{Phasor form} \quad \mathbf{V} = V\angle 0^\circ$$

$$\text{By ohm's law}, \quad I = \frac{V\angle 0^\circ}{X_L\angle\theta_L} = \frac{V}{X_L}\angle(0^\circ - \theta_L)$$

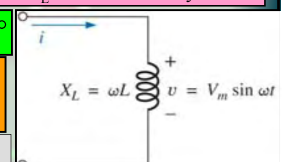

Figure 15.7 Inductive ac circuit.

Since v leads i by 90°, i must have an angle of – 90° associated with it. To satisfy this condition, θ_L must equal + 90°. Substituting $\theta_L = 90^\circ$, we obtain

$$\mathbf{I} = \frac{V\angle 0^\circ}{X_L\angle 90^\circ} = \frac{V}{X_L}\angle(0^\circ - 90^\circ) = \frac{V}{X_L}\angle -90^\circ$$

so that in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_L}\right)\sin(\omega i - 90^\circ)$$

We use the fact that $\theta_L = 90^\circ$ in the following polar format for inductive reactance to ensure the proper phase relationship between the voltage and current of an inductor:

$$\mathbf{Z_L} = X_L\angle 90^\circ$$

Ex. 15-3 Using complex algebra, find the current i for the circuit in Fig. 15- 8. Sketch the v and i curves.

Note Fig.15 – 9 :

$v = 24\sin \omega t \Rightarrow$ *phasor form* $\mathbf{V} = 16.968\,V\angle 0°$

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z_L}} = \frac{V\angle\theta}{X_L\angle 90°} = \frac{16.968\,V\angle 0°}{3\,\Omega\angle 90°} = 5.656\ A\angle -90°$$

and $i = \sqrt{2}(5.656)\sin(\omega t - 90°) = 8.0\sin(\omega t - 90°)$

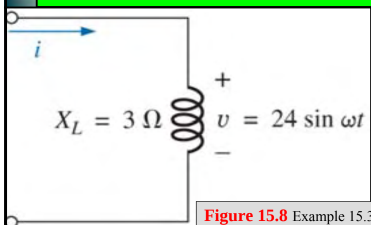

Figure 15.8 Example 15.3.

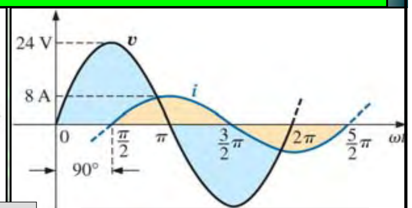

Figure 15.9 Waveform for Example 15.3.

Ex. 15-4 Using complex algebra, find the voltage v for the circuit in Fig. 15- 10. Sketch the v and i curves.

Note Fig.15 – 11 :

$i = 24\sin(\omega t + 30°) \Rightarrow$ *phasor form* $\mathbf{I} = 3.535\ A\angle 30°$

$\mathbf{V} = IZ_L = (I\angle\theta)(X_L\angle 90°) = (3.535\ A\angle 30°)(4\,\Omega\angle +90°) = 14.140\ V\angle 120°$

and $v = \sqrt{2}(14.140)\sin(\omega t + 120°) = 20\sin(\omega t + 120°)$

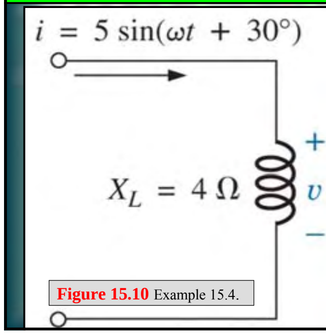

Figure 15.10 Example 15.4.

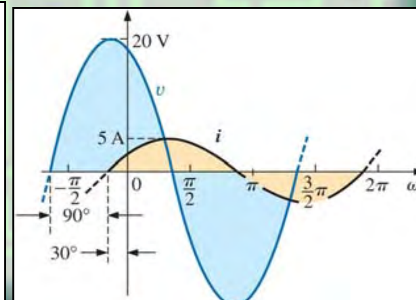

Figure 15.11 Waveforms for Example 15.4.

Capacitive Resistance

For the pure capacitor in Fig. 15.13, the current leads the voltage by 90º and that the reactance of the capacitor X_C is determined by $1/\omega C$.

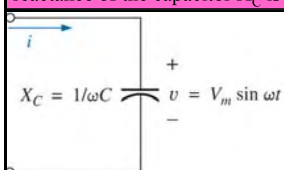

Figure 15.13 Capacitive ac circuit.

$V = V_m \sin \omega t \Rightarrow$ *Phasor form* $\mathbf{V} = V\angle 0°$

Applying Ohm's law and using phasor algebra, we find

$$\mathbf{I} = \frac{V\angle 0°}{X_C\angle\theta_C} = \frac{V}{X_C}\angle(0° - \theta_C)$$

Since i leads v by 90º, i must have an angle of +90º associated with it. To satisfy this condition, θ_C must equal –90º. Substituting θ_C = – 90º yields

$$\mathbf{I} = \frac{V\angle 0°}{X_C\angle -90°} = \frac{V}{X_C}\angle(0° - (-90°)) = \frac{V}{X_C}\angle 90°$$

so, in the time domain,

$$i = \sqrt{2}\left(\frac{V}{X_C}\right)\sin(\omega t + 90°)$$

We use the fact that θ_C = –90º in the following polar format for capacitive reactance to ensure the proper phase relationship between the voltage and current of a capacitor:

$$Z_C = X_C\angle -90°$$

Ex. 15-5 Using complex algebra, find the current i for the circuit in Fig. 15.14. Sketch the v and i curves.

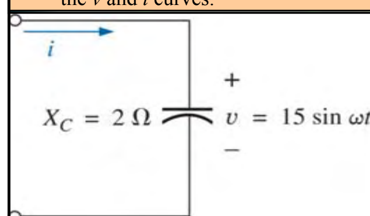

Figure 15.14 Example 15.5.

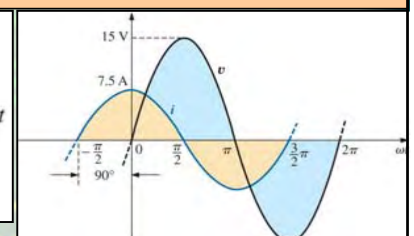

Figure 15.15 Waveforms for Example 15.5.

$v = 15\sin \omega t \Rightarrow$ *phasor notation* $\mathbf{V} = 10.605\,V\angle 0°$

$$\boldsymbol{I} = \frac{\mathbf{V}}{\boldsymbol{Z_C}} = \frac{V\angle\theta}{X_C\angle -90°} = \frac{10.605\,V\angle 0°}{2\Omega\angle -90°} = 5.303\,A\angle 90°$$

and $i = \sqrt{2}(5.303)\sin(\omega t + 90°) = 7.5\sin(\omega t + 90°)$

Ex. 15-6 Using complex algebra, find the current v for the circuit in Fig. 15.16. Sketch the v and i curves.

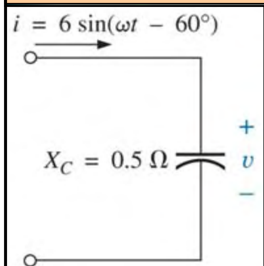

Figure 15.16 Example 15.6.

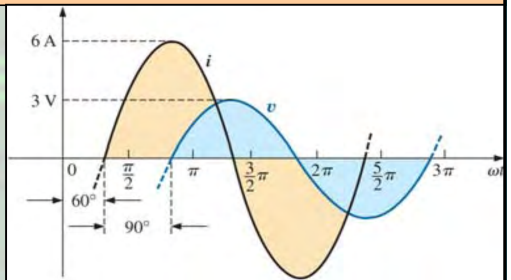

Figure 15.17 Waveforms for Example 15.6.

$$i = 6\sin(\omega t - 60°) \Rightarrow \ phasor\ \ notation \quad \mathbf{I} = 4.242\ A\angle -60°$$

$$\mathbf{I} = \mathbf{IZ}_C = (I\angle\theta)(X_C\angle -90°) = (4.242\ A\angle -60°)$$

$$= (0.5\Omega\angle -90°) = 2.121\ V\angle -150°$$

$$and \quad v = \sqrt{2}(2.121)\sin(\omega t - 150°) = 3.0\sin(\omega t - 150°)$$

Series Configuration

The overall properties of series ac circuits (Fig. 15.20) are the same as those for dc circuits. For instance, the total impedance of a system is the sum of the individual impedances:

$$Z_T = Z_1 + Z_2 + Z_3 + \cdots\cdots + Z_N$$

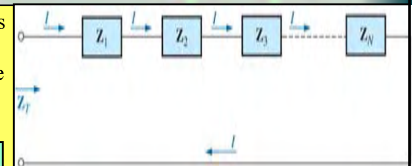

Figure 15.20 Series impedance.

Ex. 15-7 Draw the impedance diagram for the circuit in Fig. 15.21, and find the total impedance.

As indicated by Fig. 15.22, the input impedance can be found graphically from the impedance diagram by properly scaling the real and imaginary axes and finding the length of the resultant vector Z_T and angle θ_T. Or, by using vector algebra, we obtain

$$Z_T = Z_1 + Z_2 = R\angle 0° + X_L\angle 90° = R + jX_L$$
$$= 4\Omega + j8\Omega = 8.94\Omega\angle 63.43°$$

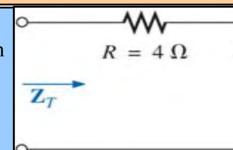

Figure 15.21 Example 15.7.

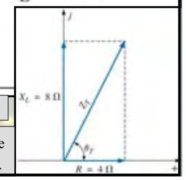

Figure 15.22 Impedance diagram for Example 15.7.

Ex. 15-8 Determine the input impedance to the series network in Fig. 15.23. Draw the impedance diagram.

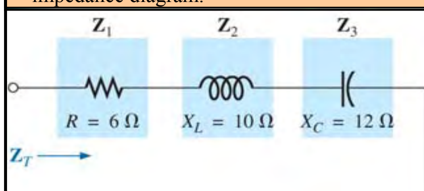

Figure 15.23 Example 15.8.

$$Z_T = Z_1 + Z_2 + Z_3$$
$$= R\angle 0° + X_L\angle 90° + X_C\angle -90°$$
$$= R + jX_L - jX_C$$
$$= R + j(X_L - X_C)$$
$$= 6\Omega + j(8\Omega - 12\Omega)$$
$$= 6.32\Omega\angle -18.43°$$

The impedance diagram appears in Fig. 15.24. Note that in this example, series inductive and capacitive reactances are in direct opposition. For the circuit in Fig. 15.23, if the inductive reactance were equal to the capacitive reactance, the input impedance would be purely resistive.

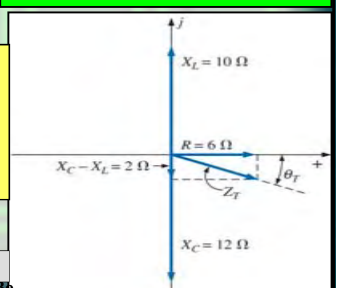

Figure 15.24 Impedance diagram for Example 15.8.

For the representative **series ac configuration** in Fig. 15.25 having two impedances, ***the currents is the same through each element*** (as it was for the series dc circuits) and is determined by Ohm's law:

$$Z_T = Z_1 + Z_2 \quad and \quad \mathbf{I} = \frac{\mathbf{E}}{\mathbf{Z}_T}$$

The voltage across each element can be found by another application of Ohm's law:

$$V_1 = I\,Z_1 \quad and \quad V_2 = I\,Z_2$$

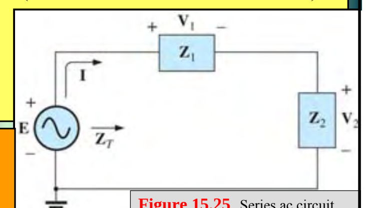

Figure 15.25 Series ac circuit.

KVL can then be applied in the same manner as it is employed for dc circuits. However, keep in mind that we are now dealing with the algebraic manipulation of quantities that have both magnitude and direction.

$$-E + V_1 + V_2 = 0 \quad or \quad E = V_1 + V_2$$

The power to the circuit can be determined by

$$P = EI\ \cos\ \theta_T$$

where θ_T is the phase angle between **E** and **I**.

Voltage Divider Rule

The basic format for the **voltage divider rule** in ac circuits is exactly the same as that for dc circuits:

$$V_x = \frac{Z_x E}{Z_T}$$

where V_x is the voltage across one or more elements in a series that have total impedance Z_x, E is the total voltage appearing across the series circuit, and Z_T is the total impedance of the series circuit.

Ex. 15-9 Using the voltage divider rule, find the voltage across each element of the circuit in Fig. 15.40.

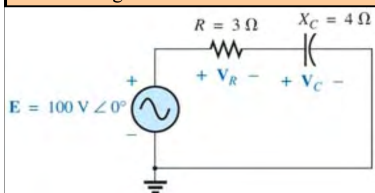

Figure 15.40 Example 15.9.

$$V_C = \frac{Z_C E}{Z_C + Z_R} = \frac{(4\Omega\angle -90^\circ)(100V\angle 0^\circ)}{4\Omega\angle -90^\circ + 3\Omega\angle 0^\circ}$$

$$= \frac{400\angle -90^\circ}{3 - j4} = \frac{400\angle -90^\circ}{5\angle -53.13^\circ} = 80V\angle -36.87^\circ$$

$$V_R = \frac{Z_R E}{Z_C + Z_R} = \frac{(3\Omega\angle 0^\circ)(100V\angle 0^\circ)}{5\Omega\angle -53.13^\circ}$$

$$= \frac{300\angle 0^\circ}{5\angle -53.13^\circ} = 60V\angle +53.13^\circ$$

Ex. 15-10 Using the voltage divider rule, find the unknown voltages V_R, V_L, V_C, and V_1 for the circuit in Fig. 15.41.

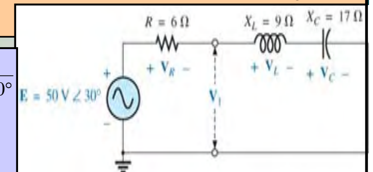

Figure 15.41 Example 15.10.

$$V_R = \frac{Z_R E}{Z_C + Z_L + Z_C} = \frac{(6\Omega\angle 0^\circ)(50V\angle 30^\circ)}{6\Omega\angle 0^\circ + 9\Omega\angle 90^\circ + 17\Omega\angle -90^\circ}$$

$$= \frac{300\angle 30^\circ}{6 + j9 - j17} = \frac{300\angle 30^\circ}{6 - j8} = \frac{300\angle 30^\circ}{10\angle -53.13}$$

$$= 30V\angle 83.13^\circ$$

$$V_L = \frac{Z_L E}{Z_T} = \frac{(9\Omega\angle 90^\circ)(50V\angle 30^\circ)}{10\Omega\angle 173.13^\circ} = \frac{450\angle 120^\circ}{10\angle -53.13} = 45V\angle 173.13^\circ$$

$$V_C = \frac{Z_C E}{Z_T} = \frac{(17\Omega\angle -90^\circ)(50V\angle 30^\circ)}{10\Omega\angle -53.13^\circ} = \frac{850\angle -60^\circ}{10\angle -53.13} = 85V\angle -6.87^\circ$$

$$V_1 = \frac{(Z_L + Z_C)E}{Z_T} = \frac{(9\Omega\angle 90^\circ + 17\Omega\angle -90^\circ)(50V\angle 30^\circ)}{10\Omega\angle -53.13^\circ}$$

$$= \frac{(8\angle -90^\circ)(50\angle 30^\circ)}{10\Omega\angle -53.13^\circ} = \frac{400\angle -60^\circ}{10\angle -53.13^\circ} = 40V\angle -6.87^\circ$$

Frequency Response for Series ac Circuits

Thus far, the analysis has been for a fixed frequency, resulting in a fixed value for the reactance of an inductor or a capacitor. We now examine how the response of a series changes as the frequency changes. We assume ideal elements throughout the discussion so that the response of each element will be shown in Fig. 15.46.

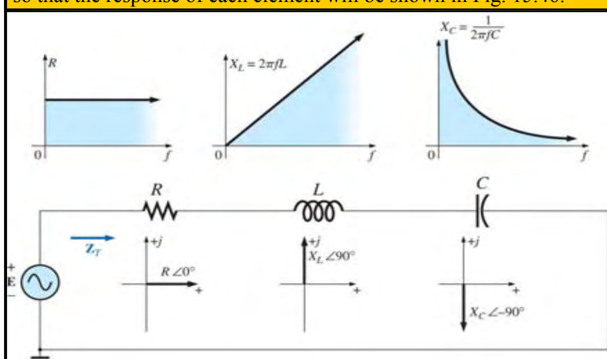

Figure 15.46 Reviewing the frequency response of the basic elements.

When considering elements in series, remember that the total impedance is the sum of the individual elements and that the reactance of an inductor is in direct opposition to that capacitor. For Fig. 15.46, we are first aware that the resistance will remain fixed for the full range of frequencies: It will always be there, but, more importantly, its magnitude will not change. ***The inductor, however, will provide increasing levels of importance as the frequency increases, while the capacitor will provide lower levels of impedance.***

In general, if we encounter **a series R-L-C circuit at very low frequencies**, we can assume that **the capacitor, with its very large impedance, will be dominant factor**. If the circuit is just an R-L series circuit, the impedance may be determined primarily by the resistive element since the reactance of the inductor is so small. As the frequency increases, the reactance of the coil increases to the point where it totally outshadows the impedance of the resistor. **For an R-L-C combination, as the frequency increases**, the reactance of the capacitor begins to approach a short-circuit equivalence, and **total impedance will be determined primarily by the inductive element**.

In total, therefore,

when encountering a series circuit of any combination of elements, always use the idealized response of each element to establish some feeling for how the circuit will response as the frequency changes.

Series R-C ac Circuits

As an example of establishing the frequency response of a circuit, consider the series R-C circuit in Fig. 15.47. As noted next to the source, the frequency range of interest is from 0 to 20 kHz.

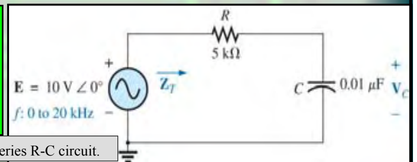

Figure 15.47 Determining the frequency response of a series R-C circuit.

The frequency at which the reactance of the capacitor drops to that of the resistor can be determined by setting the reactance of the capacitor equal to that of the resistor as follows:

$$X_C = \frac{1}{2\pi f_1 C} = R$$

Solving for the frequency yields

$$f_1 = \frac{1}{2\pi RC}$$

Now for the details. The total impedance is determined by the following equation:

$$Z_T = R - jX_C$$

$$and \quad \mathbf{Z_T} = Z_T\angle\theta_T$$

$$= \sqrt{R^2 + X_C^2}\angle -\tan^{-1}\frac{X_C}{R} \qquad (15.12)$$

The magnitude and angle of the total impedance can now be found at any frequency of interest by simply substituting into Eq. (15.12).

We now that for frequencies greater than f_1, $R > X_C$ and that for frequencies less than f_1, $X_C > R$, as shown in Fig. 15.48.

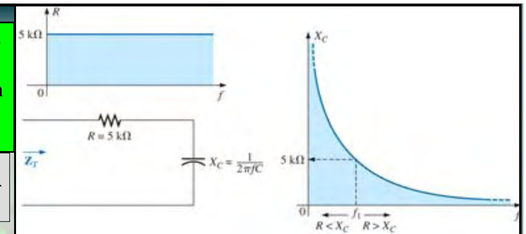

Figure 15.48 The frequency response for the individual elements of a series of a series R-C circuit.

$f = 100\ Hz$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(100\,Hz)(0.01\,\mu F)} = 159.16\,k\Omega$$

$$and \quad Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5k\Omega)^2(159.16k\Omega)^2} = 159.24\,k\Omega$$

$$with \quad \theta_T = -\tan^{-1}\frac{X_C}{R} = -\tan^{-1}\frac{X_C}{R} = -\tan\frac{159.16k\Omega}{5k\Omega} = -\tan^{-1}31.83° = -88.2°$$

$$and \quad \mathbf{Z_T} = 159.24\,k\Omega\angle -88.2°$$

$f = 1\ kHz$

$$X_C = \frac{1}{2\pi fC} = \frac{1}{2\pi(1kHz)(0.01\,\mu F)} = 15.92\,k\Omega$$

$$and \quad Z_T = \sqrt{R^2 + X_C^2} = \sqrt{(5k\Omega)^2(15.92\,k\Omega)^2} = 16.69\,k\Omega$$

$$with \quad \theta_T = -\tan^{-1}\frac{X_C}{R} = -\tan\frac{15.9.2k\Omega}{5k\Omega} = -\tan^{-1}3.18° = -72.54°$$

$$and \quad \mathbf{Z_T} = 16.69\,k\Omega\angle -72.54°$$

Ex. 15-12 For the series R-L circuit in Fig. 15.56:

a. Determine the frequency at which $X_L = R$.

b. Develop a mental image of the change in total impedance with frequency without doing any calculations.

c. Find the total impedance at $f = 100\ Hz$ and $40\ kHz$, and compare your answer with the assumptions of part (b)

d. Plot the curve of V_L versus frequency.

e. Find the phase angle of the total impedance at $f = 40\ kHz$. Can the circuit be considered inductive at this frequency? Why?

$$a. \quad X_L = 2\pi f_1 L = R \quad and$$

$$f_1 = \frac{R}{2\pi L} = \frac{2k\Omega}{2\pi(40mH)} = 7957.7\,Hz$$

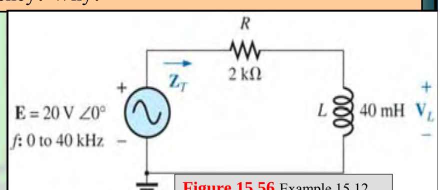

Figure 15.56 Example 15.12.

b. At low frequencies, $R > X_L$ and the impedance will be very close to that of the resistor, or 2 kΩ. As the frequency increases, X_L increases to a point where it is the predominant factor. The result is that the curve starts almost horizontal at 2 kΩ and then increases linearly to very high levels.

$$c. \quad Z_T = R + jX_L = Z_T\angle\theta_T = \sqrt{R^2 + X_L^2}\angle\tan^{-1}\frac{X_L}{R}$$

$At\ f = 100\ Hz:$

$$X_L = 2\pi fL = 2\pi(100\,Hz)(40\,mH) = 25.13\ \Omega$$

$$and \quad Z_T = \sqrt{R^2 + X_L^2} = \sqrt{(2k\Omega)^2 + (25.13\Omega)^2} = 2000.16\,\Omega \cong R$$

$At\ f = 40\ kHz:$

$$X_L = 2\pi fL = 2\pi(40\,kHz)(40\,mH) = 10.25\ k\Omega \cong X_L$$

$$and \quad Z_T = \sqrt{R^2 + X_L^2} = \sqrt{(2k\Omega)^2 + (10.05k\Omega)^2} = 10.25\,k\Omega \cong X_L$$

Both calculations support the conclusions of part (b).

d. Applying the voltage divider rule:

$$V_L = \frac{Z_L E}{Z_T}$$

From part (c), we know that at 100 Hz, $Z_T \approx R$ so that $V_R \approx X_L$ so that $V_L \approx 20V$ and $V_R \approx 0V$. The result is two plot points for the curve in Fig. 15.57.

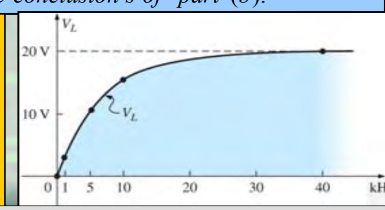

Figure 15.57 Plotting V_L versus for the series R-L circuit in Fig. 15.56.

$$At \quad 1\ kHz: \quad X_L = 2\pi fL \cong 0.25\ k\Omega$$

$$and \quad V_L = \frac{(0.25k\Omega\angle 90^\circ)(20V\angle 0^\circ)}{2k\Omega + j0.25k\Omega} = 2.48\,V\angle 82.87^\circ$$

$$At \quad 5\ kHz: \quad X_L = 2\pi fL \cong 1.26\ k\Omega$$

$$and \quad V_L = \frac{(1.26k\Omega\angle 90^\circ)(20V\angle 0^\circ)}{2k\Omega + j1.26k\Omega} = 10.68\,V\angle 57.79^\circ$$

$$At \quad 10\ kHz: \quad X_L = 2\pi fL \cong 2.5\ k\Omega$$

$$and \quad V_L = \frac{(2.5k\Omega\angle 90^\circ)(20V\angle 0^\circ)}{2.5k\Omega + j2.5k\Omega} = 15.63\,V\angle 38.66^\circ$$

The complete plot appears in Fig. 15.57.

$$e. \quad \theta_T = -\tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{10.05k\Omega}{2k\Omega} = 78.75^\circ$$

The angle θ_T is closing in on the 90º of a purely inductive network. Therefore, the network can be considered quite inductive at a frequency of 40 kHz.

HW 15-15 Calculate the voltage V_1 and V_2 for the circuits in Fig. 15.134 in Phasor form using the voltage divider rule.

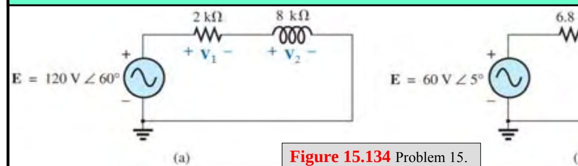

Figure 15.134 Problem 15.

Homework 15: 2-7, 9-12, 15, 16

a.
$$\mathbf{V}_1 = \frac{(2\,k\Omega\angle 0^\circ)(120\,V\angle 60^\circ)}{2\,k\Omega + j8\,k\Omega} = \frac{240\,V\angle 60^\circ}{8.25\angle 75.96^\circ} = \mathbf{29.09\,V\angle{-15.96^\circ}}$$

$$\mathbf{V}_2 = \frac{(8\,k\Omega\angle 90^\circ)(120\,V\angle 60^\circ)}{8.25\,k\Omega\angle 75.96^\circ} = \mathbf{116.36\,V\angle 74.04^\circ}$$

b.
$$\mathbf{V}_1 = \frac{(40\,\Omega\angle 90^\circ)(60\,V\angle 5^\circ)}{6.8\,\Omega + j40\,\Omega + 22\,\Omega} = \frac{2400\,V\angle 95^\circ}{28.8 + j40} = \mathbf{48.69\,V\angle 40.75^\circ}$$

$$\mathbf{V}_2 = \frac{(22\,\Omega\angle 0^\circ)(60\,V\angle 5^\circ)}{49.29\,\Omega\angle 54.25^\circ} = \frac{1.32\,kV\angle 5^\circ}{49.29\,\Omega\angle 54.25^\circ} = \mathbf{26.78\,V\angle{-49.25^\circ}}$$

EET1222/ET242 Circuit Analysis II

Parallel AC Circuits Analysis

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- **Introduction to Parallel ac Circuits Analysis**
- **Impedance and Phase Diagram**
- **Parallel Configuration**
- **Current Divider Rule**
- **Frequency Response for Parallel ac Circuits**
- **Phase Measurements**

Key Words: Parallel ac Circuit, Impedance, Phase, Frequency Response

Parallel ac Networks

For the representative parallel ac network in Fig. 15.67, the total impedance or admittance is determined as previously described, and the source current is determined by Ohm's law as follows:

$$I = \frac{E}{Z_T} = EY_T$$

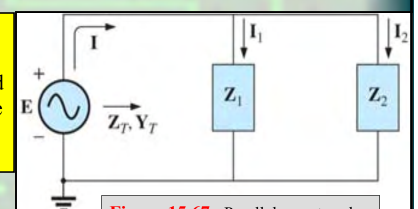


Figure 15.67 Parallel ac network.

Since the voltage is the same across parallel elements, the current through each branch can be found through another application of Ohm's law:

$$I_1 = \frac{E}{Z_1} = EY_1 \quad and \quad I_2 = \frac{E}{Z_2} = EY_2$$

KCL can then be applied in the same manner as used for dc networks with consideration of the quantities that have both magnitude and direction.

$$I - I_1 - I_2 = 0 \quad \text{or} \quad I = I_1 + I_2$$

The power to the network can be determined by $P = EI\cos\theta_T$

where θ_T is the phase angle between E and I.

Parallel ac Networks : R-L

Phasor Notation:
As shown in Fig. 15.69.

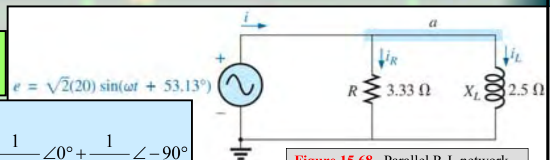


Figure 15.68 Parallel R-L network.

$Y_T = Y_R + Y_L$
$= G\angle 0° + B_L\angle -90° = \dfrac{1}{3.33\Omega}\angle 0° + \dfrac{1}{2.5\Omega}\angle -90°$
$= 0.3S\angle 0° + 0.4S\angle -90° = 0.3S - j0.4S$
$= 0.5S\angle -53.13°$
$Z_T = \dfrac{1}{Y_T} = \dfrac{1}{0.5S\angle -53.13°} = 2\Omega\angle 53.13°$

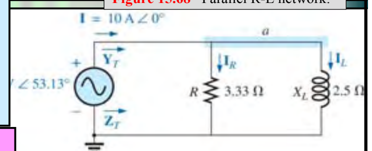


Figure 15.69 Applying phasor notation to the network in Fig. 15.68.

Admittance diagram : As shown in Fig.15.70

$I = \dfrac{E}{Z_T} = EY_T = (20V\angle 53.13°)(0.5S\angle -53.13) = 10A\angle 0°$
$I_R = \dfrac{E\angle\theta}{R\angle 0°} = (E\angle\theta)(G\angle 0°)$
$= (20V\angle 53.13°)(0.3S\angle 0°) = 6A\angle 53.13°$
$I_L = \dfrac{E\angle\theta}{X_L\angle 90°} = (E\angle\theta)(B_L\angle -90°)$
$= (20V\angle 53.13°)(0.4S\angle -90°) = 8A\angle -36.87°$

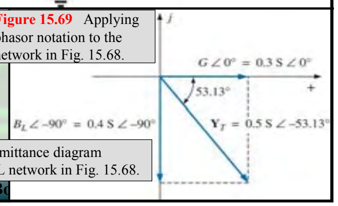


Figure 15.70 Admittance diagram for the parallel R-L network in Fig. 15.68.

$KCL:$ At node a,
$I - I_R - I_L = 0$ or $I = I_R + I_L$
$10A\angle 0° = 6A\angle 53.13° + 8A\angle -36.87°$
$10A\angle 0° = (3.60A + j4.8A) + (6.40A - j4.80A)$
$= 10A + j0$
and $10A\angle 0° = 10A\angle 0°$ (checks)

Phasor diagram: The phasor diagram in Fig. 15.71 indicates that the applied voltage E is in phase with the current I_R and leads the current I_L by 90°.

Power : The total power in watts delivered to the circuit is
$P_T = EI\cos\theta_T$
$= (20V)(10A)\cos 53.13° = (200W)(0.6)$
$= 120\ W$

Power factor : The power factor of the circuit is
$F_p = \cos\theta_T = \cos 53.13° = 0.6$ lagging

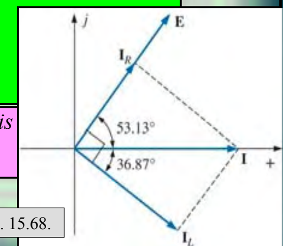


Figure 15.71 Phasor diagram for the parallel R-L network in Fig. 15.68.

Parallel ac Networks : R-C

Phasor Notation:
As shown in Fig. 15.73.

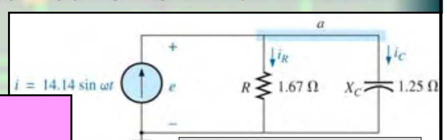


Figure 15.72 Parallel R-C network.

$Y_T = Y_R + Y_C = G\angle 0° + B_C\angle 90°$
$= \dfrac{1}{1.67\ \Omega}\angle 0° + \dfrac{1}{1.25\ \Omega}\angle 90°$
$= 0.6S\angle 0° + 0.8S\angle 90°$
$= 0.6S + j0.8S = 1.0S\angle 53.13°$
$Z_T = \dfrac{1}{Y_T} = \dfrac{1}{1.0S\angle 53.13°} = 1\Omega\angle -53.13°$

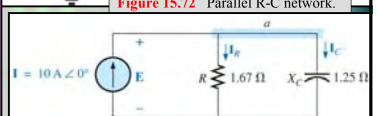


Figure 15.73 Applying phasor notation to the network in Fig. 15.72.

Admittance diagram : As shown in Fig.15.74.
$E = IZ_T = \dfrac{I}{Y_T} = \dfrac{10A\angle 0°}{1S\angle 53.13°} = 10V\angle -53.13°$
$I_R = (E\angle\theta)(G\angle 0°)$
$= (10V\angle -53.13°)(0.6S\angle 0°) = 6A\angle -53.13°$
$I_C = (E\angle\theta)(B_C\angle 90°)$
$= (10V\angle -53.13°)(0.8S\angle 90°) = 8A\angle 36.87°$

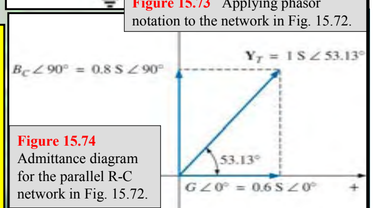


Figure 15.74 Admittance diagram for the parallel R-C network in Fig. 15.72.

$KCL:$ At node a,
$I - I_R - I_C = 0$ or $I = I_R + I_C$

Power factor : The power factor of the circuit is
$F_p = \cos\theta_T = \cos 53.13° = 0.6$ leading

Power : $P_T = EI\cos\theta_T = (10V)(10A)\cos 53.13° = (100W)(0.6S) = 60\ W$

Parallel ac Networks : R-L-C

Phasor Notation:
As shown in Fig. 15.78.

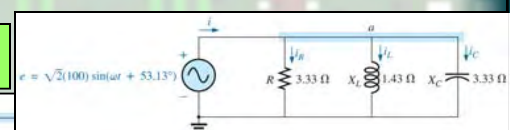


Figure 15.77 Parallel R-L-C network.

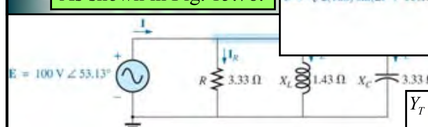


Figure 15.78 Applying phasor notation to the network in Fig. 15.77.

$Y_T = Y_R + Y_L + Y_C = G\angle 0° + B_L\angle -90° + B_C\angle 90°$
$= \dfrac{1}{3.33\Omega}\angle 0° + \dfrac{1}{1.43\Omega}\angle -90° + \dfrac{1}{3.33\Omega}\angle 90°$
$= 0.3S\angle 0° + 0.7S\angle -90° + 0.3S\angle 90°$
$= 0.3S - j0.7S + j0.3S$
$= 0.3S - j0.4S = 0.5S\angle -53.13°$
$Z_T = \dfrac{1}{Y_T} = \dfrac{1}{0.5S\angle -53.13°} = 2\Omega\angle 53.13°$

Admittance diagram : As shown in Fig.15.79.

$$I = \frac{E}{Z_T} = EY_T = (100V\angle 53.13°)(0.5S\angle -53.13) = 50A\angle 0°$$

$$I_R = (E\angle\theta)(G\angle 0°)$$
$$= (100V\angle 53.13°)(0.3S\angle 0°) = 30A\angle 53.13°$$
$$I_L = (E\angle\theta)(B_L\angle -90°)$$
$$= (100V\angle 53.13°)(0.7S\angle -90°) = 70A\angle -36.87°$$
$$I_C = (E\angle\theta)(B_C\angle 90°)$$
$$= (100V\angle 53.13°)(0.3S\angle +90°) = 30A\angle 143.13°$$

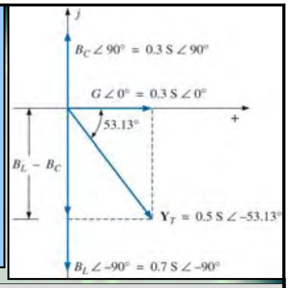


Figure 15.79 Admittance diagram for the parallel R-L-C network in Fig. 15.77.

KCL : At node a,

$$I - I_R - I_L - I_C = 0 \quad or \quad I = I_R + I_L + I_C$$

Power :

$$P_T = EI\cos\theta_T = (100V)(50A)\cos 53.13° = (5000W)(0.6S) = 3000\ W$$

Power factor : The power factor of the circuit is

$$F_p = \cos\theta_T = \cos 53.13° = 0.6\ lagging$$

Current Divider Rule

The basic format for the current divider rule in ac circuit exactly the same as that dc circuits; for two parallel branches with impedance in Fig. 15.82.

$$I_1 = \frac{Z_2 I_T}{Z_1 + Z_2} \quad or \quad I_2 = \frac{Z_1 I_T}{Z_1 + Z_2}$$

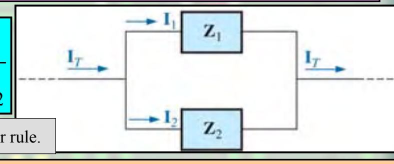


Figure 15.82 Applying the current divider rule.

Ex. 15-16 Using the current divider rule, find the current through each parallel branch in Fig. 15.83.

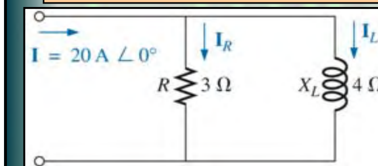


FIGURE 15.83

$$I_R = \frac{Z_L I_T}{Z_R + Z_L} = \frac{(4\Omega\angle 90°)(20A\angle 0°)}{3\Omega\angle 0° + 4\Omega\angle 90°}$$
$$= \frac{80A\angle 90°}{5\angle 53.13°} = 16A\angle 36.87°$$
$$I_L = \frac{Z_R I_T}{Z_R + Z_L} = \frac{(3\Omega\angle 0°)(20A\angle 0°)}{5\angle 53.13°}$$
$$= \frac{60A\angle 0°}{5\angle 53.13°} = 12A\angle -53.13°$$

Ex. 15-17 Using the current divider rule, find the current through each parallel branch in Fig. 15.84.

$$I_{R\text{-}L} = \frac{Z_C I_T}{Z_C + Z_{R\text{-}L}} = \frac{(2\Omega\angle -90°)(5A\angle 30°)}{-j2\Omega + 1\Omega + j8\Omega} = \frac{10A\angle -60°}{1 + j6}$$
$$= \frac{10A\angle -60°}{6.083\angle 80.54°} \cong 1.64A\angle -140.54°$$
$$I_C = \frac{Z_{R\text{-}L} I_T}{Z_{R\text{-}L} + Z_C} = \frac{(1\Omega + j8\Omega 8\Omega)(\angle 30°)}{6.083\angle 80.54°} = \frac{10A\angle -60°}{1 + j6}$$
$$= \frac{(8.06A\angle 82.87°)(5A\angle 30°)}{6.083\angle 80.54°}$$
$$= \frac{40.30A\angle 112.87°}{6.083\angle 80.54°} = 6.63A\angle 32.33°$$

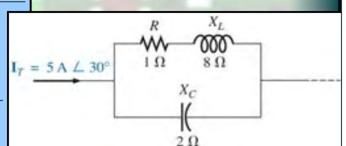


FIGURE 15.84

Frequency Response of Parallel Elements

For parallel elements, it is important to remember that ***the smallest parallel resistor or the smallest parallel reactance will have the most impact on the real or imaginary component, respectively, of the total impedance.***

In Fig. 15.85, the frequency response has been included for each element of a parallel R-L-C combination. At very low frequencies, the importance of the coil will be less than that of the resistor or capacitor, resulting in an inductive network in which the reactance of the inductor will have the most impact on the total impedance. As the frequency increases, the impedance of the inductor will increase while the impedance of the capacitor will decrease.

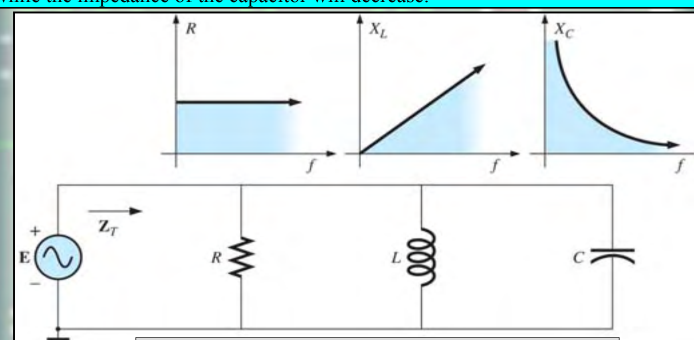


FIGURE 15.85 Frequency response for parallel R-L-C elements.

Let us now note the impact of frequency on the total impedance and inductive current for the parallel R-L network in Fig. 15.86 for a frequency range through 40 kHz.

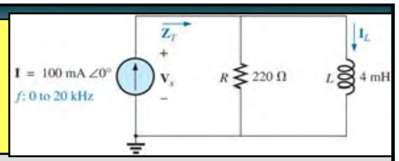
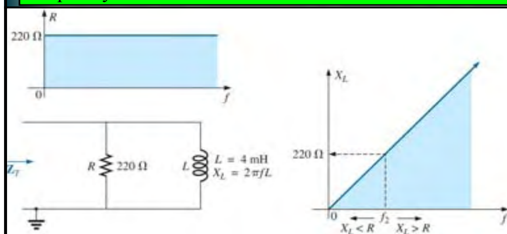


FIGURE 15.86 Determining the frequency response of a parallel R-L network.

Z_T In Fig. 15.87, X_L is very small at low frequencies compared to R, establishing X_L as the predominant factor in this frequency range. As the frequency increases, X_L increases until it equals the impedance of resistor (220 Ω). The frequency at which this situation occurs can be determined in the following manner:



$$X_L = 2\pi f_2 L = R$$

$$\text{and} \quad f_2 = \frac{R}{2\pi L} = \frac{220\Omega}{2\pi(4\times10^{-3}\,H)} \cong 8.75\ kHz$$

FIGURE 15.87 The frequency response of the individual elements of a parallel R-L network.

A general equation for the total impedance in vector form can be developed in the following manner:

$$Z_T = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{(R\angle 0°)(X_L\angle 90°)}{R + jX_L} = \frac{RX_L\angle 90°}{\sqrt{R^2 + X_L^2}\angle \tan^{-1} X_L/R}$$

$$\text{and} \quad Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}}\angle(90° - \tan^{-1} X_L/R)$$

$$Z_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}}$$

$$\text{and} \quad \theta_T = 90° - \tan^{-1} R/X_L = \tan^{-1}\frac{R}{X_L}$$

I_L Applying the current divider rule to the network in Fig. 15.86 results in the following:

$$I_L = \frac{Z_R I}{Z_R + Z_L} = \frac{(R\angle 0°)(I\angle 0°)}{R + jX_L} = \frac{RI\angle 0°}{\sqrt{R^2 + X_L^2}\angle \tan^{-1} X_L/R}$$

$$\text{and} \quad I_L = I_L\angle\theta_L = \frac{RI\angle 0°}{\sqrt{R^2 + X_L^2}}\angle - \tan^{-1} X_L/R$$

The magnitude of I_L is determined

$$I_T = \frac{RX_L}{\sqrt{R^2 + X_L^2}}$$

and the phase angle θ_L, by which I_L leads I is given by

$$\theta_L = -\tan^{-1} X_L/R$$

Equivalent Circuits

In a series circuit, the total impedance of two or more elements in series is often equivalent to an impedance that can be achieved with fewer elements of different values, the elements and their values being determined by frequency applied. This is also true for parallel circuits. For the circuit in Fig. 15.94 (a),

$$Z_T = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(5\Omega\angle -90°)(10\Omega\angle 90°)}{5\Omega\angle -90° + 10\Omega\angle 90°} = \frac{50\angle 0°}{5\angle 90°} = 10\Omega\angle -90°$$

The total impedance at the frequency applied is equivalent to a capacitor with a reactance of 10 Ω, as shown in Fig. 15.94 (b).

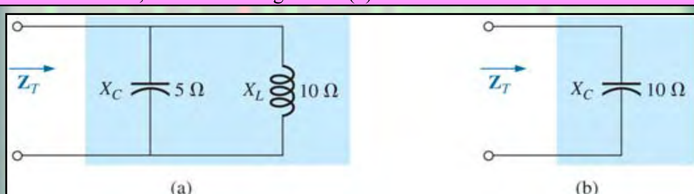


FIGURE 15.94 Defining the equivalence between two networks at a specific frequency.

Another interesting development appears if the impedance of a parallel circuit, such as the one in Fig. 15.95(a), is found in rectangular form. In this case,

$$Z_T = \frac{Z_L Z_R}{Z_L + Z_R} = \frac{(4\Omega\angle 90°)(3\Omega\angle 0°)}{4\Omega\angle 90° + 3\Omega\angle 0°} = \frac{12\angle 90°}{5\angle 53.13°} = 2.40\Omega\angle 36.87° = 1.92\Omega + j1.44\Omega$$

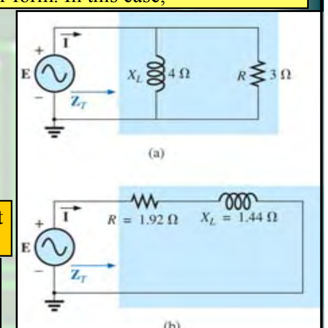


FIGURE 15.95 Finding the series equivalent circuit for a parallel R-L network.

There is an alternative method to find same result by using formulas

$$R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(3\Omega)(4\Omega)^2}{(4\Omega)^2 + (3\Omega)^2} = \frac{48\,\Omega}{25} = 1.92\,\Omega$$

and

$$X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(3\Omega)^2(4\Omega)}{(4\Omega)^2 + (3\Omega)^2} = \frac{36\,\Omega}{25} = 1.44\,\Omega$$

$$R_p = \frac{R_s^2 + X_s^2}{R_s} = \frac{(1.92\Omega)^2 + (1.44\Omega)^2}{1.92\Omega} = \frac{5.76\Omega}{1.92} = 3.0\Omega \quad \text{and} \quad X_p = \frac{R_s^2 + X_s^2}{X_s} = \frac{5.76\Omega}{1.44} = 4.0\Omega$$

Ex. 15-18 Determine the series equivalent circuit for the network in Fig. 15.97.

$$R_p = 8k\Omega$$

$$X_p(resultant) = |X_L - X_C| = |9k\Omega - 4k\Omega| = 5k\Omega$$

$$and \quad R_s = \frac{R_p X_p^2}{X_p^2 + R_p^2} = \frac{(8k\Omega)(5k\Omega)^2}{(5k\Omega)^2 + (8k\Omega)^2} = \frac{200k\Omega}{89} = 2.25k\Omega$$

$$with \quad X_s = \frac{R_p^2 X_p}{X_p^2 + R_p^2} = \frac{(8k\Omega)^2(5k\Omega)}{(5k\Omega)^2 + (8k\Omega)^2} = \frac{320k\Omega}{89} = 3.6k\Omega \ (inductive)$$

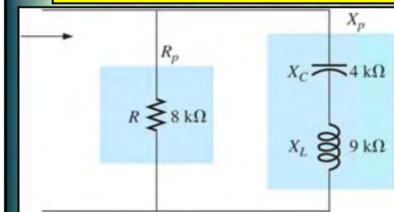


FIGURE 15.97 Example 15.18.

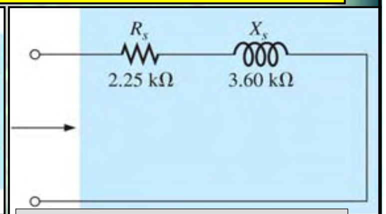


FIGURE 15.98 The equivalent series circuit for the parallel network in Fig. 15.97.

Phase Measurement

Measuring the phase angle between quantities is one of the most important functions that an oscilloscope can perform. Whenever you are using the dual-trace capability of an oscilloscope, the most important thing to remember is that ***both channel of a dual-trace oscilloscope must be connected to the same ground***.

Measuring Z_T and θ_T

For ac parallel networks, the total impedance can be found in the same manner as described for dc circuits: Simply remove the source and place an ohmmeter across the network terminals. However,

For parallel ac networks with reactive elements, the total impedance cannot be measured with an ohmmeter.

The phase angle between the applied voltage and the resulting source current is one of the most important because (a) it is also the phase angle associated with the total impedance; (b) it provides an instant indication of whether the network is resistive or reactive; (c) it reveals whether a network is inductive or capacitive; and (d) it can be used to find the power delivered to the network.

In Fig. 15.104, a resistor has been added to the configuration between the source and the network to permit measuring the current and finding the phase angle between the applied voltage and the source current. In Fig. 15.104, channel 1 is displaying the applied voltage, and channel 2 the voltage across the sensing resistor. Sensitivities for each channel are chosen to establishes the waveforms appearing on the screen in Fig. 15.105.

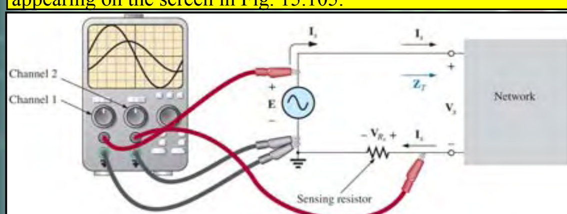


FIGURE 15.104 Using an oscilloscope to measure Z_T and Θ_T.

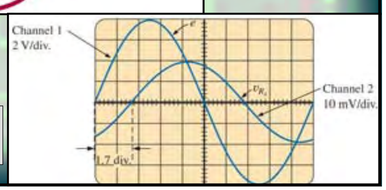


FIGURE 15.105 e and v_R for the configuration in Fig. 15.104.

Using the sensitivities, the peak voltage across the sensing resistor is

E_m = (4div.)(2V/div.) = 8 V

while the peak value of the voltage across the sensing resistor is

$V_{R_s(peak)}$ = (2div.)(10mV/div.) = 20 mV

Using Ohm's law, the peak value of the current is

$$I_T = \frac{V_{R_s(peak)}}{R_s} = \frac{20\ mV}{10\Omega} = 2\ mA$$

The magnitude of the input impedance is then

$$Z_T = \frac{V_x}{I_s} \cong \frac{E}{I_s} = \frac{8\ V}{2\ mA} = 4\ k\Omega$$

For the chosen horizontal sensitivity, each waveform in Fig. 15.105 has a period T defined by ten horizontal divisions, and the phase angle between the two waveforms is 1.7 divisions. Using the fact that each period of a sinusoidal waveform encompasses 360º, the following ratios can be set up to determine the phase angle θ:

$$\frac{10div.}{360°} = \frac{1.7div.}{\theta}$$

$$and \quad \theta = \left(\frac{1.7}{10}\right)360° = 61.2°$$

In general,

$$\theta = \frac{(div.\ for\ \theta)}{(div.\ for\ T)} \times 360°$$

Therefore, the total impedance is

$$Z_T = 4k\Omega\angle 61.2° = 1.93k\Omega + j3.51k\Omega = R + jX_L$$

HW 15-25 Find the total admittance and impedance of the circuits in Fig. 15.142. Identify the values of conductance and susceptance, and draw the admittance diagram.

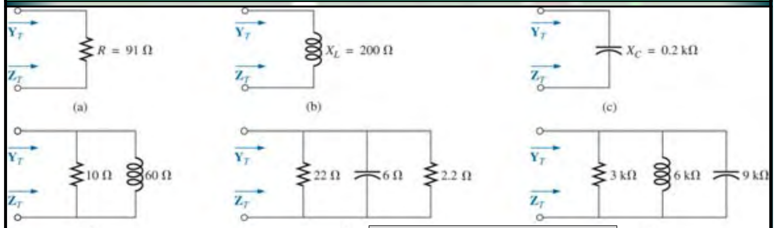


Figure 15.142 Problem 25.

a. $\mathbf{Z}_T = \mathbf{91\ \Omega\ \angle 0°} = R\ \angle 0°$, $\mathbf{Y}_T = \mathbf{10.99\ mS\ \angle 0°} = G\ \angle 0°$

b. $\mathbf{Z}_T = \mathbf{200\ \Omega\ \angle 90°} = X_L\ \angle 90°$, $\mathbf{Y}_T = \mathbf{5\ mS\ \angle{-90°}} = B_L\ \angle{-90°}$

c. $\mathbf{Z}_T = \mathbf{0.2\ k\Omega\ \angle{-90°}} = X_C\ \angle{-90°}$, $\mathbf{Y}_T = \mathbf{5.00\ mS\ \angle 90°} = B_C\ \angle 90°$

d. $\mathbf{Z}_T = \dfrac{(10\ \Omega\angle 0°)(60\ \Omega\ \angle 90°)}{10\ \Omega + j60\ \Omega} = \mathbf{9.86\ \Omega\ \angle 9.46° = 9.73\ \Omega + \mathit{j}1.62\ \Omega} = R + jX_L$

$\mathbf{Y}_T = \mathbf{0.10\ S\ \angle{-9.46°} = 0.1\ S - \mathit{j}0.02\ S} = G - jB_L$

e. $22\ \Omega \parallel 2.2\ \Omega = 2\ \Omega$

$\mathbf{Z}_T = \dfrac{(2\ \Omega\angle 0°)(6\ \Omega\ \angle -90°)}{2\ \Omega - j6\ \Omega} = \dfrac{12\ \Omega\ \angle -90°}{6.32\ \Omega\ \angle -71.57°} = \mathbf{1.90\ \Omega\ \angle{-18.43°}}$

$= \mathbf{1.80\ \Omega - \mathit{j}0.6\ \Omega} = R - jX_C$

$\mathbf{Y}_T = \mathbf{0.53\ S\ \angle 18.43° = 0.5\ S + \mathit{j}0.17\ S} = G + jB_C$

f. $\mathbf{Y}_T = \dfrac{1}{3\ \text{k}\Omega\ \angle 0°} + \dfrac{1}{6\ \text{k}\Omega\ \angle 90°} + \dfrac{1}{9\ \text{k}\Omega\ \angle -90°}$

$= 0.333 \times 10^{-3}\ \angle 0° + 0.167 \times 10^{-3}\ \angle{-90°} + 0.111 \times 10^{-3}\ \angle 90°$

$= \mathbf{0.333 \times 10^{-3}\ S - \mathit{j}0.056 \times 10^{-3}\ S = 0.34\ mS\ \angle{-9.55°}}$

$= G - jB_L$

$\mathbf{Z}_T = \dfrac{1}{\mathbf{Y}_T} = \mathbf{2.94\ k\Omega\ \angle 9.55° = 2.90\ k\Omega + \mathit{j}0.49\ k\Omega}$

Homework 15: 25, 27-32, 33, 39, 40, 47, 48

EET1222/ET242 Circuit Analysis II

Series & Parallel AC Circuits Analysis

Electrical and Telecommunication Engineering Technology

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

Acknowledgement

I want to express my gratitude to Prentice Hall giving me the permission to use instructor's material for developing this module. I would like to thank the Department of Electrical and Telecommunications Engineering Technology of NYCCT for giving me support to commence and complete this module. I hope this module is helpful to enhance our students' academic performance.

Sunghoon Jang

OUTLINES

- **Introduction to Series - Parallel ac Circuits Analysis**
- **Reduction of series parallel Circuits to Series Circuits**
- **Analysis of Ladder Circuits**

Key Words: ac Circuit Analysis, Series Parallel Circuit, Ladder Circuit

Series & Parallel ac Networks - Introduction

In general, when working with series-parallel ac networks, consider the following approach:

1. *Redraw the network, using block impedances to combine obvious series and parallel elements, which will reduce the network to one that clearly reveals the fundamental structure of the system.*
2. *Study the problem and make a brief mental sketch of the overall approach you plan to use. In some cases, a lengthy, drawn-out analysis may not be necessary. A single application of a fundamental law of circuit analysis may result in the desired solution.*
3. *After the overall approach has been determined, it is usually best to consider each branch involved in your method independently before tying them together in series-parallel combinations. In most cases, work back from the obvious series and parallel combinations to the source to determine the total impedance of the network.*
4. *When you have arrived a solution, check to see that it is reasonable by considering the magnitudes of the energy source and the elements in the circuit.*

Ex. 16-1 For the network in Fig. 16.1:

a. Calculate Z_T. b. Determine I_s. c. Calculate V_R and V_C. d. Find I_C.
e. Compute the power delivered. f. Find F_p of the network.

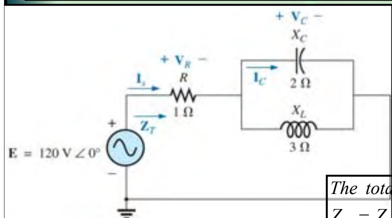


Figure 16.1 Example 16.1.

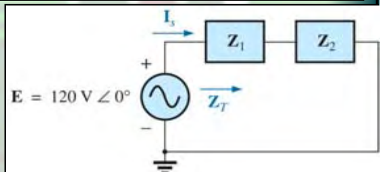


Figure 16.2 Network in Fig. 16.1 after assigning the block impedances.

a. As suggested in the introduction, the network has been redrawn with block impedances, as shown in Fig. 16.2. Impedance Z_1 is simply the resistor R of 1Ω, and Z_2 is the parallel combination of X_C and X_L.

The total impedance is defined by

$Z_T = Z_1 + Z_2$ *with*

$Z_1 = R\angle 0°$

$$Z_2 = Z_C \,//\, Z_L = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-jX_C + jX_L}$$

$$= \frac{(2\Omega\angle -90°)(3\Omega\angle 90°)}{-j2\Omega + j3\Omega}$$

$$= \frac{6\Omega\angle 0°}{j1} = \frac{6\Omega\angle 0°}{1\angle 90°} = 6\Omega\angle -90°$$

and $Z_T = Z_1 + Z_2 = 1\Omega - j6\Omega = 6.08\Omega\angle -80.54°$

$$b.\quad I_s = \frac{E}{Z_T} = \frac{120\angle 0°}{6.08\Omega\angle -80.54°} = 19.74A\angle 80.54°$$

c. Referring to Fig.16.2. we find that V_R *and* V_C *can be found by a direct application of Ohm's law:*

$$V_R = I_s Z_1 = (19.74A\angle 80.54°)(1\Omega\angle 0°) = 19.74V\angle 80.54°$$

$$V_C = I_s Z_2 = (19.74A\angle 80.54°)(6\Omega\angle -90°) = 118.44V\angle -9.46°$$

d. Now that V_C *is known, the current* I_C *can be also found using Ohm's law:*

$$I_C = \frac{V_C}{Z_C} = \frac{118.44V\angle -9.46°}{2\Omega\angle -90°} = 59.22A\angle 80.54°$$

$$e.\quad P_{del} = I_s^2 R = (19.74A)^2(1\Omega) = 389.67\ W$$

$$f.\quad F_p = \cos\theta = \cos 80.54° = 0.164\ \text{leading}$$

Ex. 16-2 For the network in Fig. 16.3:

a. If I is 50A ∠ 30°, calculate I_1 using the current divider rule.
b. Repeat part (a) for I_2.
c. Verify Kirchhoff's current law at one node.

a. Redrawing the circuit as in Fig. 16.4, we have

$Z_1 = R + jX_L = 3\Omega + j4\Omega = 5\Omega\angle 53.13°$

$Z_2 = -jX_C = -j8\Omega = 8\Omega\angle -90°$

Using the current divider rule yields

$$I = \frac{Z_2 I}{Z_2 + Z_1} = \frac{(8\Omega\angle -90°)(50A\angle 30°)}{(-j8\Omega + (3\Omega + j4\Omega))} = \frac{400\angle -60°}{3 - j4}$$

$$= \frac{400\angle -60°}{5\angle -53.13°} = 80A\angle -6.87°$$

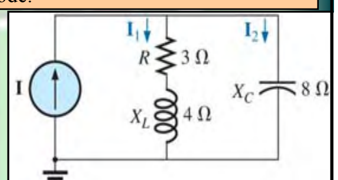


Figure 16.3 Example 16.2.

$$b.\quad I_2 = \frac{Z_1 I}{Z_2 + Z_1} = \frac{(5\Omega\angle 53.13°)(50A\angle 30°)}{5\Omega\angle -53.13°}$$

$$= \frac{250\angle 83.13°}{5\angle -53.13°} = 50A\angle 136.26°$$

$c.\quad I = I_1 + I_2$

$50A\angle 30° = 80A\angle -6.87° + 50A\angle 136.26°$

$= (79.43 - j9.57) + (-36.12 + j34.57)$

$= 43.31 + j25.0$

$50A\angle 30° = 50A\angle 30°$ (checks)

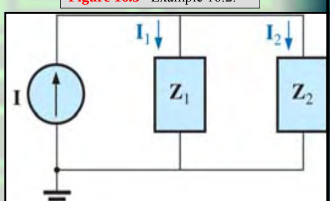


Figure 16.4 Network in Fig. 16.3 after assigning the block impedances.

Ex. 16-3 For the network in Fig. 16.5:

a. Calculate the voltage V_C using the voltage divider rule.
b. Calculate the current I_s.

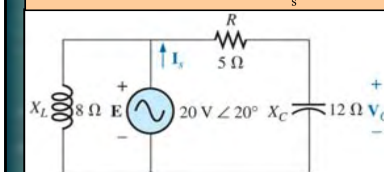


Figure 16.5 Example 16.3.

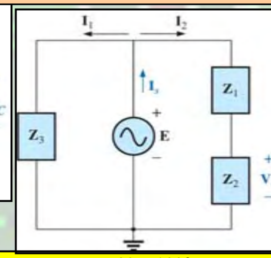


Figure 16.6 Network in Fig. 16.5 after assigning the block impedances.

a. The network is redrawn as shown in Fig.16.6, with

$Z_1 = 5\Omega = 5\Omega\angle 0°$

$Z_2 = -j12\Omega = 12\Omega\angle -90°$

$Z_3 = +j8\Omega = 8\Omega\angle 90°$

$$V_C = \frac{Z_2 E}{Z_1 + Z_2} = \frac{(12\Omega\angle -90°)(20V\angle 20°)}{5\Omega - j12\Omega}$$

$$= \frac{240V\angle -70°}{13\angle -67.38°} = 18.46V\angle -2.62°$$

$$b.\quad I_1 = \frac{E}{Z_3} = \frac{20V\angle 20°}{8\Omega\angle -53.13°} = 2.5A\angle -70°$$

$$I_2 = \frac{E}{Z_1 + Z_2} = \frac{20V\angle 20°}{13\Omega\angle -67.38°} = 1.54A\angle 87.38°$$

and

$I_s = I_1 + I_2 = 2.5A\angle -70° + 1.54A\angle 87.38°$

$= (0.86 - j2.35) + (0.07 + j1.54)$

$I_s = 0.93 - j0.81 = 1.23A\angle -41.05°$

Ex. 16-4 For Fig. 16.7:

a. Calculate the current I_s.

b. Find the voltage V_{ab}.

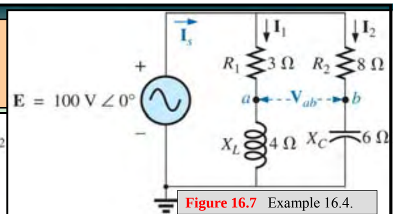


Figure 16.7 Example 16.4.

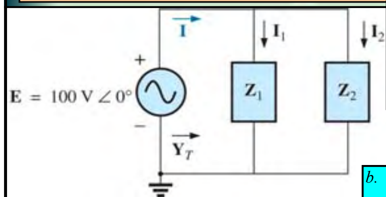


Figure 16.8 Network in Fig. 16.7 after assigning the block impedances.

a. Redrawing the circuit as in Fig.16.8, we obtai

$$Z_1 = R_1 + jX_L = 3\Omega + j4\Omega = 5\Omega\angle 53.13°$$

$$Z_2 = R_2 - jX_C = 8\Omega - j6\Omega = 10\Omega\angle -36.87°$$

$$Z_T = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(5\Omega\angle 53.13°)(10\Omega\angle -36.87°)}{(3\Omega + j4\Omega) + (8\Omega - j6\Omega)}$$

$$= \frac{50\Omega\angle 16.26°}{11 - j2} = \frac{50\Omega\angle 16.26°}{11.18\angle -10.30°} = 4.472\,\Omega\angle 26.56°$$

$$\text{and} \quad I_s = \frac{E}{Z_T} = \frac{100V\angle 0°}{4.472\,\Omega\angle 26.56°} = 22.36A\angle -26.56°$$

b. By Ohm's law,

$$I_1 = \frac{E}{Z_1} = \frac{100V\angle 0°}{5\Omega\angle 53.13°} = 20A\angle -53.13°$$

$$I_2 = \frac{E}{Z_2} = \frac{100V\angle 0°}{10\Omega\angle -36.87°} = 10A\angle 36.87°$$

$$V_{R_1} = I_1 Z_{R_1} = (20A\angle -53.13°)(3\Omega\angle 0°) = 60V\angle -53.13°$$

$$V_{R_2} = I_1 Z_{R_2} = (10A\angle +36.87°)(8\Omega\angle 0°) = 80V\angle +36.87°$$

$$V_{ab} + V_{R_1} - V_{R_2} = 0$$

$$\text{or} \quad V_{ab} = V_{R_2} - V_{R_1} = 80V\angle 36.87° - 60V\angle -53.13°$$

$$= (64 + j48) - (36 - j48) = 28 + j96 = 100V\angle 73.74°$$

Ex. 16-6 For the network in Fig. 16.12:

a. Determine the current I.

b. Find the voltage V.

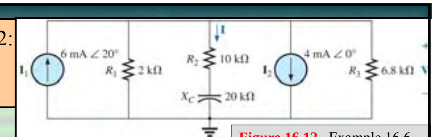


Figure 16.12 Example 16.6.

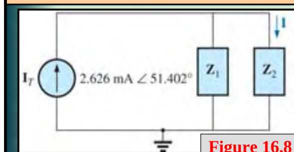


Figure 16.8

a. The equivalent current source is their sum or difference (as phasors).

$$I_T = 6mA\angle 20° - 4mA\angle 0° = 5.638mA + j2.052mA - 4mA$$

$$= 1.638mA + j2.052mA = 2.626mA\angle 51.402°$$

Redrawing the circuit as in Fig.16.13, we obtain

$$Z_1 = 2k\Omega\angle 0° // 6.8k\Omega\angle 0°$$

$$= 1.545k\Omega\angle 0°$$

$$Z_2 = 10k\Omega - j20k\Omega$$

$$= 22.361k\Omega\angle -63.435°$$

Current divider rule :

$$I = \frac{Z_1 I_T}{Z_1 + Z_2} = \frac{(1.545k\,\Omega\angle 0°)(2.626mA\ \angle 51.402°)}{1.545k\,\Omega + 10k\,\Omega - j20k\,\Omega}$$

$$= \frac{4.057A\ \angle 51.402°}{11.545\times 10^3 - j20\times 10^3} = \frac{4.057A\ \angle 51.402°}{23.093\times 10^3\angle -60.004°}$$

$$= 0.18mA\ \angle 111.41°$$

$$\text{b.} \quad V = IZ_2 = (0.176mA\ \angle 111.406°)(22.36k\ \Omega\angle -63.435°)$$

$$= 3.94V\ \angle 47.97°$$

Ex. 16-7 For the network in Fig. 16.14:

a. Compute I. b. Find I_1, I_2, and I_3 c. Verify KCL by showing that $I = I_1 + I_2 + I_3$. d. Find total impedance of the circuit;.

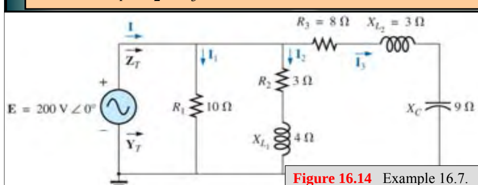


Figure 16.14 Example 16.7.

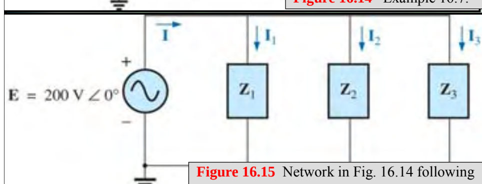


Figure 16.15 Network in Fig. 16.14 following the assignment of the subscripted impedances.

a. *Redrawing the circuit as in Fig.16.15 reveals a strictly parallel network where*

$$Z_1 = R_1 = 10\Omega\angle 0°$$

$$Z_2 = R_2 + jX_{L_1} = 3\Omega + j4\Omega$$

$$Z_2 = R_3 + jX_{L_2} - jX_C$$

$$= 8\Omega + j3\Omega - j9\Omega$$

$$= 8\Omega - j6\Omega$$

The total admittance is

$$Y_T = Y_1 + Y_2 + Y_3 = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}$$

$$= \frac{1}{10\Omega} + \frac{1}{3\Omega + j4\Omega} + \frac{1}{8\Omega - j6\Omega}$$

$$= 0.1S + \frac{1}{5\Omega\angle 53.13°} + \frac{1}{10\Omega\angle -36.87°}$$

$$= 0.1S + 0.2S\angle -53.13° + 0.1S\angle 36.87°$$

$$= 0.1S + 0.12S - j0.16S + 0.08S + j0.06S$$

$$= 0.3S - j0.1S = 0.316S\angle -18.43°$$

The current I :

$$I = EY_T = (200V\angle 0°)(0.326\angle -18.435°)$$

$$= 63.2A\angle -18.435°$$

b. *Since the voltage is the same across parallel branches,*

$$I_1 = \frac{E}{Z_1} = \frac{200V\angle 0°}{10\Omega\angle 0°} = 20A\angle 0°$$

$$I_2 = \frac{E}{Z_2} = \frac{200V\angle 0°}{5\Omega\angle 53.13°} = 40A\angle -53.13°$$

$$I_3 = \frac{E}{Z_3} = \frac{200V\angle 0°}{10\Omega\angle -36.87°} = 20A\angle +36.87°$$

$$\text{c.} \quad I = I_1 + I_2 + I_3$$

$$60 - j20 = 20\angle 0° + 40\angle -53.13° + 20\angle +36.87°$$

$$= (20 + j0) + (24 - j32) + (16 + j12)$$

$$60 - j20 = 60 - j20 \quad (checks)$$

$$\text{d.} \quad Z_T = \frac{1}{Y_T} = \frac{1}{0.316S\angle -18.435°} = 3.17\Omega\angle 18.44°$$

Ex. 16-8 For the network in Fig. 16.18: a. Calculate the total impedance Z_T. b. Compute I. c. Find the total power factor. d. Calculate I_1 and I_2. e. Find the average power delivered to the circuit.

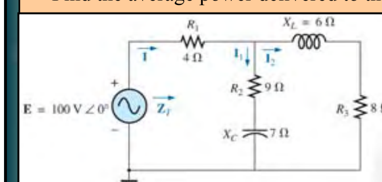


Figure 16.18 Example 16.8.

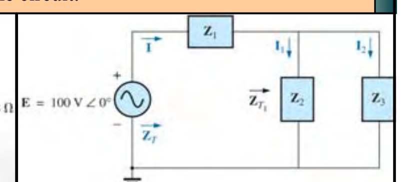


Figure 16.19 Network in Fig. 16.14 following the assignment of the subscripted impedances.

a. *Redrawing the circuit as in Fig.16.19, we have*

$$Z_1 = R_1 = 4\Omega\angle 0°$$

$$Z_2 = R_2 - jX_C = 9\Omega - j7\Omega = 11.40\,\Omega\angle -37.87°$$

$$Z_3 = R_3 + jX_L = 8\Omega + j6\Omega = 10\Omega\angle +36.87°$$

Notice that all the desired quantities were conserved in the redrawn network. The total impedance:

$$Z_T = Z_1 + Z_2 = Z_1 + \frac{Z_2 Z_3}{Z_2 + Z_3}$$
$$= 4\Omega + \frac{(11.4\Omega\angle -37.87^\circ)(10\Omega\angle 36.87^\circ)}{(9\Omega - j7\Omega) + (8\Omega + j6\Omega)}$$
$$= 4\Omega + \frac{114\Omega\angle -1.00^\circ}{17.03\angle -3.37^\circ} = 4\Omega + 6.69\Omega\angle 2.37^\circ$$
$$= 4\Omega + 6.68\Omega + j0.28\Omega = 10.68\Omega + j0.28\Omega$$
$$Z_T = 10.68\Omega\angle 1.5^\circ$$

b. $I = \frac{E}{Z_T} = \frac{100V\angle 0^\circ}{10.684\Omega\angle 1.5^\circ} = 9.36A\angle -1.5^\circ$

c. $F_p = \cos\theta_T = \frac{R}{Z_T} = \frac{10.68\Omega}{10.684^\circ}$

d. *current divider rule:*

$$I_2 = \frac{Z_2 I}{Z_2 + Z_3} = \frac{(11.40\Omega\angle -37.87^\circ)(9.36A\angle -1.5^\circ)}{(9\Omega - j7\Omega) + (8\Omega + j6\Omega)}$$
$$= \frac{106.7A\angle -39.37^\circ}{17 - j1} = \frac{106.7A\angle -39.37^\circ}{17.03\angle -3.37^\circ}$$
$$= 6.27A\angle -36^\circ$$

Applying KCL yields

$$I_1 = I - I_2$$
$$= (9.36A\angle -1.5^\circ) - (6.27A\angle -36^\circ)$$
$$= (9.36A - j0.25) - (5.07 - j3.69A)$$
$$= 4.29A + j3.44 = 5.5A\angle 38.72^\circ$$

e. $P_T = EI\cos\theta_T$

$$= (100V)(9.36A)\cos 1.5^\circ$$
$$= (936)(0.99966)$$
$$= 935.68\,W$$

Ladder Networks

Ladder networks were discussed in some detail in Chapter 7. This section will simply apply the first method described in Section 7.6 to the general sinusoidal ac ladder network in Fig. 16.22. The current I_6 is desired.

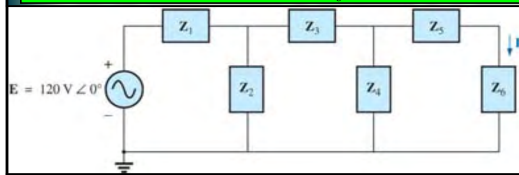


Figure 16.22 Ladder network.

Independance Z_T, $Z_T^{'}$, and $Z_T^{''}$ and currents I_1 and I_3 are defined in Fig. 16.23.

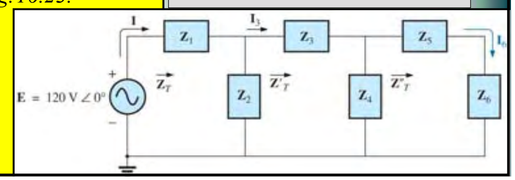
$$Z_T^{''} = Z_5 + Z_6$$

and $Z_T^{'} = Z_3 + Z_4 // Z_T^{''}$

with $Z_T = Z_1 + Z_2 // Z_T^{'}$

Then $I = E / Z_T$

Figure 16.23 Defining an approach to the analysis of ladder networks.



HW 16-13 Find the average power delivered to R_4 in Fig. 16.51.

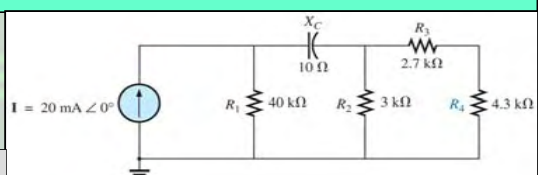


Figure 16.51 Problem 13.

$R_3 + R_4 = 2.7\text{ k}\Omega + 4.3\text{ k}\Omega = 7\text{ k}\Omega$

$R' = 3\text{ k}\Omega \parallel 7\text{ k}\Omega = 2.1\text{ k}\Omega$

$Z' = 2.1\text{ k}\Omega - j10\ \Omega$

(CDR) $\quad \mathbf{I}'(\text{of } 10\ \Omega\text{ cap.}) = \frac{(40\text{ k}\Omega\angle 0^\circ)(20\text{ mA}\angle 0^\circ)}{40\text{ k}\Omega + 2.1\text{ k}\Omega - j10\ \Omega}$

$= 19\text{ mA}\angle{+0.014^\circ}$ as expected since $R_1 \gg \mathbf{Z}'$

(CDR) $\quad \mathbf{I}_4 = \frac{(3\text{ k}\Omega\angle 0^\circ)(19\text{ mA}\angle 0.014^\circ)}{3\text{ k}\Omega + 7\text{ k}\Omega} = \frac{57\text{ mA}\angle 0.014^\circ}{10}$

$= 5.7\text{ mA}\angle 0.014^\circ$

$P = I^2R = (5.7\text{ mA})^2\,4.3\text{ k}\Omega = \mathbf{139.71\ mW}$

Homework 16: 1-8, 10, 12-14

EET1222/ET242 Circuit Analysis II

Selected Network Theorems for AC Circuits

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on “Introduction to Circuit Analysis” by Robert Boylestad, Prentice Hall, 11th edition.

OUTLINES

Key Words: Dependent Source, Source Conversion, Mesh Analysis, Nodal Analysis

Methods of Analysis and Selected Topics (AC)

For the net works with two or more sources that are not in series or parallel, the methods described the methods previously described can not be applied. Rather, methods such as **mesh analysis** or **nodal analysis** to ac circuits must be used.

Independent Versus Dependent (Controlled) Sources

In the previous modules, each source appearing in the analysis of dc or ac networks was an **independent source**, such as E and I (or **E** and **I**) in Fig. 17.1.

The term independent specifies that the magnitude of the source is independent of the network to which it is applied and that the source display its terminal characteristics even if completely isolated.

A dependent or controlled source is one whose magnitude is determined (or controlled) by a current or voltage of the system in which it appears.

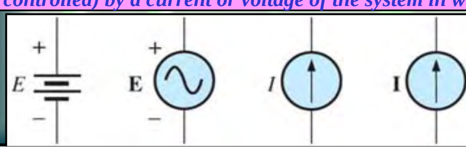


Figure 17.1 Independent sources.

Currently two symbols are used for controlled sources. One simply uses the independent symbol with an indication of the controlling element, as shown in Fig. 17.2(a). In Fig. 17.2(a), the magnitude and phase of the voltage are controlled by a voltage **V** elsewhere in the system, with the magnitude further controlled by the constant k_1. In Fig. 17.2(b), the magnitude and phase of the current source are controlled by a current **I** elsewhere in the system, with the magnitude with further controlled by k_2. To distinguish between the dependent and independent sources, the notation in Fig. 17.3 was introduced. Possible combinations for controlled sources are indicated in Fig. 17.4. Note that the magnitude of current sources or voltage sources can be controlled by voltage and a current.

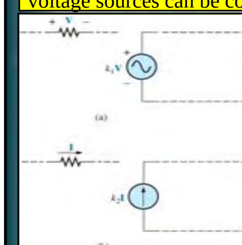


Figure 17.2 Controlled or dependent sources.

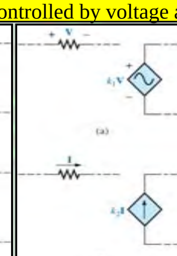


Figure 17.3 Special notation for controlled or dependent sources.

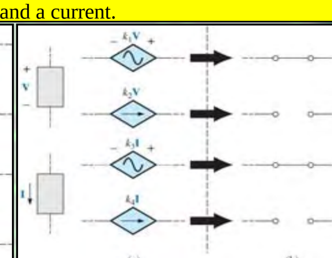


Figure 17.4 Conditions of **V** = 0V and **I** = 0A for a controlled source.

Source Conversions

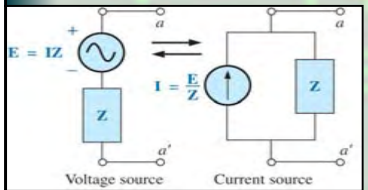


Figure 17.5 Source Conversion.

When applying the methods to be discussed, it may be necessary to convert a current source to a voltage source, or a voltage source to a current source. This source conversion can be accomplished in much the same manner as for dc circuits, except dealing with phasors and impedances instead of just real numbers and resistors.

Independent Sources In general, the format for converting one type of independent source to another is as shown in Fig. 17.5.

Ex. 17-1 Convert the voltage source in Fig. 17.6(a) to a current source.

$$I = \frac{E}{Z} = \frac{100V\angle 0°}{5\Omega\angle 53.13°} = 20A\angle -53.13° \quad [Fig.17.6(b)]$$

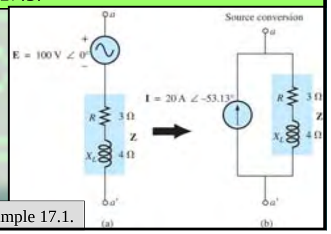


Figure 17.6 Example 17.1.

Ex. 17-2 Convert the current source in Fig. 17.7(a) to a voltage source.

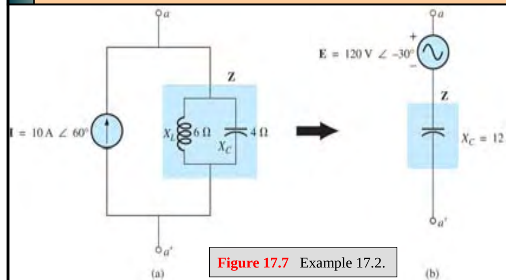


Figure 17.7 Example 17.2.

$$Z = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{(X_C\angle -90°)(X_L\angle 90°)}{-jX_C + jX_L} = \frac{(4\Omega\angle -90°)(6\Omega\angle 90°)}{-j4\Omega + j6\Omega} = \frac{24\Omega\angle 0°}{2\angle 90°} = 12\Omega\angle -90°$$

$$E = IZ = (10A\angle 60°)(12\Omega\angle -90°) = 120V\angle -30° \quad [Fig.17.7(b)]$$

Dependent Sources For the dependent sources, direct conversion in Fig. 17.5 can be applied if the controlling variable (**V** or **I** in Fig.17.4) is not determined by a portion of the network to which the conversion is to be applied. For example, in Figs. 17.8 and 17.9, V and I, respectively, are controlled by an external portion of the network.

Ex. 17-3 Convert the voltage source in Fig. 17.7(a) to a current source.

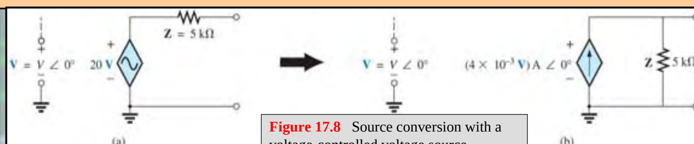
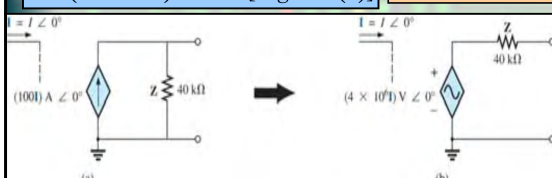


Figure 17.8 Source conversion with a voltage-controlled voltage source.

$$I = \frac{E}{Z} = \frac{(20V)V\angle 0°}{5k\Omega\angle 0°} = (4\times 10^{-3}V)A\angle 0° \quad [Fig.17.8(b)]$$

Ex. 17-4 Convert the current source in Fig. 17.9(a) to a voltage source.



$$E = IZ = (100I)A\angle 0°)(40k\Omega\angle 0°) = (4\times 10^{6}I)V\angle 0° \quad [Fig.17.9(b)]$$

Figure 17.9 Source conversion with a current-controlled current source.

Mesh Analysis

Independent Voltage Sources The general approach to mesh analysis for independent sources includes the same sequence of steps appearing in previous module. In fact, throughout this section the only change from the dc coverage is to substitute impedance for resistance and admittance for conductance in the general procedure.

1. ***Assign a distinct current in the clockwise direction to each independent closed loop of the network.***
2. ***Indicate the polarities within each loop for each impedance as determined by the assumed direction of loop current for that loop.***
3. ***Apply KVL around each closed loop in the clockwise direction. Again, the clockwise direction was chosen to establish uniformity and to prepare us for the formed approach to follow.***
   - a. ***If an impedance has two or more assumed currents through it, the total current through the impedance is the assumed current of the loop in which KVL law is being applied.***
   - b. ***The polarity of a voltage source is unaffected by the direction of the assigned loop currents.***
4. ***Solve the resulting simultaneous linear equations for the assumed loop currents.***

The technique is applied as above for all networks with independent sources or for networks with *dependent sources where the controlling variable is not a part of the network under investigation.*

Ex. 17-5 Using the general approach to mesh analysis, find the current I_1 in Fig.17.10.

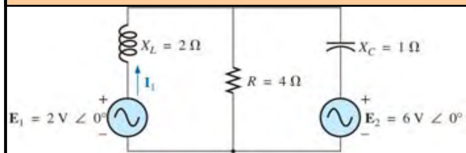


Figure 17.10 Example 17.5.

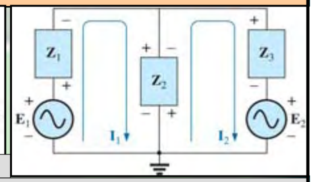


Figure 17.11 Assigning the mesh currents and subscripted impedances for the network in Fig.17.10.

The network is redrawn in Fig.17.11 with subscripted impedances:

$$Z_1 = +jX_L = +j2\Omega \quad E_1 = 2V\angle 0°$$
$$Z_2 = R = 4\Omega \quad E_2 = 6V\angle 0°$$
$$Z_3 = -jX_C = -j1\Omega$$

Steps 1 and 2 are as indicated in Fig.17.11.

Step 3:

$$+E_1 - I_1Z_1 - Z_2(I_1 - I_2) = 0$$
$$-Z_2(I_2 - I_1) - I_2Z_3 - E_2 = 0$$

or $\quad E_1 - I_1Z_1 - I_1Z_2 + I_2Z_2 = 0$

$$-I_2Z_2 + I_1Z_2 - I_2Z_3 - E_2 = 0$$

so that $\quad I_1(Z_1 + Z_2) - I_2Z_2 = E_1$

$$-I_1Z_2 + I_2(Z_2 + Z_3) = -E_2$$

which are rewritten as

$$I_1(Z_1 + Z_2) - I_2Z_2 = E_1$$
$$-I_1Z_2 + I_2(Z_2 + Z_3) = -E_2$$

Step 4:

Using determinan ts, we obtain

$$I_1 = \frac{\begin{matrix} E_1 & -Z_2 \\ -E_2 & Z_2 + Z_3 \end{matrix}}{\begin{matrix} Z_1 + Z_3 & -Z_2 \\ -Z_2 & Z_2 + Z_3 \end{matrix}} = \frac{E_1(Z_2 + Z_3) - E_2(Z_2)}{(Z_1 + Z_3)(Z_2 + Z_3) - (Z_2)^2} = \frac{(E_1 - E_2)Z_2 + E_1Z_3}{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}$$

Substituti ng numerical values yields,

$$I_1 = \frac{(2V - 6V)(4\Omega) + (2V)(-j1\Omega)}{(+j2\Omega)(4\Omega) + (+j2\Omega)(-j2\Omega) + (4\Omega)(-j2\Omega)}$$
$$= \frac{-16 - j2}{j8 - j^2 - j4} = \frac{-16 - j2}{2 + j4} = \frac{16.12A\angle -172.87°}{4.47\angle 63.43°}$$
$$= 3.61A\angle -236.30° \quad or \quad 3.61A\angle 123.70°$$

Dependent Voltage Sources For dependent voltage sources, the procedure is modified as follow:

1. ***Step 1 and 2 are the same as those applied for independent sources.***
2. ***Step 3 is modifies as follows: Treat each dependent source like an independent when KVL is applied to each independent loop. However, once the equation is written, substitute the equation for the controlling quantity to ensure that the unknowns are limited solely to the chosen mesh currents.***
3. ***Step 4 is as before.***

Ex. 17-6 Write the mesh currents for the network in Fig. 17.12 having a dependent voltage source.

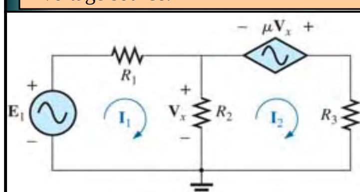


Figure 17.12 Applying mesh analysis to a network with a voltage-controlled voltage source.

Steps 1 *and* 2 *are defined in Fig.*17.12.

Step 3: $\quad E_1 - I_1R_1 + R_2(I_1 - I_2) = 0$

$$R_2(I_2 - I_1) + \mu V_x - I_2R_3 = 0$$

Then substituting $V_x = (I_1 - I_2)R_2$

The result is two equations and two unknowns.

$$E_1 - I_1R_1 - R_2(I - I_2) = 0$$
$$R_2(I_2 - I_1) + \mu R_2(I_1 - I_2) - I_2R_3 = 0$$

Independent Current Sources For independent current sources, the procedure is modified as follow:

1. ***Step 1 and 2 are the same as those applied for independent sources.***
2. ***Step 3 is modifies as follows: Treat each current source as an open circuit and write the mesh equations for each remaining independent path. Then relate the chosen mesh currents to the dependent sources to ensure that the unknowns of the final equations are limited to the mesh currents.***
3. ***Step 4 is as before.***

Ex. 17-7 Write the mesh currents for the network in Fig. 17.13 having an independent current source.

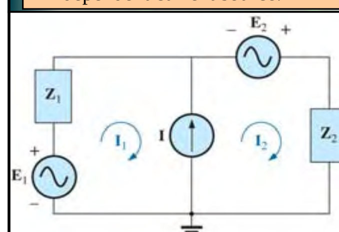


Figure 17.13 Applying mesh analysis to a network with an independent current source.

Steps 1 *and* 2 *are defined in Fig.*17.13.

Step 3: $\quad E_1 - I_1Z_1 + E_2 - I_2Z_2 = 0$

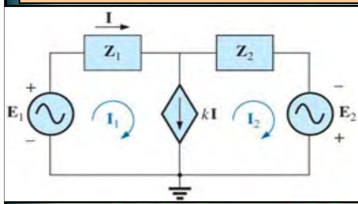
with $\quad I_1 + I = I_2$

The result is two equations and two unknowns.

Dependent Current Sources For dependent current sources, the procedure is modified as follow:

1. ***Step 1 and 2 are the same as those applied for independent sources.***
2. ***Step 3 is modifies as follows: The procedure is essentially the same as that applied for dependent current sources, except now the dependent sources have to be defined in terms of the chosen mesh currents to ensure that the final equations have only mesh currents as the unknown quantities.***
3. ***Step 4 is as before.***

Ex. 17-8 Write the mesh currents for the network in Fig. 17.14 having an dependent current source.



Steps 1 *and* 2 *are defined in Fig*.17.14.

Step 3: $E_1 - I_1Z_1 + I_2Z_2 - I_2Z_2 = 0$

and $kI = I_1 - I_2$

Now $I = I_1$ *so that* $kI_1 = I_1 - I_2$ *or* $I_2 = I_1(1-k)$

The result is two equations and two unknowns.

Figure 17.14 Applying mesh analysis to a network with an current-controlled current source.

Nodal Analysis

Independent Sources Before examining the application of the method to ac networks, a review of the appropriate sections on nodal analysis of dc circuits is suggested since the content of this section is limited to the general conclusions. The fundamental steps are the following:

1. *Determine the number of nodes within the network.*
2. *Pick a reference node and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Applying KCL at each node except the reference.*
4. *Solve the resulting equations for the nodal voltages.*

Ex. 17-12 Determine the voltage across the inductor for the network in Fig. 17.23.

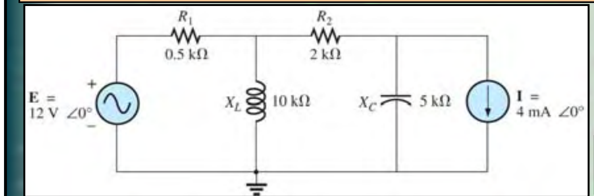
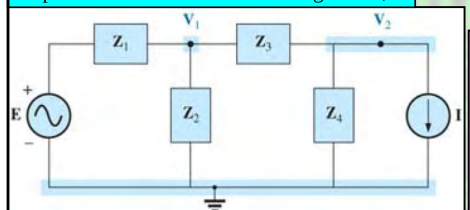


FIGURE 17.23 Example 17.12.

Steps 1 and 2 are as indicated in Fig. 17.24,



Step 3: *Note Fig*.17.25 *for the application of KCL to node* V_1:

$$\sum I_i = \sum I_o$$

$$0 = I_1 + I_2 + I_3$$

$$\frac{V_1 - E}{Z_1} + \frac{V_1}{Z_2} + \frac{V_1 - V_2}{Z_3} = 0$$

Figure 17.24 Assigning the nodal voltages and subscripted impedances to the network in Fig. 17.23.

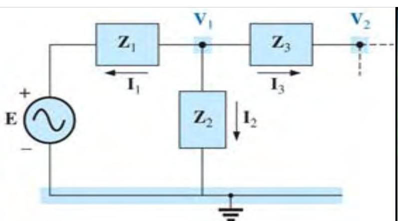


Figure 17.25 Applying KCL to the nodes V_1 in Fig. 17.24.

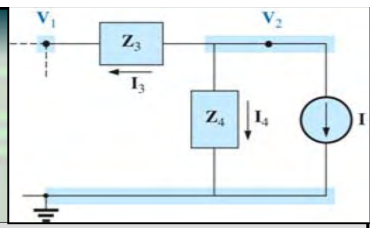


Figure 17.26 Applying KCL to the nodes V_2 in Fig. 17.24.

Rearranging terms :

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E_1}{Z_1}$$

Note Fig.17.26 *for the application of KCL to the node* V_2.

$$0 = I_3 + I_4 + I$$

$$\frac{V_2 - V_1}{Z_3} + \frac{V_2}{Z_4} + I = 0$$

Regarding terms :

$$V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] - V_1\left[\frac{1}{Z_3}\right] = -I$$

Grouping equations :

$$V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3}\right] - V_2\left[\frac{1}{Z_3}\right] = \frac{E}{Z_1}$$

$$V_1\left[\frac{1}{Z_3}\right] \quad - V_2\left[\frac{1}{Z_3} + \frac{1}{Z_4}\right] \quad = I$$

$$\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{0.5k\Omega} + \frac{1}{j10k\Omega} + \frac{1}{2k\Omega} = 2.5mS\angle -2.29°$$

$$\frac{1}{Z_3} + \frac{1}{Z_4} = \frac{1}{2k\Omega} + \frac{1}{-j5k\Omega} = 0.539mS\angle 21.80°$$

and

$$V_1[2.5mS\angle -2.29°] - V_2[0.5mS\angle 0°] = 24\,mA\angle 21.80°$$

$$V_1[0.5mS\angle 0°] - V_2[0.539\,mS\angle 21.80°] = 4mS\angle 0°$$

with

$$V_1 = \frac{\begin{vmatrix} 24\,mA\angle 0° & -0.5mS\angle 0° \\ 4mA\angle 0° & -0.539\,mS\angle 21.80° \end{vmatrix}}{\begin{vmatrix} 2.5mS\angle -2.29° & -0.5mS\angle 0° \\ 0.5mS\angle 0° & -0.539\,mS\angle 21.80° \end{vmatrix}}$$

$$= \frac{(24\,mA\angle 0°)(-0.539\,mS\angle 21.80°) + (0.5mS\angle 0°)(4mA\angle 0°)}{(2.5mS\angle -2.29°)(-0.539\,mS\angle 21.80°) + (0.5mS\angle 0°)(0.5mS\angle 0°)}$$

$$= \frac{-12.94\times 10^{-6}V\angle 21.80° + 2\times 10^{-6}V\angle 0°}{-1.348\times 10^{-6}\angle 19.51° + 0.25\times 10^{-6}\angle 0°}$$

$$= \frac{-(12.01 + j4.81)\times 10^{-6}V + 2\times 10^{-6}V}{-(1.271 + j0.45)\times 10^{-6} + 0.25\times 10^{-6}}$$

$$= \frac{-10.01V - j4.81V}{-1.021 - j0.45} = \frac{11.106\,V\angle -154.33°}{1.116\angle -156.21°}$$

$$V_1 = 9.95V\angle 1.88°$$

Dependent Current Sources

For dependent current sources, the procedure is modified as follow:

1. *Step 1 and 2 are the same as those applied for independent sources.*
2. *Step 3 is modifies as follows: Treat each dependent current source like an independent source when KCL is applied to each defined node.*
3. *Step 4 is as before.*

Ex. 17-13 Write the nodal equations for the network in Fig. 17.28 having a dependent current source.

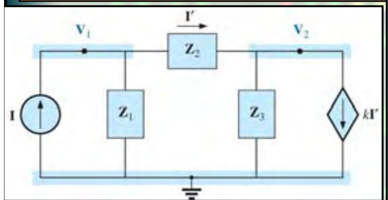


Figure 17.28 Applying nodal analysis to a network with a current-controlled current source.

Step 1 and 2 are as defined in Fig .17 .28 .

Step 3 : At node V_1 , $I = I_1 + I_2$

$$\frac{V_1}{Z_1} + \frac{V_1 - V_2}{Z_2} - I = 0$$

$$and \quad V_1\left[\frac{1}{Z_1} + \frac{1}{Z_2}\right] - V_2\left[\frac{1}{Z_2}\right] = I$$

At node V_2, $I_2 + I_3 + kI = 0$

$$\frac{V_2 - V_1}{Z_2} + \frac{V_2}{Z_3} + k\left[\frac{V_1 - V_2}{Z_2}\right] = 0$$

$$and \quad V_1\left[\frac{1-k}{Z_2}\right] - V_2\left[\frac{1-k}{Z_2} + \frac{1}{Z_3}\right] = 0$$

Ex. 17-14 Write the nodal equations for the network in Fig. 17.29 having an independent source between two assigned nodes.

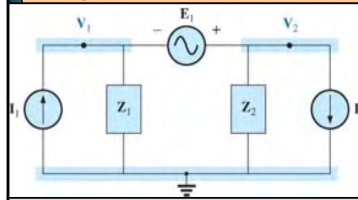


Figure 17.29 Applying nodal analysis to a network with an independent voltage source between defined nodes.

Steps 1 and 2 are as indicated in Fig. 17.29.

Step 3: Replacing the independent source E_1 with a short-circuit equivalent results in a super-node that generates the following equation when KCL is applied to node V_1:

$$I_1 = \frac{V_1}{Z_1} + \frac{V_2}{Z_2} + I_2$$

$$with \quad V_2 - V_1 = E_1$$

and we have two equations and two unknowns.

Dependent Voltage Sources between Defined Nodes

For dependent voltage sources between defined nodes, the procedure is modified as follow:

1. *Step 1 and 2 are the same as those applied for independent voltage sources.*
2. *Step 3 is modifies as follows: The procedure is essentially the same as that applied for independent voltage sources, except now the dependent sources have to be defined in terms of the chosen voltages to ensure that the final equations have only nodal voltages as their unknown quantities.*
3. *Step 4 is as before.*

Ex. 17-15 Write the nodal equations for the network in Fig. 17.30 having an dependent voltage source between two defined nodes.

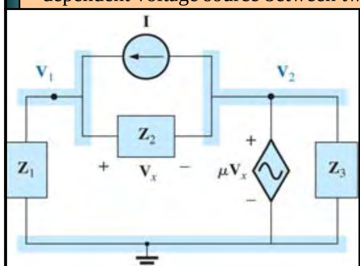


Figure 17.30 Applying nodal analysis to a network with a voltage-controlled voltage source.

Steps 1 and 2 are as indicated in Fig. 17.30.

Step 3: Replacing the dependent source μV_x with a short-circuit equivalent results in the following equation when KCL is applied to at node V_1:

$$I = I_1 + I_2$$

$$\frac{V_1}{Z_1} + \frac{(V_1 - V_2)}{Z_2} - I = 0$$

$$and \quad V_2 = \mu V_x = \mu|V_1 - V_2|$$

$$or \quad V_2 = \frac{\mu}{1+\mu} V_1$$

resulting in two equations and two unknown. Note that because the impedance Z_3 is in parallel with a voltage source, it does not appear in the analysis. It will, however, affect the current through the dependent voltage source.

HW 17-10 An electrical system is rated 10 kVA, 200V at a leading power factor.

a. Determine the impedance of the system in rectangular coordinates.
b. Find the average power delivered to the system.

Homework 17: 2-4, 5, 6, 14, 15

EET1222/ET242 Circuit Analysis II

Network Theorems (AC)

Electrical and Telecommunications
Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

OUTLINES

- Introduction to Network Theorems (AC)
- Thevenin Theorem
- Superposition Theorem
- Maximum Power Transfer Theorem

Key Words: Network Theorem, Thevinin, Superposition, Maximum Power

Network Theorems (AC) - Introduction

This module will deal with network theorems of ac circuit rather than dc circuits previously discussed. Due to the need for developing confidence in the application of the various theorems to networks with controlled (dependent) sources include independent sources and dependent sources. Theorems to be considered in detail include the superposition theorem, Thevinin's theorem, maximum power transform theorem.

Superposition Theorem

The **superposition theorem** eliminated the need for solving simultaneous linear equations by considering the effects of each source independently in previous module with dc circuits. To consider the effects of each source, we had to remove the remaining sources . This was accomplished by setting ***voltage sources to zero (short-circuit representation)*** and ***current sources to zero (open-circuit representation).*** The current through, or voltage across, a portion of the network produced by each source was then added algebraically to find the total solution for the current or voltage.

The only variation in applying this method to ac networks with independent sources is that we are now working with impedances and phasors instead of just resistors and real numbers.

Independent Sources

Ex. 18-1 Using the superposition theorem, find the current **I** through the 4Ω resistance (X_{L_2}) in Fig. 18.1.

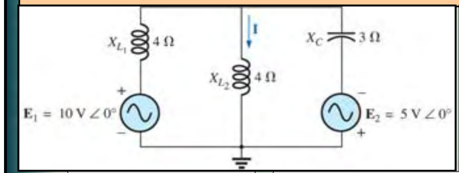


Figure 18.1 Example 18.1.

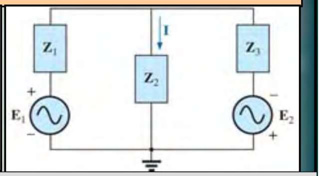


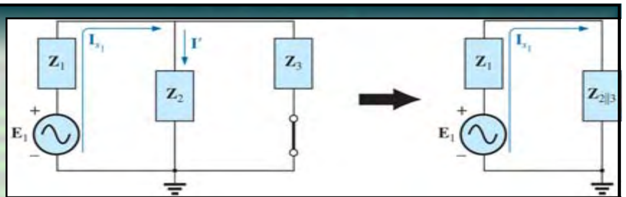
Figure 18.2 Assigning the subscripted impedances to the network in Fig.18.1.

For the redrawn circuit (Fig.18.2),

$$Z_1 = +jX_{L_1} = j4\Omega$$
$$Z_2 = +jX_{L_2} = j4\Omega$$
$$Z_3 = -jX_C = -j3\Omega$$

Considering the effects of the voltage source E_1 (Fig.18.3), we have

$$Z_{2//3} = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(j4\Omega)(-j3\Omega)}{j4\Omega - j3\Omega}$$
$$= \frac{12\Omega}{j} = -j12\Omega = 12\Omega\angle -90°$$
$$I_{s_1} = \frac{E_1}{Z_{2//3} + Z_1} = \frac{10V\angle 0°}{-j12\Omega + j4\Omega}$$
$$= \frac{10V\angle 0°}{-j12\Omega + j4\Omega} = \frac{10V\angle 0°}{8\Omega\angle -90°} = 1.25A\angle 90°$$



and $I' = \dfrac{Z_3 I_{s_1}}{Z_2 + Z_3}$ (current divider rule)

$$= \frac{(-j3\Omega)(j1.25A)}{j4\Omega - j3\Omega} = \frac{3.75A}{j1} = 3.75A\angle -90^\circ$$

Figure 18.3 Determining the effect of the voltage source $\mathbf{E}_1$ on the current **I** of the network in Fig. 18.1.

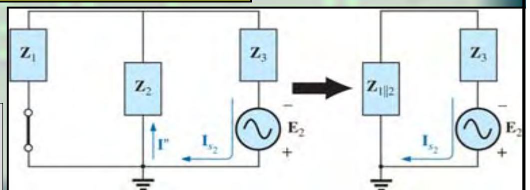


Figure 18.4 Determining the effect of the voltage source $\mathbf{E}_2$ on the current **I** of the network in Fig. 18.1.

Considering the effects of the voltage source E_2 (Fig.18.4), we have

$$Z_{1//2} = \frac{Z_1}{N} = \frac{j4\Omega}{2} = j2\Omega$$

$$I_{s_2} = \frac{E_2}{Z_{1//2} + Z_3} = \frac{5V\angle 0^\circ}{j2\Omega - j3\Omega} = \frac{5V\angle 0^\circ}{1\Omega\angle -90^\circ} = 5A\angle 90^\circ$$

and $I'' = \dfrac{I_{s_2}}{2} = 2.5A\angle 90^\circ$

The resultant current through the 4Ω reactance X_{L_2} (Fig.18.5) is

$$I = I' - I'' = 3.75A\angle -90^\circ$$
$$= -j3.75A - j2.50A$$
$$= -j6.25A = 6.25A\angle -90^\circ$$

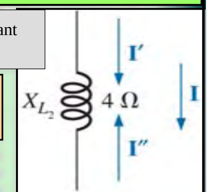


Figure 18.5 Determining the resultant current for the network in Fig. 18.1.

Ex. 18-2 Using the superposition, find the current **I** through the 6Ω resistor in Fig.18.6.

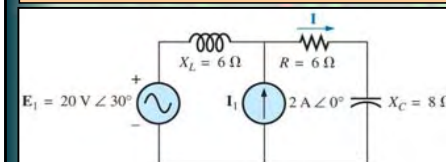


Figure 18.6 Example 18.2.

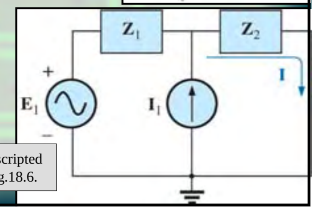
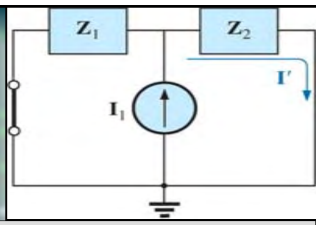
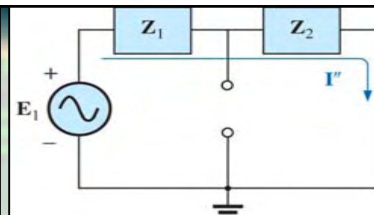


Figure 18.7 Assigning the subscripted impedances to the network in Fig.18.6.





For the redrawn circuit (Fig.18.7),

$Z_1 = j6\Omega \qquad Z_2 = 6\Omega - j8\Omega$

Conseder the effects of the voltage source (Fig.18.8). Applying the current divider rule, we have

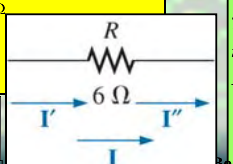
$$I' = \frac{Z_1 I_1}{Z_1 + Z_2} = \frac{(6\Omega)(2A)}{j6\Omega + 6\Omega - j8\Omega}$$
$$= \frac{j12A}{6 - j2} = \frac{12A\angle 90^\circ}{6.32\angle -18.43^\circ}$$
$$= 1.9A\angle 48.43^\circ$$

Conseder the effects of the voltage source (Fig.18.9). Applying Ohm's law gives us

$$I'' = \frac{E_1}{Z_T} = \frac{E_1}{Z_1 + Z_2} = \frac{20V\angle 30^\circ}{6.32\Omega\angle -18.43^\circ}$$
$$= 3.16A\angle 48.43^\circ$$

The total current through the 6Ω resistor (Fig.18.10) is

$$I = I' + I''$$
$$= 1.9A\angle 108.43^\circ + 3.16A\angle 48.43^\circ$$
$$= (-0.60A + j1.80A) + (2.10A + j2.36A)$$
$$= 1.50A + j4.16A = 4.42A\angle 70.2^\circ$$



Ex. 18-3 Using the superposition, find the voltage across the 6Ω resistor in Fig.18.6. Check the results against $V_{6\Omega} = I(6\Omega)$, where I is the current found through the 6Ω resistor in Example 18.2.

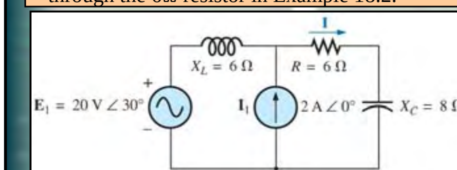


Figure 18.6

For the current source,

$$V'_{6\Omega} = I'(6\Omega)$$
$$= (1.9A\angle 108.43^\circ)(6\Omega)$$
$$= 11.4V\angle 108.43^\circ$$

For the voltage source,

$$V''_{6\Omega} = I''(6\Omega)$$
$$= (3.16A\angle 48.43^\circ)(6\Omega)$$
$$= 18.96V\angle 48.43^\circ$$

For the total voltage the 6Ω resistor (Fig.18.11) is

$$V_{6\Omega} = V'(6\Omega) + V''(6\Omega)$$
$$= 11.4V\angle 108.43^\circ + 18.96V\angle 48.43^\circ$$
$$= (-3.60V + j10.82V) + (12.58V + j14.18V)$$
$$= 8.98V + j25.0V = 26.5V\angle 70.2^\circ$$

Check the result, we have

$$V_{6\Omega} = I(6\Omega) = (4.42A\angle 70.2^\circ)(6\Omega)$$
$$= 26.5V\angle 70.2^\circ \quad (checks)$$

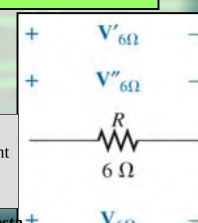


Figure 18.11 Determining the resultant voltage $V_{6\Omega}$ for the network in Fig. 18.6.

Dependent Sources

For dependent sources in which ***the controlling variable is not determined by the network to which the superposition is to be applied***, the application of the theorem is basically the same as for independent sources.

Ex. 18-5 Using the superposition, determine the current $\mathbf{I}_2$ for the network in Fig.18.18. The quantities μ and h are constants.

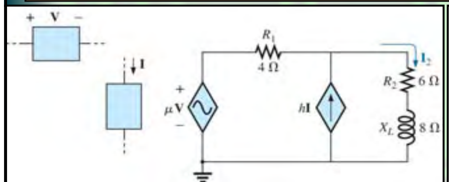


Figure 18.18 Example 18.5.

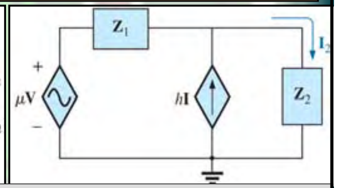


Figure 18.19 Assigning the subscripted impedances to the network in Fig.18.18.

With a portion of the system (*Fig*.18.19),

$Z_1 = R_1 = 4\Omega \qquad Z_2 = R_2 + jX_L = 6\Omega + j8\Omega$

For the voltage source (*Fig*.18.20),

$$I' = \frac{\mu V}{Z_1 + Z_2} = \frac{\mu V}{4\Omega + 6\Omega + j8\Omega} = \frac{\mu V}{10\Omega + j8\Omega} = \frac{\mu V}{12.8\Omega\angle 38.66^\circ} = 0.078\,\mu V / \Omega\angle -38.66^\circ$$

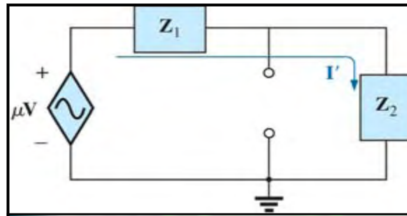


Figure 18.20 Determining the effect of the voltage-controlled voltage source on the current $\mathbf{I}_2$ for the network in Fig.18.18.

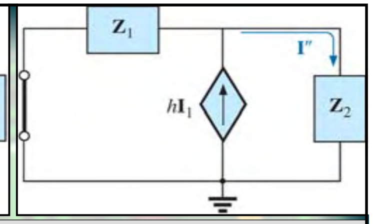


Figure 18.21 Determining the effect of the current-controlled current source on the current $\mathbf{I}_2$ for the network in Fig.18.18.

For the current source (*Fig*.18.21),

$$I'' = \frac{Z_1(hI)}{Z_1 + Z_2} = \frac{(4\Omega)(hI)}{12.8\Omega\angle 38.66^\circ} = 4(0.078)hI\angle -38.66^\circ = 0.312hI\angle -38.66^\circ$$

For the current I_2 *is*

$I_2 = I' + I'' = 0.078\,\mu V/\Omega\angle -38.66^\circ + 0.312\,hI\angle -38.66^\circ$

For $V = 10V\angle 0^\circ,\ \mu = 20,$ *and* $h = 100,$

$I_2 = 0.078(20)(10V\angle 0^\circ)/\Omega\angle -38.66^\circ + 0.312(100)(20mA\angle 0^\circ)\angle -38.66^\circ$

$= 15.60\,A\angle -38.66^\circ + 0.62\,A\angle -38.66^\circ = 16.22\,A\angle -38.66^\circ$

Thevenin's Theorem

Thevenin's theorem, as stated for sinusoidal ac circuits, is changed only to include the term impedance instead of resistance, that is,

any two-terminal linear ac network can be replaced with an equivalent circuit consisting of a voltage source and an importance in series, as shown in Fig. 18.23.

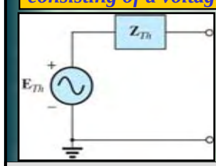


Figure 18.23 Thevenin equivalent circuit for ac networks.

Since the reactances of a circuit are frequency dependent, the Thevinin circuit found for a particular network is applicable only at one frequency. The steps required to apply this method to dc circuits are repeated here with changes for sinusoidal ac circuits. As before, the only change is the replacement of the term resistance with impedance. Again, dependent and independent sources are treated separately.

Independent Sources

1. ***Remove that portion of the network across which the Thevenin equivalent circuit is to be found.***
2. ***Mark (o, •, and so on) the terminal of the remaining two-terminal network.***
3. ***Calculate Z_{TH} by first setting all voltage and current sources to zero (short circuit and open circuit, respectively) and then finding the resulting impedance between the marked terminals.***
4. ***Calculate E_{TH} by first replacing the voltage and current sources and then finding the open-circu voltage between the marked terminals.***
5. ***Draw the Thevenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the Thevinin equivalent circuit.***

Ex. 18-7 Find the Thevenin equivalent circuit for the network external to resistor R in Fig. 18.24.

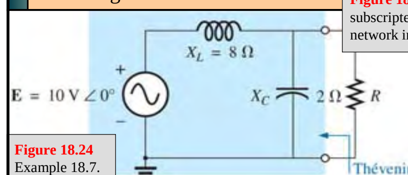


Figure 18.24 Example 18.7.

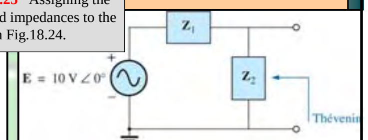


Figure 18.25 Assigning the subscripted impedances to the network in Fig.18.24.

Steps 1 *and* 2 (*Fig*.18.25):

$Z_1 = jX_L = j8\Omega \qquad Z_2 = -jX_L = -j2\Omega$

Step 3 (*Fig*.18.26):

$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(j8\Omega)(-j2\Omega)}{j8\Omega - j2\Omega}$$

$$= \frac{-j^2 16\Omega}{j6\Omega} = \frac{16\Omega}{6\angle 90^\circ} = 2.67\Omega\angle -90^\circ$$

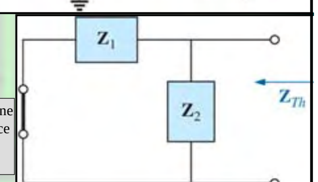


Figure 18.26 Determine the Thevenin impedance for the network in Fig.18.24.

Step 4 (*Fig*.18.27):

$$E_{Th} = \frac{Z_2 E}{Z_1 + Z_2} \quad (voltage\ divider\ rule)$$

$$= \frac{(-j2\Omega)(10V)}{j8\Omega - j2\Omega} = \frac{-j20V}{j6} = 3.33V\angle -180^\circ$$

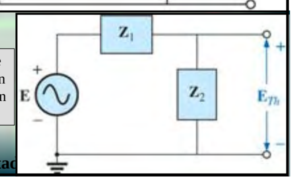


Figure 18.27 Determine the open-circuit Thevenin voltage for the network in Fig.18.24.

Step 5: The Thevenin equivalent circuit is shown in Fig. 18.28.

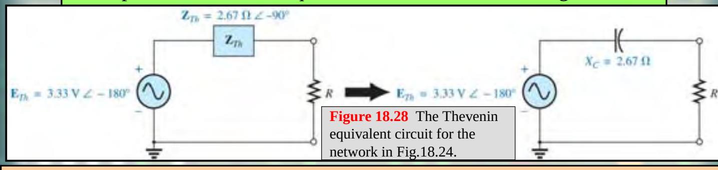


Figure 18.28 The Thevenin equivalent circuit for the network in Fig.18.24.

Ex. 18-8 Find the Thevenin equivalent circuit for the network external to resistor to branch a-a′ in Fig. 18.24.

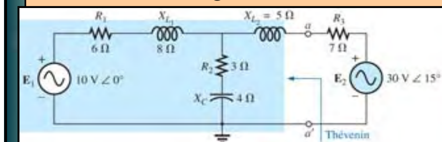


Figure 18.29 Example 18.8.

Steps 1 *and* 2 (*Fig*.18.30) : *Note the reduced complexity with subscripted impedances* :

$$Z_1 = R_1 + jX_{L_1} = 6\Omega + j8\Omega$$
$$Z_2 = R_2 - jX_C = 3\Omega - j4\Omega$$
$$Z_3 = +jX_{L_2} = j5\Omega$$

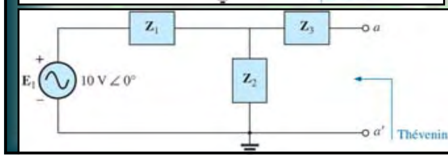


Figure 18.30 Assigning the subscripted impedances for the network in Fig.18.29.

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Step 3 (*Fig*.18.31) :

$$Z_{Th} = Z_3 + \frac{Z_1 Z_2}{Z_1 + Z_2}$$
$$= j5\Omega + \frac{(10\Omega\angle 53.13°)(5\Omega\angle -53.13°)}{(6\Omega + j8\Omega) + (3\Omega - j4\Omega)}$$
$$= j5 + \frac{50\angle 0°}{9 + j4} = j5 + \frac{50\angle 0°}{9.85\angle 23.96°}$$
$$= j5 + 5.08\angle -23.96° = j5 + 4.64 - j2.06$$
$$= 4.64\Omega + j2.94\Omega = 5.49\Omega\angle 32.36°$$

Figure 18.26 Determine the Thevenin impedance for the network in Fig.18.29.

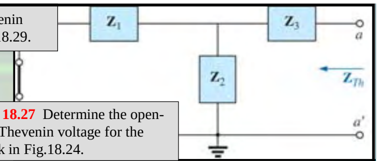
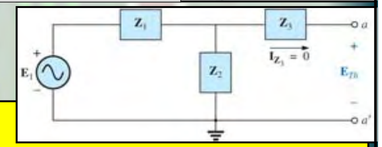


Figure 18.27 Determine the open-circuit Thevenin voltage for the network in Fig.18.24.



Step 4 (*Fig*.18.32) : *Since a – a′ is an open circuit*, $I_{Z_3} = 0$. *Then* E_{Th} *is the voltage drop across* Z_2 :

$$E_{Th} = \frac{Z_2 E}{Z_2 + Z_1} \quad (\text{voltage divider rule}) = \frac{(5\Omega\angle -53.13°)(10V\angle 0°)}{9.85\Omega\angle 23.96°} = \frac{50V\angle -53.13°}{9.85\angle 23.96°} = 5.08V\angle -77.09°$$

Step 5: The Thevenin equivalent circuit is shown in Fig. 18.33.

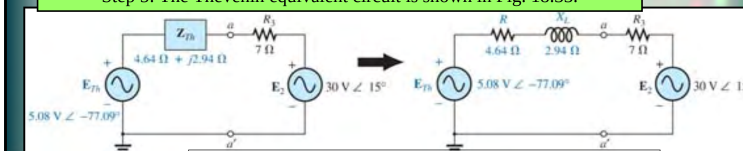


 Figure 18.33 The Thevenin equivalent circuit for the network in Fig.18.29.

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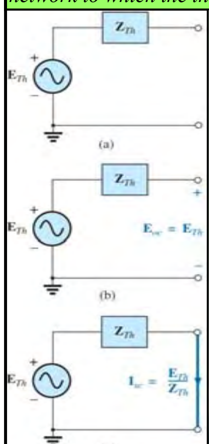
Dependent Sources

For dependent sources with a *controlling variable not in the network under investigation*, the procedure indicated above can be applied. However, for dependent sources of the other type, where the *controlling variable is part of the network to which the theorem is to be applied*, another approach must be used.



The new approach to Thevenin's theorem can best be introduced at this stage in the development by considering the Thevenin equivalent circuit in Fig. 18.39(a). As indicated in fig. 18.39(b), the open-circuit terminal voltage (**E_{oc}**) of the Thevenin equivalent circuit is the Thevenin equivalent voltage; that is

$$E_{oc} = E_{Th}$$

If the external terminals are short circuited as in Fig. 18.39(c), the resulting short-circuit current is determined by

$$I_{sc} = \frac{E_{Th}}{Z_{Th}}$$

or, rearranged,

$$Z_{Th} = \frac{E_{Th}}{I_{sc}}$$

and

$$Z_{Th} = \frac{E_{oc}}{I_{sc}}$$

Figure 18.39 Defining an alternative approach for determining the Thevenin impedance.

Ex. 18-11 Determine the Thevenin equivalent circuit for the network in Fig. 18.24.

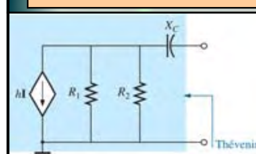


Figure 18.47 Example 18.11.

From Fig.18.47, E_{Th} *is*

$$E_{Th} = E_{oc} = -hI\,(R_1 // R_2) = -\frac{hR_1R_2I}{R_1 + R_2}$$

Method 1: *See Fig*.18.48. $\quad Z_{Th} = R_1 // R_2 - jX_C$

Method 2: *See Fig*.18.49. $\quad I_{sc} = \frac{-(R_1 // R_2)hI}{(R_1 // R_2) - jX_C}$

and $\quad Z_{Th} = \frac{E_{oc}}{I_{sc}} = \frac{-hI\,(R_1 // R_2)}{\frac{-(R_1 // R_2)hI}{(R_1 // R_2) - jX_C}} = R_1 // R_2 - jX_C$

Method 3: *See Fig*.18.50. $\quad I_g = \frac{E_g}{(R_1 // R_2) - jX_C}$

and $\quad Z_{Th} = \frac{E_g}{I_g} = R_1 // R_2 - jX_C$

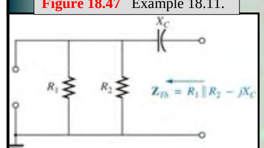


Figure 18.48 Determine the Thevenin impedance for the network in Fig.18.47.

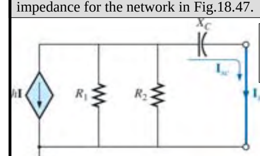


Figure 18.49 Determine the short-circuit current for the network in Fig.18.47.

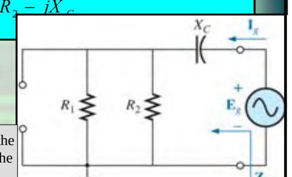


Figure 18.50 Determining the Thevenin impedance using the approach $Z_{Th} = E_g/I_g$.

Maximum Power Transfer Theorem

When applied to ac circuits, the maximum power transfer theorem states that

maximum power will be delivered to a load when the load impedance is the conjugate of the Thevenin impedance across its terminals.

That is, for Fig. 18.81, for maximum power transfer to the load,

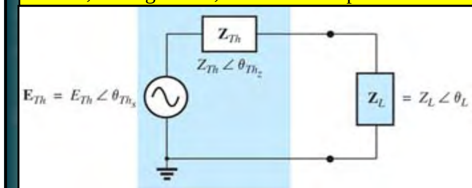


Figure 18.81 Defining the conditions for maximum power transfer to a load.

$$Z_L = Z_{Th} \quad and \quad \theta_L = -\theta_{Th_Z}$$

or, in rectangular form,

$$R_L = R_{Th} \quad and \quad \pm jX_{load} = \mp jX_{Th}$$

The conditions just mentioned will make the total impedance of the circuit appear purely resistive, as indicated in Fig.18.82 :

$$Z_T = (R \pm jX) + (R \mp jX)$$

$$and \quad Z_T = 2R$$

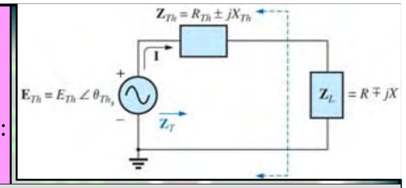


Figure 18.82 Conditions for maximum power transfer to Z_L.

Since the circuit is purely resistive, the power factor of the circuit under maximum power conditions is 1 : that is,

$$F_p = 1 \quad (maximum \ \ power \ \ transfer)$$

The magnitude of the current I in Fig.18.82 is

$$I = \frac{E_{Th}}{Z_T} = \frac{E_{Th}}{2R}$$

The maximum power to the load is

$$P_{max} = I^2 R = \left(\frac{E_{Th}}{2R}\right)^2 R$$

$$and \quad P_{max} = \frac{E_{Th}^2}{4R}$$

Ex. 18-19 Find the load impedance in Fig. 18.83 for maximum power to the load, and find the maximum power.

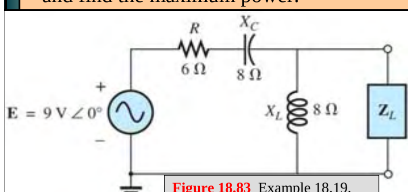


Figure 18.83 Example 18.19.

Determine Z_{Th} *[Fig.18.84(a)] :*

$$Z_1 = R - jX_C = 6\Omega - j8\Omega = 10\Omega\angle -53.13°$$

$$Z_2 = +jX_L = j8\Omega$$

$$Z_{Th} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(10\Omega\angle -53.13°)(8\Omega\angle 90°)}{6\Omega - j8\Omega + j8\Omega}$$

$$= \frac{80\Omega\angle 36.87°}{6\angle 0°} = 13.33\Omega\angle 36.87° = 10.66\Omega + j8\Omega$$

$$and \quad Z_L = 13.33\Omega\angle -36.87° = 10.66\Omega - j8\Omega$$

To find the maximum power, we must find

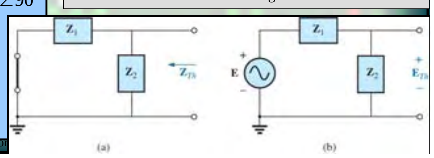
$$E_{Th} = \frac{Z_2 E}{Z_2 + Z_1} \quad (voltage \ \ divider \ \ rule)$$

$$= \frac{(8\Omega\angle 90°)(9V\angle 0°)}{j8\Omega + 6\Omega - j8\Omega} = \frac{72V\angle 90°}{6\angle 0°} = 12V\angle 90°$$

$$Then \quad P_{max} = \frac{E_{Th}^2}{4R} = \frac{(12V)^2}{4(10.66\Omega)}$$

$$= \frac{144}{42.64} = 3.38\,W$$

Figure 18.84 Determining (a) Z_{Th} and (b) E_{Th} for the network external to the load in Fig. 18.83.



HW 18-6 Using superposition, determine the current $\mathbf{I}_L$ *(h = 100)* for the network in Fig. 18.112.

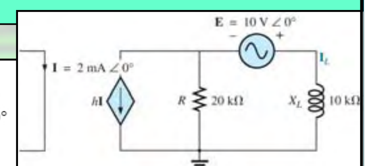
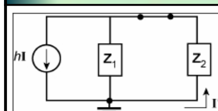
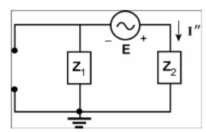


Figure 18.112 Problems 6 and 20.



$\mathbf{Z}_1 = 20\text{ k}\Omega \angle 0°$
$\mathbf{Z}_2 = 10\text{ k}\Omega \angle 90°$
$\mathbf{I} = 2\text{ mA} \angle 0°$
$\mathbf{E} = 10\text{ V} \angle 0°$

$$\mathbf{I}' = \frac{\mathbf{Z}_1(h\mathbf{I})}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{(20\text{ k}\Omega\angle 0°)(100)(2\text{ mA}\angle 0°)}{20\text{ k}\Omega + j10\text{ k}\Omega} = 0.179\text{ A}\angle -26.57°$$



$$\mathbf{I}'' = \frac{\mathbf{E}}{\mathbf{Z}_1 + \mathbf{Z}_2} = \frac{10\text{ V}\angle 0°}{22.36\text{ k}\Omega\angle 26.57°}$$

$= 0.447\text{ mA}\angle -26.57°$

$\mathbf{I}_L = \mathbf{I}' - \mathbf{I}''$ (direction of $\mathbf{I}'$)

$= 179\text{ mA}\angle -26.57° - 0.447\text{ mA}\angle -26.57°$

$= \mathbf{178.55\text{ mA}\angle -26.57°}$

Homework 18: 1, 2, 6, 12, 13, 19, 39

EET1222/ET242 Circuit Analysis II

Power (AC)

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

Acknowledgement

I want to express my gratitude to Prentice Hall giving me the permission to use instructor's material for developing this module. I would like to thank the Department of Electrical and Telecommunications Engineering Technology of NYCCT for giving me support to commence and complete this module. I hope this module is helpful to enhance our students' academic performance.

Sunghoon Jang

OUTLINES

- Introduction to Power (AC)
- Apparent Power
- Power Factor Correction
- Inductive Circuit and Reactive Power
- Power Meter
- Effective Resistance

Key Words: Power, Apparent Power, Power Factor, Power Meter, Effective Resistance

Power (AC) - Introduction

The discussion of power in earlier module of response of basic elements included only the average or real power delivered to an ac network. We now examine the total power equation in a slightly different form and introduce two additional types of power: **apparent** and **reactive**.

General Equation

For any system as in Fig. 19.1, the power delivered to a load at any instant is defined by the product of the applied voltage and the resulting current; that is,

$$p = vi$$

In this case, since v and i are sinusoidal quantities, let us establish a general case where $v = V_m \sin(\omega t + \theta)$

and $i = I_m \sin \omega t$

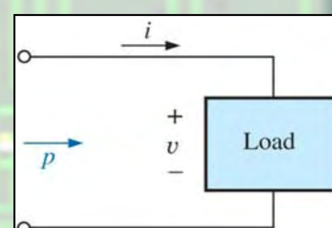


Figure 19.1 Defining the power delivered to a load.

The chosen v and i include all possibilities because, if the load is purely resistive, θ = 0°. If the load is purely inductive or capacitive, θ = 90° or θ = – 90°, respectively.

Substituting the above equations for v and I into the power equation results in

$$p = V_m I_m \sin\omega t \sin(\omega t + \theta)$$

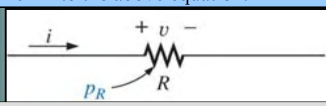
If we now apply a number of trigonometric identities, the following form for the power equation results in:

$$p = VI\cos\theta(1 - \cos 2\omega t) + VI\sin\theta(\sin 2\omega t)$$

where V and I are rms values.

Resistive Circuit

For a purely resistive circuit (such as that in Fig. 19.2), v and i are in phase, and θ = 0°, as appearing in Fig. 19.3. Substituting θ = 0° into the above equation.



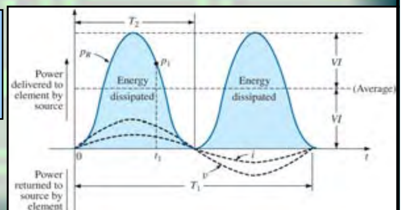


Figure 19.2 Determining the power delivered to a purely resistive load

$$p_R = VI\cos(0°)(1 - \cos 2\omega t) + VI\sin(0°)\sin 2\omega t$$
$$= VI(1 - \cos 2\omega t) + 0$$
$$\text{or} \quad p_R = VI - VI\cos 2\omega t$$

where VI is the average or dc term and $-VI\cos 2\omega t$ is a negative cosine wave with twice the frequency of either input quantity and a peak value of VI.

Note that T_1 = period of input quantities

T_2 = period of power curve p_R

Consider also that since the peak and average values of the power curve are the same, the curve is always above the horizontal axis. This indicates that

the total power delivered to a resistor will be dissipated in the form of heat.

The average (real) power is VI; or, as a summary,

$$p = VI = \frac{V_m I_m}{2} = I^2 R = \frac{V^2}{R} \quad (watts, W)$$

The energy dissipated by the resistor (WR) over one full cycle of the applied voltage is the area under the power curve in Fig. 19.3. It can be found using the following equation: $W = pt$

where p is the average value and t is the period of the applied voltage, that is

$$W_R = VIT_1 \quad \text{or} \quad W_R = VI/f_1 \quad \text{(joules, J)}$$

Ex. 19-1 For the resistive circuit in Fig. 19.4,

a. Find the instantaneous power delivered to the resistor at times t_1 through t_6.

b. Plot the results of part (a) for one full period of the applied voltage.

c. Find the average value of the curve of part (b) and compare the level to that determined by Eq. (19.3).

d. Find the energy dissipated by the resistor over one full period of the applied voltage.

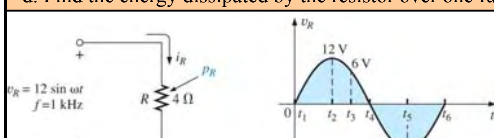


Figure 19.4 Example 19.1.

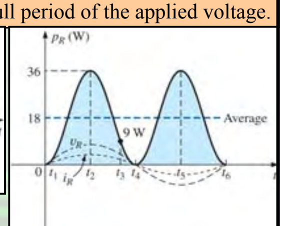


Figure 19.5 Assigning the subscripted impedances to the network in Fig.18.6.

a. $t_1 : v_R = 0\,V \quad and \quad p_R = v_R i_R = 0\,W$

$t_2 : v_R = 12\,V \quad and \quad i_R = 12\,V/3\Omega = 3\,A$

$p_R = v_R i_R = (12V)(3A) = 36\,W$

$t_3 : v_R = 6\,V \quad and \quad i_R = 6\,V/4\Omega = 1.5\,A$

$p_R = v_R i_R = (6V)(1.5A) = 9\,W$

$t_4 : v_R = 0\,V \quad and \quad p_R = v_R i_R = 0\,W$

$t_5 : v_R = -12\,V \quad and \quad i_R = -12\,V/4\Omega = -3\,A$

$p_R = v_R i_R = (-12V)(-3A) = 36\,W$

$t_6 : v_R = 0\,V \quad and \quad p_R = v_R i_R = 0\,W$

b. The resulting plot of v_R, i_R, and p_R appears in Fig. 19.5.

c. The average values of the curve in Fig. 19.5 is 18W, which is an exact match with that obtained using Eq.(19.3). that is,

$$p = \frac{V_m I_m}{2} = \frac{(12V)(3A)}{2} = 18\,W$$

d. The area under the curve is determined by Eq.(19.5) :

$$W_R = \frac{VI}{f_1} = \frac{V_m I_m}{2f_1} = \frac{(12V)(3A)}{2(1kHz)} = 18\,mJ$$

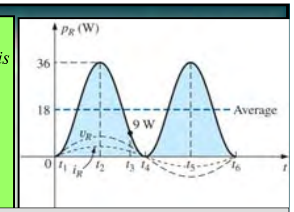


Figure 19.5 Power curve for Example 19.1.

Apparent Power

From our analysis of dc networks (and resistive elements earlier), it would seen *apparent* that the power delivered to the load in Fig. 19.6 is determined by the product of the applied voltage and current, with no concern for the components of the load; that is, P = VI. However, the power dissipated, less pronounced for more reactive loads. Although the product of the voltage and current is not always the power delivered, it is a power rating of significant usefulness in the description and analysis of sinusoidal ac networks and in the maximum rating of a number of electrical components and systems. It is called the **apparent power** and is represented symbolically by S*. Since it is simply the product of voltage and current, its units are volt-amperes (VA).

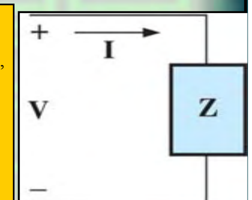


Figure 19.6 Defining the apparent power to a load.

Its magnitude is determined by

$$S = VI \quad (volt\text{-}amperes, VA)$$

or, since $V = IZ$ *and* $I = \frac{V}{Z}$

then $S = I^2 Z \quad (VA)$

and $S = \frac{V^2}{Z} \quad (VA)$

The average power to the load in Fig.19.4 is

$$P = VI\cos\theta$$

However, $S = VI$

Therefore, $P = S\cos\theta \quad (W)$

and the power factor of a system F_p *is*

$$F_p = \cos\theta = \frac{P}{S} \quad (unitless)$$

The reason for rating some electrical equipment in kilovolt-amperes rather than in kilowatts can be described using the configuration in Fig. 19.7. The load has an apparent power rating of 10 kVA and a current demand of 70 A is above the rated value and could damage the load element, yet the reading on the wattmeter is relatively low since the load is highly reactive.

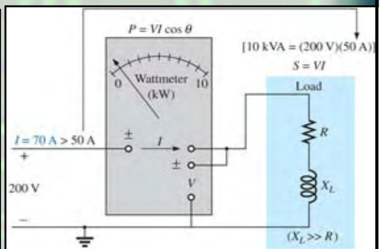


Figure 19.7 Demonstrating the reason for rating a load in kVA rather than kW.

Inductive Circuit and Reactive Power

For a purely inductive circuit (such as that in Fig. 19.8), *v* leads *i* by 90°, as shown in Fig. 19.9. Therefore, in Eq. (19.1), Θ = – 90°. Substituting Θ = – 90°, into Eq. (19.1), yields

$$P_L = VI\cos(90°)(1-\cos 2\omega t) + VI\sin(90°)(\sin 2\omega t)$$

$$= 0 + VI\sin 2\omega t$$

or $P_L = VI\sin 2\omega t \qquad (19.11)$

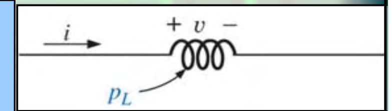


Figure 19.6 Defining the power level for a purely inductive load.

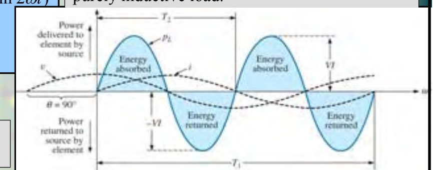


Figure 19.9 The power curve for a purely inductive load.

where *VI sin2ωt* is a sine wave with twice the frequency of either input quantity and a peak value of *VI*. Note that the absence of an average or constant term in the equation.

Plotting the waveform for p_L (Fig. 19.9), we obtain

T_1 = period of either input quantity

T_2 = period of p_L curve

Note that over one full cycle of p_L (T_2), the area above the horizontal axis in Fig. 19.9 is exactly equal to that below the axis. This indicates that over a full cycle of p_L, the power delivered by the sources to the inductor is exactly equal to that returned to the source by the inductor.

The net flow of power to the pure (ideal) inductor is zero over a full cycle, and no energy is lost in the transaction.

The power absorbed or returned by the inductor at any instant of time t_1 can be found simply by substituting t_1 into Eq. (19.11). The peak value of the curve *VI* is defined as the reactive power associated with a pure inductor. The symbol for reactive power is *Q*, and its unit of measure is the *volt-ampere reactive* (VAR).

$$Q_L = VI\sin\theta \quad (volt-ampere\ reactive, VAR)$$

where θ *is the phase angle between* V *and* I.

For the inductor,

$$Q_L = VI \quad (VAR) \qquad (19.13)$$

or, since $V = IX_L$ *or* $I = V / X_L$

$$Q_L = I^2 X_L \quad (VAR) \quad or \quad Q_L = \frac{V^2}{X_L} \quad (VAR)$$

The energy stored by the inductor during the positive portion of the cycle (Fig.19.9) is equal to that returned during the negative portion and can be determined using the following equation: *W = Pt*

Where P is the average value for the interval and t is the associated interval of time. The average value of the positive portion of a sinusoid equals 2(peak value/π) and t = $T_2/2$.

$$W_L = \left(\frac{2VI}{\pi}\right)\times\left(\frac{T_2}{2}\right) \quad and \quad W_L = \frac{VIT_2}{\pi} \quad (J)$$

or, since $T_2 = 1/f_2$, *where* f_2 *is the frequency of the* p_L *curve, we have*

$$W_L = \left(\frac{VI}{\pi f_2}\right) \quad (J) \qquad (19.17)$$

Since the frequency f_2 *of power curve is twice that of the input quantity, if we substitute the frequency* f_1 *of the input voltage or current, Eq.(19.17) becomes*

$$W_L = \frac{VI}{\pi(2f_1)} = \frac{VI}{\omega_1} \quad where\ V = IX_L = I\omega_1 L$$

so that $W_L = \frac{(I\omega_1 L)I}{\omega_1}$ *and* $W_L = LI^2\ (J)$

Ex. 19-2 For the inductive circuit in Fig. 19.10,

a. Find the instantaneous power level for the inductor at times t_1 through t_5.

b. Plot the results of part (a) for one full period of the applied voltage.

c. Find the average value of the curve of part (b) over one full cycle of the applied voltag and compare the peak value of each pulse with the value determined by Eq. (19.13).

d. Find the energy stored or released for any one pulse of the power curve.

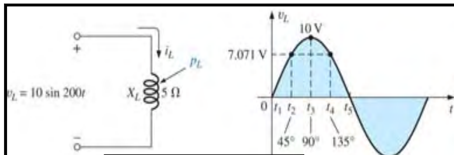


Figure 19.10 Example 19.2.

b. The resulting plot of v_L, i_L, and p_L appears in Fig. 19.11.

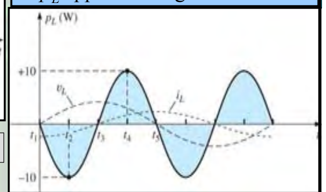


Figure 19.11 Power curve for Example 19.2.

$$a. \quad t_1 : v_L = 0\,V \quad and \quad p_L = v_L i_L = 0\,W$$

$$t_2 : v_L = 7.071\,V \quad and \quad i_L = \frac{V_m}{X_L}\sin(\alpha - 90°)$$

$$= \frac{10V}{5\Omega}\sin(\alpha - 90°) = 2\sin(\alpha - 90°)$$

$$at\ \alpha = 45°, i_L = 2\sin(45° - 90°) = 2\sin(-45°) = -1.414\,A$$

$$p_L = v_L i_L = (7.071V)(-1.414A) = -10\,W$$

$$t_3 : i_L = 0\ A, \quad p_L = v_L i_L = 0\,W$$

$$t_4 : v_L = 7.071\,V, \quad i_L = 2\sin(\alpha - 90°) = 2\sin(135° - 90°)$$

$$= 2\sin 45° = 1.414\,A$$

$$p_L = v_L i_L = (7.071V)(1.414A) = +10\,W$$

$$t_5 : v_L = 0\,V, \quad p_L = v_L i_L = 0\,W$$

c. The average value for the curve in Fig. 19.11 is 0W over one full cycle of the applied voltage. The peak value of the curve is 10W which compares directly with that obtained from the product

$$VI = \frac{V_m I_m}{2} = \frac{(10V)(2A)}{2} = 10\,W$$

d. The average stored or released during each pulse of the power curve is:

$$W_L = \frac{VI}{\omega_1} = \frac{V_m I_m}{2\omega_2} = \frac{(10V)(2A)}{2(200\,rad/s)} = 50\,mJ$$

Power-Factor Correction

The design of any power transmission system is very sensitive to the magnitude of the current in the lines as determined by the applied loads. Increased currents result in increased power losses (by squared factor since $P = I^2R$) in the transmission lines due to the resistance of the lines. Heavier currents also require larger conductors, increasing the amount of copper needed for the system.

Every effort must therefore be made to keep current levels at a minimum. Since the line voltage of a transmission system is fixed, the apparent power is directly related to the current level. In turn, the smaller the net apparent power, the smaller the current drawn from the supply. Minimum current is therefore drawn from a supply when S = P and QT = 0. Note the effect of decreasing levels of QT on the length (and magnitude) of S in Fig. 19.28 for the same real power.

The process of introducing reactive elements to bring the power-factor closer to unity is called is called power-factor correction. Since most loads are inductive, the process normally involves introducing elements with capacitive terminal characteristics having the sole purpose of improving the power factor.

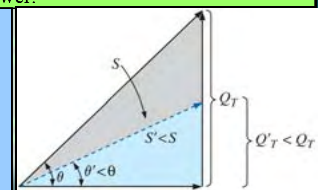


Figure 19.28 Demonstrating the impact of power-factor correction on the power triangle of a network.

In Fig. 19.29(a), for instance, an inductive load is drawing a current IL that has a real and an imaginary component. In Fig. 19.29(b), a capacitive load was added in parallel with original load to raise the power factor of the total system to the unity power-factor level. Note that by placing all the elements in parallel, the load still receives the same terminal voltage and draws same current I_L. In other words, the load is unaware of and unconcerned about whether it is hooked up as shown in Fig. 19.29(a) or (b).

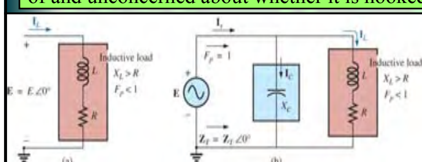


Figure 19.29 Demonstrating the impact of a capacitive element on the power factor of a network.

Solving for the source current in Fig.19.29(b):

$$I_s = I_C + I_L$$

$$= jI_C(I_{mag}) + I_L(R_e) + jI_L(I_{mag}) = jI_C + jI_L - jI_L$$

$$= I_L(R_e) + j[I_L(I_{mag}) + I_C(I_{mag})] = I_L + j[I_C + I_L]$$

If X_C is chosen such that $I_C = I_L$, then

$$I_s = I_L + j(0) = I_L\angle 0°$$

The result is a source current whose magnitude is simply equal to the real part of the inductive load current, which can be considerably less than the magnitude of the load current in Fig. 19.29(a). In addition, since the phase angle associated with both the applied voltage and the source current is same, the system appears "resistive" at the input terminals, and all of power supplied is absorbed, ***creating maximum efficiency for a generating utility.***

Power Meter

The **power meter** in Fig. 19.34 uses a sophisticated electronic package to sense the voltage and current levels and has an analog-to-digital conversion unit that display the levels in digital form. It is capable of providing a digital readout for distorted nonsinusoidal waveforms, and it can provide the phase power, total power, apparent power, reactive power, and power factor. The **power quality analyzer** in Fig. 19.35 can also display the real, reactive, and apparent power levels along with the power factor. However, it has a board range of other options, including providing the harmonic content of up to 51 terms for the voltage, current, and power.

Figure 19.34 Digital single-phase and three-phase power meter.

Figure 19.35 Power quality analyzer capable of displaying the power in watts, the current in amperes, and the voltage in volts.

Effective Resistance

The resistance of a conductor as determined by the equation R = ρ(l/A) is often called the dc, ohmic or geometric resistance. It is a constant quantity determined only by the material used and its physical dimensions. In ac circuits, the actual resistance of a conductor (called **effective resistance**) differs from the dc resistance because of the varying currents and voltages that introduce effects not present in dc circuits. ***These effects include radiation losses, skin effect, eddy currents, and hysteresis losses.***

Effective Resistance – Experimental Procedure

The effective resistance of an ac circuit cannot be measured by the ratio V/I since this ratio is now the impedance of a circuit that may have both resistance and reactance. The effective resistance can be found, however, by using the power equation $P = I^2R$, where

$$R_{eff} = \frac{P}{I^2}$$

A wattmeter and ammeter are therefore necessary for measuring the effective resistance of an ac circuit.

Effective Resistance – Radiation Losses

The **radiation loss** is the loss of energy in the form of electromagnetic waves during the transfer of energy in the from one element to another. This loss in energy requires that the input power be larger to establish the same current I, causing R to increases as determined by Eq. (19.31). At a frequency of 60Hz, the effects of radiation losses can be completely ignored. However, at radio frequencies, this is important effect and may in fact become the main effect in an electromagnetic device such as an antenna.

Effective Resistance – Skin Effect

The explanation of **skin effect** requires the use of some basic concepts previously described. A magnetic field exist around every current-carrying conductor. Since the amount of charge flowing in ac circuits changes with time, the magnetic field surrounding the moving charge (current) also changes. Recall also that a wire placed in a changing magnetic field will have an induced voltage across its terminals as determined by Faraday's law, $e = N \times (d\Phi/dt)$. The higher the frequency of the changing flux as determined by an alternating current, the greater the induced voltage.

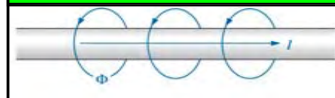


Figure 19.36 Demonstrating the skin effect on the effective resistance of a conductor.

Effective Resistance – Hysteresis and Eddy current losses

As mentioned earlier, **hysteresis and eddy current losses** appear when a ferromagnetic material is placed in the region of a changing magnetic field. To describe eddy current losses in greater detail, we consider the effects of an alternating current passing through coil wrapped around a ferromagnetic core. As the alternating current passes through the coil, it develops a changing magnetic flux Φ linking both coil and the core that develops an induced voltage and geometric resistance of the core $R_C = \rho(l/A)$ cause currents to be developed within the core, $i_{core} = (e_{ind}/R_C)$, called **eddy currents**.

The currents flow in circular paths, as shown in fig. 19.37, changing direction with the applied ac potential. The **eddy current losses** are determined by $P_{eddy} = i^2_{eddy}R_{core}$

The **eddy current loss** is proportional to the square of the frequency times the square of magnetic field strength:

$$p_{eddy} \propto f^2\mathbf{B}^2$$

Eddy current losses can be reduced if the core is constructed of thin, laminated sheets of ferromagnetic material insulated from one another and aligned parallel to the magnetic flux.

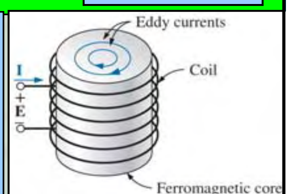


Figure 19.37 Defining the eddy current losses of a ferromagnetic core.

In terms of the frequency of the applied signal and the magnetic field strength produced, the **hysteresis loss** is proportional to the frequency to the 1st power times the magnetic field strength to the nth power: $p_{hys} \propto f^1\mathbf{B}^n$

Where n can vary from 1.4 to 2.6, depending on the material under consideration. **Hysteresis losses** can be effectively reduced by the injection of small amounts of silicon into the magnetic core, constituting some 2% or 3% of the total composition of the core.

HW 19-10 An electrical system is rated 10 kVA, 200V at a leading power factor.

a. Determine the impedance of the system in rectangular coordinates.
b. Find the average power delivered to the system.

a. $I_s = \dfrac{S_T}{E} = \dfrac{10{,}000 \text{ VA}}{200 \text{ V}} = 50 \text{ A}$

$0.5 \Rightarrow 60° \text{ leading}$

$\therefore \mathbf{I}_s \text{ leads } \mathbf{E} \text{ by } 60°$

$$\mathbf{Z}_T = \frac{\mathbf{E}}{\mathbf{I}_s} = \frac{200 \text{ V} \angle 0°}{50 \text{ A} \angle 60°} = 4\ \Omega \angle{-60°} = 2\ \Omega - j3.464\ \Omega = R - jX_C$$

b. $F_p = \dfrac{P_T}{S_T} \Rightarrow P_T = F_pS_T = (0.5)(10{,}000 \text{ VA}) = \mathbf{5000\ W}$

Homework 19: 2-6, 10-13, 16-18

EET1222/ET242 Circuit Analysis II

Series Resonance

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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I want to express my gratitude to Prentice Hall giving me the permission to use instructor's material for developing this module. I would like to thank the Department of Electrical and Telecommunications Engineering Technology of NYCCT for giving me support to commence and complete this module. I hope this module is helpful to enhance our students' academic performance.

Sunghoon Jang

OUTLINES

- Introduction to Series Resonance
- Series Resonance Circuit
- Quality Factor (Q)
- Z_T Versus Frequency
- Selectivity
- V_R, V_L, and V_C

Key Words: Series Resonance, Total Impedance, Quality Factor, Selectivity

Resonance - Introduction

The resonance circuit is a combination of R, L, and C elements having a frequency response characteristic similar to the one appearing in Fig. 20.1. Note in the figure that the response is a maximum for the frequency Fr, decreasing to the right and left of the frequency. In other words, for a particular range of frequencies, the response will be near or equal to the maximum. When the response is at or near the maximum, the circuit is said to be in a state of **resonance**.

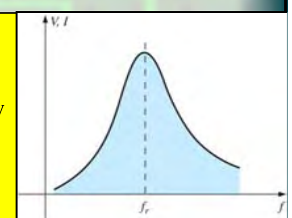

Figure 20.1 Resonance curve.

Series Resonance – Series Resonance Circuit

A **resonant circuit** must have an inductive and a capacitive element. A resistive element is always present due to the internal resistance (R_s), the internal resistance of the response curve (R_{design}). The basic configuration for the series resonant circuit appears in Fig. 20.2(a) with the resistive elements listed above. The "cleaner" appearance in Fig. 20.2(b) is a result of combining the series resistive elements into one total value. That is $R = R_s + R_l + R_d$

The total impedance of this network at any frequency is determined by

$$Z_T = R + jX_L - jX_C = R + j(X_L - X_C)$$

The resonant conditions described in the introduction occurs when

$$X_L = X_C \qquad (20.2)$$

removing the reactive component from the total impedance equation. The total impedance at resonance is then

$$Z_{T_s} = R$$

representing the minimum value of Z_T at any frequency. The subscript s is employed to indicate series resonant conditions.

The resonant frequency can be determined in terms of the inductance and capacitance by examining the defining equation for resonance [Eq. (20.2)]: $X_L = X_C$

Substituting yields

$$\omega L = \frac{1}{\omega C} \quad and \quad \omega^2 = \frac{1}{LC}$$

$$and \qquad \omega = \frac{1}{\sqrt{LC}}$$

$$or \qquad f_s = \frac{1}{2\pi\sqrt{LC}}$$

f = hertz (Hz), L = henries (H), C = farads (F)

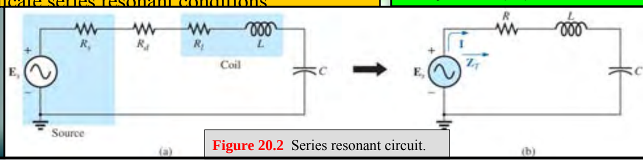

Figure 20.2 Series resonant circuit.

The current through the circuit at resonance is

$$I = \frac{E\angle 0°}{R\angle 0°} = \frac{E}{R}\angle 0°$$

which is the maximum current for the circuit in Fig. 20.2 for an applied voltage **E** since $\mathbf{Z_T}$ is a minimum value. Consider also that the input voltage and current are in phase at resonance.

Since the current is the same through the capacitor and inductor, the voltage across each is equal in magnitude but 180° out of phase at resonance:

$$\left.\begin{aligned} V_L &= (I\angle 0°)(X_L\angle 90°) = IX_L\angle 90° \\ V_L &= (I\angle 0°)(X_C\angle -90°) = IX_C\angle -90° \end{aligned}\right\} \text{180° out of phase}$$

And, since $X_L = X_C$, the magnitude of V_L equals V_C at resonance; that is,

$$V_{L_s} = V_{C_s}$$

Fig. 20.3, a phasor diagram of the voltage and current, clearly indicates that the voltage across the resistor at resonance is the input voltage, and E, I, and VR are in phase at resonance.

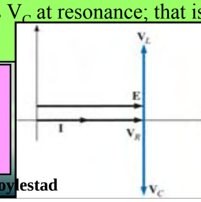

Figure 20.2 Phasor diagram for the series resonant circuit at resonance.

The average power to the resistor at resonance is equal to I^2R, and the **reactive power** to the capacitor and inductor are I^2X_C and I^2X_C, respectively. The power triangle at resonance (Fig. 20.4) shows that the total **apparent power** is equal to the average power dissipated by the resistor since $Q_L = Q_C$. The **power factor** of the circuit at resonance is

$$F_p = \cos\Theta = P/S \ \ and \qquad F_{p_s} = 1$$

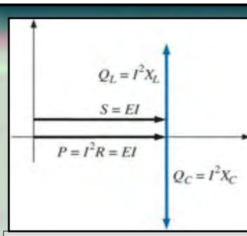

Figure 20.4 Power triangle for the series resonant circuit at resonance.

Series Resonance – Quality Factor (Q)

The **quality factor** Q of a series resonant circuit is defined as the ratio of the reactive power of either the inductor or the capacitor to the average power of the resistor at resonance; that is,

Q_s = reactive power / average power

The **quality factor** is also an indication of how much energy is placed in storage compared to that dissipated. *The lower the level of dissipation for the same reactive pow the larger the Q_s factor and the more concentrated and intense the region of resonance.*

Substituting for an inductive reactance in Eq.(20.8) at resonance gives us

$$Q_s = \frac{I^2X_L}{I^2R} \quad and \quad Q_s = \frac{X_L}{R} = \frac{\omega_s L}{R} \qquad (20.9)$$

If the resistor R is just the resistance of the coil(R_l), we can speak of the Q of the coil, where

$$Q_{coil} = Q_l = \frac{X_L}{R_l}$$

Note in Fig.20.6 that for coils of the same type, Q_l drops off more quickly for higher levels of inductance, If we substitute

$$\omega_s = 2\pi f_s \quad and\ then \quad f_s = \frac{1}{2\pi\sqrt{LC}}$$

into Eq.(20.9), we have

$$Q_s = \frac{\omega_s L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi}{R}\left(\frac{1}{2\pi\sqrt{LC}}\right)L$$

$$= \frac{L}{R}\left(\frac{1}{\sqrt{LC}}\right) = \left(\frac{\sqrt{L}}{\sqrt{L}}\right)\frac{L}{R\sqrt{LC}} = \frac{1}{R}\sqrt{\frac{L}{C}}$$

providing Q_s in terms of the circuit parameters.

For series resonant circuits used in communication systems, is usually greater than 1. By applying the voltage divider Irule to the circuit in Fig.20.2, we obtain

$$V_L = \frac{X_L E}{Z_T} = \frac{X_L E}{R} \quad (at\ resonance)$$

$$and \quad V_{L_s} = Q_s E \quad or \quad V_C = \frac{X_C E}{Z_T} = \frac{X_C E}{R}$$

$$and \quad V_{C_s} = Q_s E$$

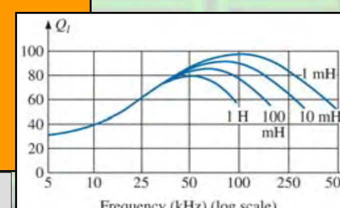

Figure 20.6 Q_l versus frequency for a series of inductor of similar construction.

Series Resonance – Z_T Versus Frequency

The total impedance of the series R – L – C circuit in Fig.20.2 at any frequency is determined by

$$Z_T = R + jX_L - jX_C \quad or \quad Z_T = R + j(X_L - X_C)$$

The magnitude of the impedance Z_T versus frequency is determined by

$$Z_T = \sqrt{R^2 + (X_L - X_C)^2}$$

The total-impedance-versus-frequency curve for the series resonant circuit in Fig. 20.2 can be found by applying the impedance-versus-frequency curve for each element of the equation just derived, written in the following form:

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2}$$

Where $Z_T(f)$ "means" the total impedance as a function of frequency. For the frequency range of interest, we assume that the resistance R does not change with frequency, resulting in the plot in Fig.20.8.

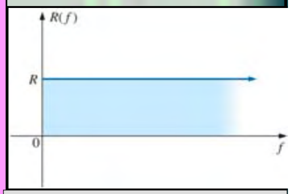

Figure 20.8 Resistance versus frequency.

The curve for the inductance, as determined by the reactance equation, is a straight line intersecting the origin with a slope equal to the inductance of the coil. The mathematical expression for any straight line in a two-dimensional plane is given by

$$y = mx + b$$

Thus, for the coil,

$$X_L = 2\pi fL + 0 = (2\pi L)(f) + 0$$

$$y = \quad m \cdot x + b$$

(where $2\pi fL$ is the slope), producing the results shown in Fig. 20.9.

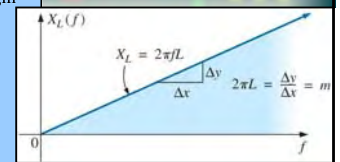

Figure 20.9 Inductive reactance versus frequency.

For the capacitor, $X_C = \dfrac{1}{2\pi fC} \quad or \quad X_C f = \dfrac{1}{2\pi C}$

which becomes $yx = k$, the equation for a hyperbola, where

$$y(variable) = X_C$$
$$x(variable) = f$$
$$k(variable) = \frac{1}{2\pi C}$$

The hyperbolic curve for $X_C(f)$ is plotted in Fig.20.10. In particular, note its very large magnitude at low frequencies and its rapid drop-off as the frequency increases.

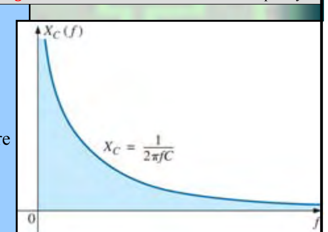

Figure 20.10 Capacitive reactance versus frequency.

If we place Figs.20.9 and 20.10 on the same set of axes, we obtain the curves in Fig.20.11. The condition of resonance is now clearly defined by the point of intersection, where $X_L = X_C$. For frequency less than Fs, it is also quite clear that the network is primarily capacitive ($X_C > X_L$). For frequencies above the resonant condition, $X_L > X_C$, and network is inductive.

Applying

$$Z_T(f) = \sqrt{[R(f)]^2 + [X_L(f) - X_C(f)]^2} = \sqrt{[R(f)]^2 + [X(f)]^2}$$

to the curves in Fig.20.11, where $X(f) = X_L(f) - X_C(f)$, we obtain the curve for $Z_T(f)$ as shown in Fig.20.12.The minimum impedance occurs at the resonant frequency and is equal to the resistance R.

The phase angle associated with the total impedance is

$$\theta = \tan^{-1}\frac{(X_L - X_C)}{R}$$

At low f ... nd Θ approaches –90° (capacitive), as shown in Fig.20.13, whereas at high frequencies, $X_L > X_C$, and Θ approaches 90°. In general, therefore, for a series resonant circuit:

$f < f_s$: network capacitive; I leads E
$f > f_s$: network capacitive; E leads I
$f = f_s$: network capacitive; E and I are in phase

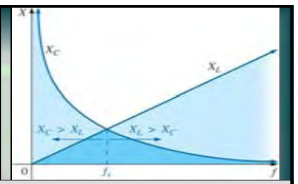

Figure 20.11 Placing the frequency response of the inductive and capacitive reactance of a series R-L-C circuit on the same set of axes.

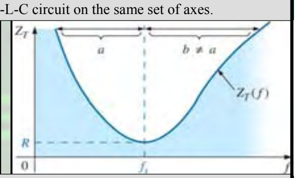

Figure 20.12 Z_T versus frequency for the series resonant circuit.

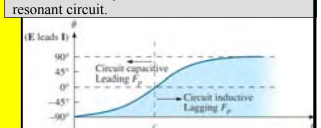

Figure 20.13 Phase plot for the series resonant circuit.

Series Resonance – Selectivity

If we now plot the magnitude of the current $I = E/Z_T$ versus frequency for a fixed applied voltage E, we obtain the curve shown in Fig. 20.14, which rises from zero to a maximum value of E/R and then drops toward to zero at a slower rate than it rose to its peak value.

There is a definite range of frequencies at which the current is near its maximum value and the impedance is at a minimum. Those frequencies corresponding to 0.707 of maximum current are called the **band frequencies**, **cutoff frequencies**, **half-power frequencies**, or **corner frequencies**. They are indicated by f_1 and f_2 in Fig.20.14. The range of frequencies between the two is referred to as **bandwidth** (**BW**) of the resonant circuit.

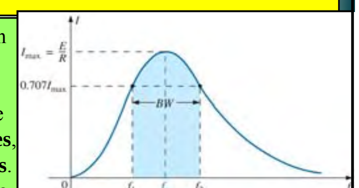

Figure 20.14 I versus frequency for the series resonant circuit.

Half-power frequencies are those frequencies at which the power delivered is one-half that delivered at the resonant frequency; that is

$$P_{HPF} = \frac{1}{2}P_{max} \quad where \quad P_{max} = I_{max}^2 R$$

Since the resonant circuit is adjusted to select a band of frequencies, the curve in Fig.20.14 is called the selective curve. The term is derived from the fact that on must be selective in choosing the frequency to ensure that is in the bandwidth. The smaller bandwidth, the higher the selectivity. The shape of the curve, as shown in Fig. 20.15, depends on each element of the series R-L-C circuit. If resistance is made smaller with a fixed inductance and capacitance, the bandwidth decreases and the selectivity increases.

The bandwidth (BW) is $BW = \frac{f_s}{Q_s}$

It can be shown through mathematical manipulations of the pertinent equations that the resonant frequency is related to the geometric mean of the band frequencies; that is $f_s = \sqrt{f_1 f_2}$

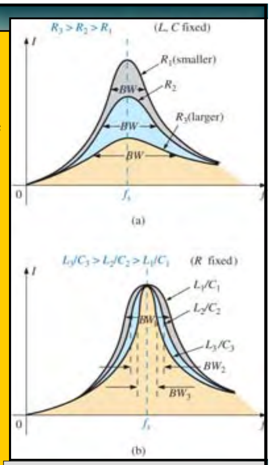

Figure 20.15 Effect of R, L, and C on the selectivity curve for the series resonant circuit.

Series Resonance – V_R, V_L, and V_C

Plotting the magnitude (effective value) of the voltage $\mathbf{V_R}$, $\mathbf{V_L}$, and $\mathbf{V_C}$ and the current **I** versus frequency for the series resonant circuit on the same set of axes, we obtain the curves shown in Fig.20.17. Note that the V_R curve has the same shape as the I curve and a peak value equal to the magnitude of the input voltage E. The V_C curve build up slowly at first from a value equal to the input voltage since the reactance of the capacitor is infinite (open circuit) at zero frequency and reactance of the inductor is zero (short circuit) at this frequency.

For the condition $Q_s \geq 10$, the curves in Fig.20.17 appear as shown in Fig.20.18. Note that they each peak at the resonant frequency and have a similar shape.

In review,

1. V_C and V_L are at their maximum values at or near resonance. (depending on Q_s).
2. At very low frequencies, VC is very close to the source voltage and V_L is very close to zero volt, whereas at very high frequencies, V_L approaches the source voltage and V_C approaches zero volts.
3. Both V_R and I peak at the resonant frequency and have the same shape.

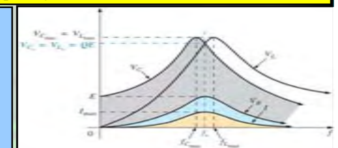

Figure 20.17 V_R, V_L, V_C, and I versus frequency for a series resonant circuit.

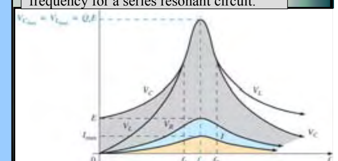

Figure 20.18 V_R, V_L, V_C, and I for a series resonant circuit where $Q_s \geq 10$.

Ex. 20-1

a. For the series resonant circuit in Fig.20.19, find I, V_R, V_L, and V_C at resonance.
b. What is the Q_s of the circuit?
c. If the resonant frequency is 5000Hz, find the bandwidth.
d. What is the power dissipated in the circuit at the half-power frequencies?

$$a.\quad Z_{T_s} = R = 2\Omega$$
$$I = \frac{E}{Z_{T_s}} = \frac{10V\angle 0°}{2\Omega\angle 0°} = 5A\angle 0°$$
$$V_R = E = 10V\angle 0°$$
$$V_L = (I\angle 0°)(X_L\angle 90°) = (5A\angle 0°)(10\Omega\angle 90°) = 50V\angle 90°$$
$$V_C = (I\angle 0°)(X_C\angle -90°) = (5A\angle 0°)(10\Omega\angle -90°) = 50V\angle -90°$$

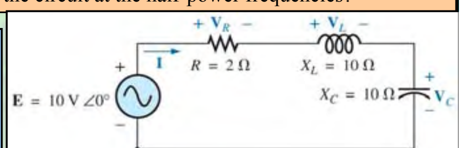

Figure 20.19 Example 20.1.

$$b.\quad Q_s = \frac{X_L}{R} = \frac{10\Omega}{2\Omega} = 5$$
$$c.\quad BW = f_2 - f_1 = \frac{f_s}{Q_s} = \frac{5000Hz}{5} = 1000\,Hz$$
$$d.\quad P_{HPF} = \frac{1}{2}P_{max} = \frac{1}{2}I^2_{max}R = \left(\frac{1}{2}\right)(5A)^2(2\Omega) = 25\,W$$

Ex. 20-2 The bandwidth of a series resonant circuit is 400 Hz.

a. If the resonant frequency is 4000 Hz, what is the value of Q_s?
b. If R = 10Ω, what is the value of X_L at resonance?
c. Find the inductance L and capacitance C of the circuit.

$$a.\quad BW = \frac{f_s}{Q_s} \quad or \quad Q_s = \frac{f_s}{BW} = \frac{4000Hz}{400Hz} = 10$$
$$b.\quad Q_s = \frac{X_L}{R} \quad or \quad X_L = Q_s R = (10)(10\Omega) = 100\,\Omega$$
$$c.\quad X_L = 2\pi f_s L \quad or \quad L = \frac{X_L}{2\pi f_s} = \frac{100\Omega}{2\pi(4000Hz)} = 3.98\,Hz$$
$$X_C = \frac{1}{2\pi f_s C} \quad or \quad C = \frac{1}{2\pi f_s X_C} = \frac{1}{2\pi(4000Hz)(100\Omega)} = 397.89\,nF$$

Ex. 20-3 A series R-L-C circuit has a series resonant frequency of 12,000 Hz.

a. If R = 5Ω, and if X_L at resonance is 300Ω, find the bandwidth.
c. Find the cutoff frequencies.

$$a.\quad Q_s = \frac{X_L}{R} = \frac{300\Omega}{5\Omega} = 60 \quad and \quad BW = \frac{f_s}{Q_s} = \frac{12{,}000Hz}{60} = 200Hz$$

b. Since $Q_s \geq 10$, the bandwidth is bisected by f_s. Therefore,

$$f_2 = f_s + \frac{BW}{2} = 12{,}000Hz + 100Hz = 12{,}100Hz$$
$$and \quad f_1 = f_s - \frac{BW}{2} = 12{,}000Hz - 100Hz = 11{,}900Hz$$

Ex. 20-4

a. Determine the Q_s and bandwidth for the response curve in Fig.20.20.

b. For C = 101.5 nF, determine L and R for the series resonant circuit.

c. Determine the applied voltage.

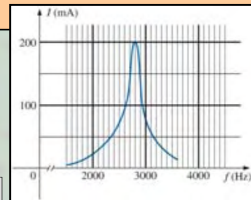

Figure 20.20 Example 20.4.

a. *The resonant frequency is* $2800\,Hz$. *At* 0.707 *times the peak value,*

$$BW = 200\,Hz \quad and \quad Q_s = \frac{f_s}{BW} = \frac{2800\,Hz}{200\,Hz} = 14$$

$$b. \quad f_s = \frac{1}{2\pi\sqrt{LC}} \quad or \quad L = \frac{1}{4\pi^2 f_s^2 C} = \frac{1}{4\pi^2(2.8kHz)^2(101.5nF)} = 31.83mH$$

$$Q_s = \frac{X_L}{R} \quad or \quad R = \frac{X_L}{Q_s} = \frac{2\pi(2800\,Hz)(31.832\,mH)}{14} = 40\,\Omega$$

$$c. \quad I_{\max} = \frac{E}{R} \quad or \quad E = I_{\max}R = (200\,mA)(40\Omega) = 8\,V$$

Ex. 20-5 A series R-L-C circuit is designed to resonate at ω_s = 105 rad/s, have a bandwidth of $0.15\omega_s$, and draw 16 W from a 120 V source at resonance.

a. Determine the value of R.

b. Find bandwidth in hertz.

c. Find the nameplate values of L and C.

d. Determine the Qs of the circuit.

e. Determine the fractional bandwidth.

$$a. \quad p = \frac{E^2}{R} \quad and \quad R = \frac{E^2}{p} = \frac{(120V)^2}{16W} = 900\Omega$$

$$b. \quad f_s = \frac{\omega_s}{2\pi} = \frac{10^5\,rad/s}{2\pi} = 15{,}915.49Hz$$

$$BW = 0.15f_s = 0.15(15{,}915.49Hz) = 2387.32Hz$$

$$c. \quad BW = \frac{R}{2\pi L} \quad and \quad L = \frac{R}{2\pi\,BW} = \frac{900\Omega}{2\pi(2387.32Hz)} = 60\ mH$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \quad and \quad C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2(15{,}915.49Hz)^2(60\times10^{-3})} = 1.67\ nF$$

$$d. \quad Q_s = \frac{X_L}{R} = \frac{2\pi f_s L}{R} = \frac{2\pi(15{,}915.49Hz)(60mH)}{900\Omega} = 6.67$$

$$e. \quad \frac{f_2 - f_1}{f_s} = \frac{BW}{f_s} = \frac{1}{Q_s} = \frac{1}{6.67} = 0.15$$

HW 20-11 A series resonant circuit is to resonate at $\omega_s = 2\pi \times 10^6$ rad/s and draw 20W from a 120 V source at resonance. If the fractional bandwidth is 0.16.

a. Determine the resonant frequency in hertz.

b. Calculate the bandwidth in hertz.

c. Determine the values of R, L, and C.

d. Find the resistance of the coil if $Q_l = 80$.

Homework 20: 1-12

$$a. \quad f_s = \frac{\omega_s}{2\pi} = \frac{2\pi\times10^6\ \text{rad/s}}{2\pi} = \mathbf{1\ MHz}$$

$$b. \quad \frac{f_2 - f_1}{f_s} = 0.16 \Rightarrow BW = f_2 - f_1 = 0.16\,f_s = 0.16(1\ \text{MHz}) = \mathbf{160\ kHz}$$

$$c. \quad P = \frac{V_R^2}{R} \Rightarrow R = \frac{V_R^2}{P} = \frac{(120\ \text{V})^2}{20\ \text{W}} = \mathbf{720\ \Omega}$$

$$BW = \frac{R}{2\pi L} \Rightarrow L = \frac{R}{2\pi BW} = \frac{720\,\Omega}{(6.28)(160\ \text{kHz})} = \mathbf{0.716\ mH}$$

$$f_s = \frac{1}{2\pi\sqrt{LC}} \Rightarrow C = \frac{1}{4\pi^2 f_s^2 L} = \frac{1}{4\pi^2(10^6\ \text{Hz})^2(0.716\ \text{mH})} = \mathbf{35.38\ pF}$$

$$d. \quad Q_\ell = \frac{X_L}{R_\ell} = 80 \Rightarrow R_P = \frac{X_L}{80} = \frac{2\pi f_s L}{80} = \frac{2\pi(10^6\ \text{Hz})(0.716\ \text{mH})}{80} = \mathbf{56.23\ \Omega}$$

EET1222/ET242 Circuit Analysis II

Parallel Resonance

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Parallel Resonance
- Parallel Resonance Circuit
- Unity Power Factor (f_p)
- Selectivity Curve
- Effect of $Q_L \geq 10$
- Examples

Key Words: Resonance, Unity Power Factor, Selective Curve, Quality Factor

Parallel Resonance Circuit - Introduction

The basic format of the series resonant circuit is a series R-L-C combination in series with an applied voltage source. The parallel resonant circuit has the basic configuration in Fig. 20.21, a **parallel R-L-C combination in parallel** with an applied current source.

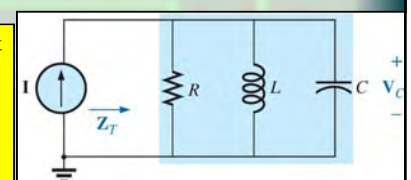


Figure 20.21 Ideal parallel resonant network.

If the practical equivalent in Fig. 20.22 had the format in Fig. 20.21, the analysis would be as direct and lucid as that experience for series resonance. However, in the practical world, the internal resistance of the coil must be placed in series with the inductor, as shown in Fig.20.22.

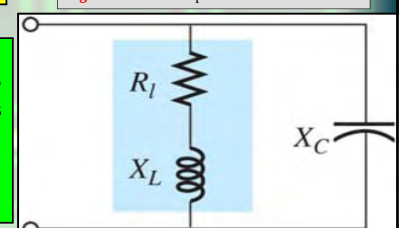


Figure 20.22 Practical parallel L-C network.

The first effort is to find a parallel network equivalent for the series R-L branch in Fig.20.22 using the technique in earlier section. That is

$$Z_{R-L} = R_l + jX_L$$

$$\text{and} \quad Y_{R-L} = \frac{1}{Z_{R-L}} = \frac{1}{R_l + jX_L} = \frac{R_l}{R_l^2 + X_L^2} - j\frac{X_L}{R_l^2 + X_L^2}$$

$$= \frac{1}{\dfrac{R_l^2 + X_L^2}{R_l}} + \frac{1}{j\left(\dfrac{R_l^2 + X_L^2}{X_L}\right)} = \frac{1}{R_p} + \frac{1}{jX_{L_p}}$$

$$\text{with} \quad X_{L_p} = \frac{R_l^2 + X_L^2}{X_l}$$

as shown in Fig.20.23.

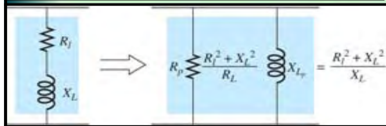


Figure 20.23 Equivalent parallel network for a series R-L combination.

Parallel Resonant Circuit – Unity Power Factor, f_p

For the network in Fig.20.25,

$$Y_T = \frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} = \frac{1}{R} + \frac{1}{jX_{L_p}} + \frac{1}{-jX_C}$$

$$= \frac{1}{R} - j\left(\frac{1}{X_{L_p}}\right) + j\left(\frac{1}{X_C}\right)$$

$$\text{and} \quad Y_T = \frac{1}{R} + j\left(\frac{1}{X_C} - \frac{1}{X_{L_p}}\right)$$

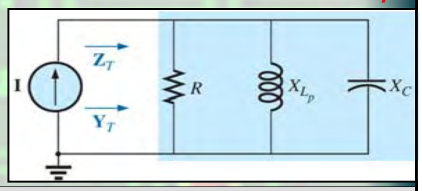


Figure 20.25 Substituting R = $R_s//R_p$ for the network in Fig. 20.24.

For unity power factor, the reactive component must be zero as defined by

$$\frac{1}{X_C} - \frac{1}{X_{L_p}} = 0$$

$$\text{Therefore,} \quad \frac{1}{X_C} = \frac{1}{X_{L_p}} \quad \text{and} \quad X_{L_p} = X_C$$

Substituting for X_{L_p} *yields*

$$\frac{R_l^2 + X_L^2}{X_L} = X_C$$

The resonant frequency, f_p*, can be determined as follow:*

$$f_p = \frac{1}{2\pi\sqrt{LC}}\sqrt{1 - \frac{R_l^2 C}{L}} \quad \text{or} \quad f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}} \qquad (20.31)$$

Where f_p is the resonant frequency of a parallel resonant circuit (for $F_p = 1$) and f_s is the resonant frequency as determined by $X_L = X_C$ for series resonance. Note that unlike a series resonant circuit, the resonant frequency f_p is a function of resistance (in this case R_l).

Parallel Resonant Circuit – Maximum Impedance, f_m

At $f = f_p$ the input impedance of a parallel resonant circuit will be near its maximum value but not quite its maximum value due to the frequency dependence of R_p. The frequency at which impedance occurs is defined by fm and is slightly more than f_p, as demonstrated in Fig. 20.26.

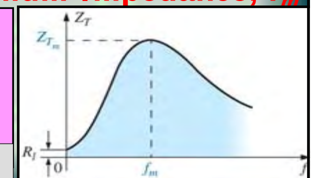


Figure 20.26 Z_T versus frequency for the parallel resonant circuit.

The frequency f_m is determined by differentiating the general equation for Z_T with respect to frequency and then determining the frequency at which the resulting equation is equal to zero. The resulting equation, however, is the following:

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left(\frac{R_l^2 C}{L}\right)}$$

Note the similarities with Eq. (20.31). Since square root factor of Eq. (20.32) is always more than the similar factor of Eq. (20.31), fm is always closer to fs and more than f_p. In general,

$$f_s > f_m > f_p$$

Once fm is determined, the network in Fig. 20.25 can be used to determine the magnitude and phase angle of the total impedance at the resonance condition simply by substituting $f = f_m$ and performing the required calculations. That is

$$Z_{T_m} = R \,//\, X_{L_p} \,//\, X_C \quad f = f_m$$

Parallel Resonant Circuit – Selectivity Curve

Since the current I of the current source is constant for any value of Z_T or frequency, the voltage across the parallel circuit will have the same shape as the total impedance Z_T, as shown in Fig. 20.27. For parallel circui the resonance curve of interest in V_C derives from electronic considerations that often place the capacitor at the input to another stage of a network.

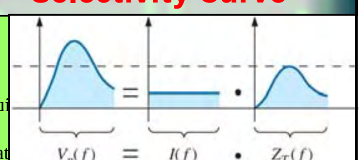


Figure 20.27 Defining the shape of the $V_p(f)$ curve.

Since the voltage across parallel elements is the same,

$$V_C = V_p = IZ_T$$

The resonant value of V_C is therefore determined by the value of Z_{T_m} and magnitude of the current source I. The **quality factor of the parallel resonant circuit** continues to be determined as following;

$$Q_p = \frac{R}{X_{L_p}} = \frac{R_s // R_p}{X_{L_p}} = \frac{R_s // R_p}{X_C} \qquad X_{L_p} = X_C \text{ at resonance}$$

For the **ideal current source** (Rs = ∞ Ω) or when R_s is sufficiently large compared to R_p, we can make the following approximation:

$$Q_p = \frac{X_L}{R_l} = Q_l \qquad R_s >> R_p$$

In general, the **bandwidth** is still related to the resonant frequency and the quality factor by

$$BW = f_2 - f_1 = \frac{f_r}{Q_p}$$

The **cutoff frequencies f_1 and f_2** can be determined using the equivalent network and the quality factor by

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] \quad \text{and} \quad f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

The effect of R_l, L, and C on the shape of the parallel resonance curve, as shown in Fig. 20.28 for the input impedance, is quite similar to their effect on the series resonance curve. Whether or not R_l is zero, the parallel resonant circuit frequently appears in a network schematic as shown in Fig. 20.28. At resonance, an increase in R_l or decrease in the ratio L/R results in a decrease in the resonant impedance, with a corresponding increase in the current.

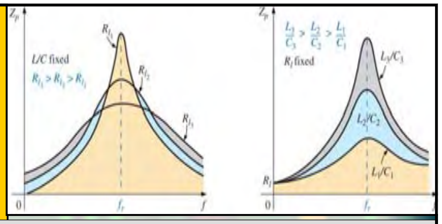


Figure 20.28 Effect of R_l, L, and, C on the parallel resonance curve.

Parallel Resonant Circuit – Effect of $Q_L \geq 10$

The analysis of parallel resonant circuits is significantly more complex than encountered for series circuits. However, this is not the case since, for the majority of parallel resonant circuits, **the quality factor of the coil Q_l** is sufficiently large to permit a number of approximations that simplify the required analysis.

Effect of $Q_L \geq 10$ – Inductive Resistance, X_{L_p}

$$X_{L_p} \cong X_L \quad _{Q_l \geq 10} \quad and \quad X_L \cong X_C \quad _{Q_l \geq 10}$$

Effect of $Q_L \geq 10$ – Resonant Frequency, f_p (Unity Power Factor)

$$f_p = f_s\sqrt{1 - \frac{1}{Q_l^2}} \quad _{Q_l \geq 10} \quad and \quad f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}} \quad _{Q_l \geq 10}$$

Effect of $Q_L \geq 10$ – Resonant Frequency, f_m (Maximum V_C)

$$f_m \cong f_s\sqrt{1 - \frac{1}{4}\left(\frac{1}{Q_l^2}\right)} \quad _{Q_l \geq 10} \quad and \quad f_p \cong f_s = \frac{1}{2\pi\sqrt{LC}} \quad _{Q_l \geq 10} \quad In\ total \quad f_p \cong f_m \cong f_s \quad _{Q_l \geq 10}$$

R_p

$$R_p \cong Q_l^2 R_l \quad _{Q_l \geq 10} \quad substituting \quad Q_l = \frac{X_L}{R_l} \quad then \quad R_p \cong \frac{L}{R_l C} \quad _{Q_l \geq 10}$$

Z_{T_p}

$$Z_{T_p} \cong R_s // R_p = R_s // Q_l^2 R_l \quad _{Q_l \geq 10} \quad and \quad Q_l = \frac{X_L}{R_l} \quad then \quad Z_{T_p} \cong Q_l^2 R_l \quad _{Q_l \geq 10,\ R_s >> R_p}$$

Q_p

$$Q_p = \frac{R}{X_{L_p}} \cong \frac{R_s // Q_l^2 R_l}{X_L} \quad and \quad Q_p \cong Q_l \quad _{Q_l \geq 10,\ R_s >> R_p}$$

BW

$$BW = f_2 - f_1 = \frac{f_p}{Q_p} \cong \frac{1}{2\pi}\left[\frac{R_l}{L} + \frac{1}{R_s C}\right] \quad and \quad BW = f_2 - f_1 \cong \frac{R_l}{2\pi\pi} \quad _{R_s = \infty\Omega}$$

I_L and I_C

A portion of Fig. 20.30 is reproduced in Fig. 20.31, with I_T defined as shown

$$I_C \cong Q_l I_T \quad _{Q_l \geq 10}$$

$$and \quad I_L \cong Q_l I_T \quad _{Q_l \geq 10}$$

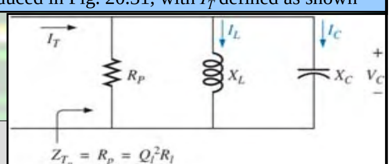


Figure 20.31 Establishing the relationship between I_C and I_L and current I_T.

Ex. 20-6 Given the parallel network in Fig. 20.32 composed of "ideal" elements:

a. Determine the resonant frequency f_p.
b. Find the total impedance at resonance
c. Calculate the quality factor, bandwidth, and cutoff frequencies f_1 and f_2 of the system.
d. Find the voltage V_C at resonance.
e. Determine the currents I_L and I_C at resonance.

a. The fact that R_l is zero ohms results in a very high Q_l $(= X_L / R_l)$, permitting the use of the following equation for f_p:

$$f_p = f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(1mH)(1\mu F)}} = 5.03kHz$$

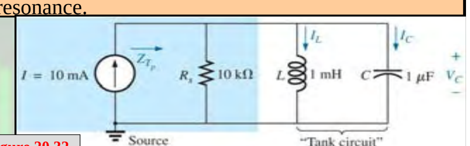


Figure 20.32 Example 20.6.

b. For the parallel reactive elements:

$$Z_L // Z_C = \frac{(X_L\angle 90°)(X_C\angle -90°)}{+j(X_L - X_C)}$$

but $X_L = X_C$ at resonance, resulting in a zero in the denominato r of the equation and a very high impedance that can be approximat ed by an open circuit. Therefore,

$$Z_{T_p} = R_s // Z_L // Z_C = R_s = 10\ k\Omega$$

$$c. \quad Q_p = \frac{R_s}{X_{L_p}} = \frac{R_s}{2\pi f_p L} = \frac{10k\Omega}{2\pi(5.03k\Omega)(1mH)} = 316.41$$

$$BW = \frac{f_p}{Q_p} = \frac{5.03kHz}{316.41} = 15.90\ Hz$$

$$f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right]$$

$$= \frac{1}{4\pi(1\mu F)}\left[\frac{1}{10k\Omega} - \sqrt{\frac{1}{(10k\Omega)^2} + \frac{4(1\mu F)}{1mH}}\right]$$

$$= 5.03\ kHz$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] = 5.04\ kHz$$

$$d. \quad V_C = IZ_{T_p} = (10mA)(10k\Omega) = 100V$$

$$e. \quad I_L = \frac{V_L}{X_L} = \frac{V_C}{2\pi f_p L} = \frac{100V}{2\pi(5.03kHz)(1mH)} = \frac{100V}{31.6\Omega} = 3.16A$$

$$I_C = \frac{V_C}{X_C} = \frac{100V}{31.6\Omega} = 3.16A \quad (= Q_p I)$$

Ex. 20-7 For the parallel resonant circuit in Fig. 20.33 with $R_s = \infty\ \Omega$:

a. Determine f_p, f_m, and f_p, and compare their levels.

b. Calculate the maximum impedance and the magnitude of the voltage V_C at f_m.

c. Determine the quality factor Q_p.

d. Calculate the bandwidth.

e. Compare the above results with those obtained using the equations associated with $Q_l \geq 10$.

$$a.\quad f_s = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{(0.3mH)(100\ \mu F)}} = 29.06\ kHz$$

$$f_m = f_s\sqrt{1 - \frac{1}{4}\left[\frac{R_l^2 C}{L}\right]} = (29.06\,kHz)\sqrt{1 - \frac{1}{4}\left[\frac{(20\,\Omega)^2(100\ nF)}{0.3mH}\right]} = 25.58\ kHz$$

$$f_p = f_s\sqrt{1 - \frac{R_l^2 C}{L}} = (29.06\,kHz)\sqrt{1 - \left[\frac{(20\,\Omega)^2(100\ nF)}{0.3mH}\right]} = 27.06\ kHz$$

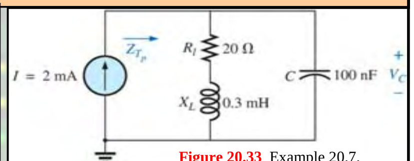


Figure 20.33 Example 20.7.

$$b.\quad Z_{T_m} = (R_l + jX_L) \,//\, -jX_C \quad at\ f = f_m$$

$$X_L = 2\pi f_m L = 2\pi(28.58\,kHz)(0.3mH) = 53.87\ \Omega$$

$$X_C = \frac{1}{2\pi f_m C} = \frac{1}{2\pi(28.58\,kHz)(100\ nF)} = 55.69\ \Omega$$

$$R_l + jX_L = 20\ \Omega + j53.87\ \Omega = 57.46\,\Omega\angle 69.63°$$

$$Z_{T_m} = \frac{(57.46\,\Omega\angle 69.63°)(55.69°\angle -90°)}{159.34\,\Omega\angle -15.17°}$$

$$V_{C_{max}} = IZ_{T_m} = (2mA)(159.34\ \Omega) = 318.68\ mV$$

$$c.\quad R_s = \infty\,\Omega;\ therefore$$

$$Q_p = \frac{R_s // R_p}{X_{L_p}} = \frac{R_p}{X_{L_p}} = Q_l = \frac{X_L}{R_l} = \frac{2\pi(29.06kHz)(0.3mH)}{20\Omega} = \frac{51\Omega}{20\Omega} = 2.55$$

$$d.\quad BW = \frac{f_p}{Q_p} = \frac{27.06kHz}{2.55} = 10.61\,Hz$$

$$e.\quad For\ Q_l \geq 10,\ f_m = f_p = f_s = 29.06\ kHz$$

$$Q_p = Q_l = \frac{2\pi f_s L}{R_l} = \frac{2\pi(29.06kHz)(0.3mH)}{20\Omega} = 2.74 \quad (versus\ 2.55\ above)$$

$$Z_{T_p} = Q_l^2 R_l = (2.74)^2 \cdot 20\Omega = 150.15\Omega\angle 0° \quad (versus\ 159.34\Omega\angle -15.17°\ above)$$

$$V_{C_{max}} = IZ_{T_p} = (2mA)(150.15\Omega) = 300.3\ mV \quad (versus\ 318.68\ mV\ above)$$

$$BW = \frac{f_p}{Q_p} = \frac{29.06kHz}{2.74} = 10.61\,kHz \quad (versus\ 10.61kHz\ above)$$

Ex. 20-8 For the network in Fig. 20.34 with f_p provided:

a. Determine Q_l. b. Determine R_p. c. Calculate Z_{T_p}. d. Find C at resonance.

e. Find Q_p. f. Calculate the BW and cutoff frequencies.

$$a.\quad Q_l = \frac{X_L}{R_l} = \frac{2\pi f_p L}{R_l} = \frac{2\pi(0.04MHz)(1mH)}{10\Omega} = 25.12$$

$$b.\quad Q_l \geq 10.\quad Therefore,$$

$$R_p \cong Q_l^2 R_l = (25.12)^2(10\Omega) = 6.31k\Omega$$

$$c.\quad Z_{T_p} = R_s // R_p = 40\ k\Omega // 6.31\ k\Omega = 5.45\ k\Omega$$

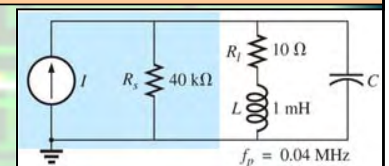


Figure 20.34 Example 20.8.

$$d.\quad Q_l \geq 10.\quad Therefore,$$

$$f_p \cong \frac{1}{2\pi\sqrt{LC}}$$

$$and\quad C = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2(0.04MHz)^2(1mH)} = 15.83\ nF$$

$$e.\quad Q_l \geq 10.\quad Therefore,$$

$$Q_p = \frac{Z_{T_p}}{X_L} = \frac{R_s // Q_l^2 R_l}{2\pi f_p L} = \frac{5.45k\Omega}{2\pi(0.04MHz)^2(1mH)} = 21.68$$

$$c.\quad f_1 = \frac{1}{4\pi C}\left[\frac{1}{R} - \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] = \frac{1}{4\pi(15.9mF)}\left[\frac{1}{5.45k\Omega} - \sqrt{\frac{1}{(5.45k\Omega)^2} + \frac{4(15.9mF)}{1mH}}\right] = 39\ kHz$$

$$f_2 = \frac{1}{4\pi C}\left[\frac{1}{R} + \sqrt{\frac{1}{R^2} + \frac{4C}{L}}\right] = 5.005\times 10^6[183.486\times 10^{-6} + 7.977\times 10^{-3}] = 40.84\ kHz$$

Ex. 20-10 Repeat Example 20.9, but ignore the effects of R_s, and compare results.

a. f_p is the same, 318.31 kHz.

b. For $R_s = \infty\ \Omega$

$$Q_p = Q_l = 100 \quad (versus\ 4.76)$$

$$c.\quad BW = \frac{f_p}{Q_p} = \frac{318.31kHz}{100} = 3.18\ kHz \quad (versus\ 66.87\ kHz)$$

$$d.\quad Z_{T_p} = R_p = 1\ M\Omega \quad (versus\ 47.62\ k\Omega)$$

$$V_p = IZ_{T_p} = (2mA)(1M\Omega) = 2000\ V \quad (versus\ 95.24\ V)$$

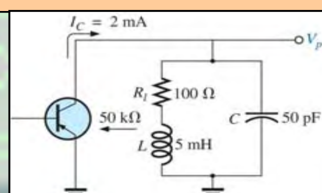


Figure 20.35 Example 20.9.

HW 20-13 For the "ideal" parallel resonant circuit in Fig. 20.52:

a. Determine the resonant frequency (f_p).

b. Find the voltage V_C at resonance.

c. Determine the currents I_L and I_C at resonance.

d. Find Q_p.

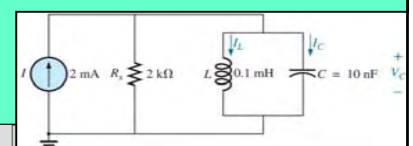
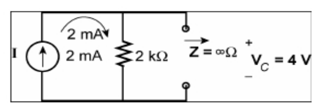


Figure 20.52 Problem 13.

$$a.\quad f_p = \frac{1}{2\pi\sqrt{LC}} = \frac{2}{2\pi\sqrt{(0.1\,mH)(10\,nF)}} = \mathbf{159.16\ kHz}$$

b.



$$d.\quad Q_p = \frac{R_s}{X_{L_p}} = \frac{2\,k\Omega}{2\pi f_p L} = \frac{2\,k\Omega}{100\,\Omega} = \mathbf{20}$$

$$c.\quad I_L = \frac{V_L}{X_L} = \frac{4\,V}{2\pi f_p L} = \frac{4\,V}{100\,\Omega} = \mathbf{40\ mA}$$

$$I_C = \frac{V_L}{X_C} = \frac{4\,V}{1/2\pi f_p C} = \frac{4\,V}{100\,\Omega} = \mathbf{40\ mA}$$

Homework 20: 13-21

EET1222/ET242 Circuit Analysis II

Transformers

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Transformers
- Mutual Inductance
- The Iron-Core Transformer
- Reflected Impedance and Power

Key Words: Transformer, Mutual Inductance, Coupling Coefficient, Reflected Impedanc

Transformers - Introduction

Mutual inductance is a phenomenon basic to the operation of the transformer, an electrical device used today in almost every field of electrical engineering. This device plays an integral part in power distribution systems and can be found in many electronic circuits and measuring instruments. In this module, we discuss three of the basic applications of a transformer: ***to build up or step down the voltage or current, to act as an impedance matching device, and to isolate one portion of a circuit from another***.

Transformers – Mutual Inductance

A transformer is constructed of two coils placed so that the changing flux developed by one links the other, as shown in Fig. 22.1. This results in an induced voltage across each coil. To distinguish between the coils, we apply the transformer convention that

the coil to which the source is applied is called the primary, and the coil to which the load is applied is called the secondary.

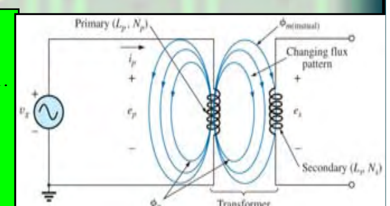


Figure 22.1 Defining the components of the transformer.

For the primary of the transformer in Fig.22.1, an application of Faraday's law result in

$$e_p = N_p \frac{d\phi_p}{dt} \quad (volts, V)$$

revealing that the voltage induced across the primary is directly related to the number of turns in the primary and the rate of change of magnetic flux linking the primary coil.

$$e_p = L_p \frac{di_p}{dt} \quad (volts, V) \qquad (22.2)$$

revealing that the induced voltage across the primary is also directly related to the self-inductance of the primary and rate of change of current through the primary winding. The magnitude of e_s, the voltage induced across the secondary, is determined by

$$e_p = N_s \frac{d\phi_m}{dt} \quad (volts, V)$$

Where Ns is the number of turns in the secondary winding and Φm is the portion of primary flux Φ_p that links the secondary, then $\Phi_m = \Phi_p$

and

$$e_s = N_s \frac{d\phi_p}{dt} \quad (volts, V)$$

The **coefficient of coupling (k)** between two coil is determined by

$$k(coefficient\ of\ coupling) = \frac{\phi_m}{\phi_p}$$

Since the maximum level of Φm is Φp, the coefficient of coupling between two coils can never be greater than 1.

The coefficient of coupling between various coils is indicated in Fig. 22.2. In Fig. 22.2(a), the ferromagnetic steel core ensures that most of the flux linking the primary also links the secondary, establishing a coupling coefficient very close to 1. In Fig. 22.2(b), the fact that both coils are overlapping results in the coil linking the other coil, with the result that the coefficient of coupling is again very close to 1. In Fig. 22.2(c), the absence of a ferromagnetic core results in low levels of flux linkage between the coils. For the secondary, we have

$$e_s = kN_s \frac{d\phi_p}{dt} \quad (volts, V)$$

The mutual inductance between the two coils in Fig. 22.1 is determined by

$$M = N_s \frac{d\phi_m}{di_p} \quad (henries, H) \quad or \quad M = N_p \frac{d\phi_p}{di_s} \quad (henries, H)$$

Note in the above equations that the symbol for mutual inductance is the capital letter M and that its unit of measurement, like that of self-inductance, is the *henry*.

mutual inductance between two coils is proportional to the instantaneous change in flux linking one coil due to an instantaneous change in current through the other coil.

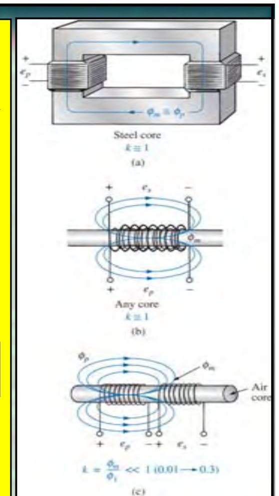


Figure 22.2 Windings having different coefficients of coupling.

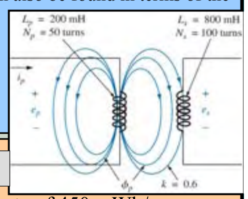
In terms of the inductance of each coil and the coefficient of coupling, the mutual inductance is determined by

$$M = k\sqrt{L_p L_s} \quad (henries, H)$$

The greater the coefficient of coupling, or the greater the inductance of either coil, the higher the mutual inductance between the coils. The secondary voltage e_s can also be found in terms of the mutual inductance if we rewrite Eq. (22.3) as

and, since $M = N_s(d\Phi_m/di_p)$, it can also be written

$$e_s = M \frac{di_p}{dt} \quad (volts, V) \quad and \quad e_p = M \frac{di_s}{dt} \quad (volts, V)$$



Ex. 22-1 For the transformer in Fig. 22.3:

a. Find the mutual inductance M.
b. Find the induced voltage e_p if the flux Φ_p changes at the rate of 450 mWb/s.
c. Find the induced voltage e_s for the same rate of change indicated in part (b).
d. Find the induced voltages e_p and e_s if the current i_p changes at the rate of 0.2 A/ms.

$$a.\quad M = k\sqrt{L_p L_s} = 0.6\sqrt{(200mH)(800mH)}$$
$$= 0.6\sqrt{16\times 10^{-2}} = 240\ mH$$
$$b.\quad e_p = N_p \frac{d\phi_p}{dt} = (50)(450mWb/s) = 22.5V$$

$$c.\quad e_s = kN_s \frac{d\phi_p}{dt} = (0.6)(100)(450mWb/s) = 27V$$
$$d.\quad e_p = L_p \frac{di_p}{dt} = (200mH)(0.2A/ms) = 40V$$
$$e_s = M \frac{di_p}{dt} = (240mH)(200A/s) = 48V$$

Transformers – The Iron-Core Transformer

An **iron-core transformer** under loaded conditions is shown in Fig. 22.4. The iron core will serve to increase the coefficient of coupling between the coils by increasing the mutual flux Φ_m.

The effective value of e_p is $E_p = 4.44fN_p\Phi_m$ which is an equation for the rms value of the voltage across the primary coil in terms of the frequency of the input current or voltage, the number turns of the primary, and the maximum value of the magnetic flux linking the primary.

The flux linking the secondary is $E_p = 4.44fN_p\Phi_m$

Dividing equations, we obtain

$$\frac{E_p}{E_s} = \frac{4.44fN_p\phi_m}{4.44fN_s\phi_m} = \frac{N_p}{N_s}$$

Revealing an important relationship for transformers:

The ratio of the magnitudes of the induced voltages is the same as the ratio of the corresponding turns.

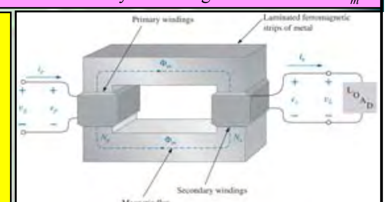


Figure 22.4 Iron-core transformer.

The ratio N_p / N_s, a, is referred to as the transformation ratio: $a = \frac{N_p}{N_s}$

*If $a < 1$, the transformer is called a **step-up transformer** and if $a > 1$, the transformer is called a **step-down transformer**.*

Ex. 22-2 For the iron-core transformer in Fig. 22.5:

a. Find the maximum flux Φ_m.

b. Find the secondary turn N_s.

$$a.\quad E_p = 4.44 N_p f \phi_m \quad Therfore,$$

$$\phi_m = \frac{E_p}{4.44 N_p f} = \frac{200V}{(4.44)(50t)(60Hz)} = 15.02\, mWb$$

$$b.\quad \frac{E_p}{E_s} = \frac{N_p}{N_s} \quad Therefore,$$

$$N_s = \frac{N_p E_s}{E_p} = \frac{(50t)(2400V)}{200V} = 600\ turns$$

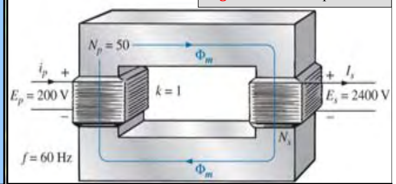


Figure 22.5 Example 22.2.

The induced voltage across the secondary of the transformer in Fig. 22.4 establish a current i_s through the load Z_L and the secondary windings. This current and the turns N_s develop an mmf $N_s i_s$ that are not present under no-load conditions since $i_s = 0$ and $N_s i_s = 0$.

Since the instantaneous values of ip and is are related by the turns ratio, the phasor quantities I_p and I_s are also related by the same ratio:

$$N_p I_p = N_s I_s \quad or \quad \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

The primary and secondary currents of a transformer are therefore related by the inverse ratios of the turns.

Transformers – Reflected Impedance and Power

In previous section we found that

$$\frac{V_g}{V_L} = \frac{N_p}{N_s} = a \quad and \quad \frac{I_p}{I_s} = \frac{N_s}{N_p} = \frac{1}{a}$$

Dividing the first by the second, we have

$$\frac{V_g/V_L}{I_p/I_s} = \frac{a}{1/a} \quad or \quad \frac{V_g/I_p}{V_L/I_s} = a^2 \quad and \quad \frac{V_g}{I_p} = a^2 \frac{V_L}{I_s}$$

However, since

$$Z_p = \frac{V_g}{I_p} \quad and \quad Z_L = \frac{V_L}{I_s}$$

$$then \quad Z_p = a^2 Z_L$$

That is, the impedance of the primary circuit of an ideal transformer is the transformation ratio squared times the impedance of the load. Note that if the load is capacitive or inductive, the **reflected impedance** is also capacitive or inductive. For the ideal iron-core transformer,

$$\frac{E_p}{E_s} = a = \frac{I_s}{I_p} \quad or \quad E_p I_p = E_s I_s$$

$$and \quad P_{in} = P_{out} \quad (ideal\ condition)$$

Ex. 22-3 For the iron-core transformer in Fig. 22.6:

a. Find the magnitude of the current in the primary and the impressed voltage across the primary.

b. Find the input resistance of the transformer.

$$a.\quad \frac{I_p}{I_s} = \frac{N_s}{N_p}$$

$$I_p = \frac{N_s}{N_p} I_s = \left(\frac{5t}{40t}\right)(0.1A) = 12.5\ mA$$

$$V_L = I_s Z_L = (0.1A)(2k\Omega) = 200\ V$$

$$also \quad \frac{V_g}{V_L} = \frac{N_p}{N_s}$$

$$V_g = \frac{N_p}{N_s} V_L = \left(\frac{40t}{5t}\right)(200V) = 1600\, V$$

$$b.\quad Z_p = a^2 Z_L$$

$$a = \frac{N_p}{N_s} = 8$$

$$Z_p = (8)^2(2\,k\Omega) = R_p = 128\ k\Omega$$

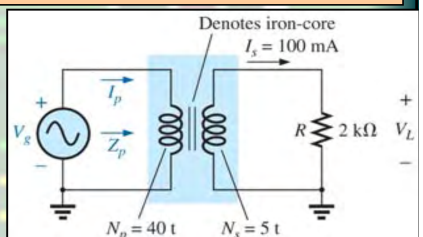


Figure 22.6 Example 22.3.

HW 12-12

a. If N_p = 400 V, V_s = 1200, and V_g = 100 V, find the magnitude of I_p for the iron-core transformer in Fig. 22.58 if $Z_L = 9\ \Omega + j12\ \Omega$.

b. Find the magnitude of the voltage V_L and the current I_L for the conditions of part (a).

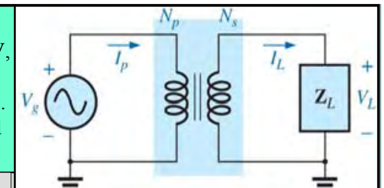


Figure 22.12 Problem 12.

$$a.\quad a = \frac{N_p}{N_s} = \frac{400t}{1200t} = \frac{1}{3}$$

$$Z_i = a^2 Z_L = \left(\frac{1}{3}\right)^2 (9\Omega + j12\Omega) = 1\Omega + j1.333\Omega = 1.667\Omega\angle 53.13°$$

$$I_p = \frac{V_g}{Z_i} = \frac{100\ V}{1.667\,\Omega} = 60\ A$$

$$b.\quad I_L = aI_p = \frac{1}{3}(60A) = 20A$$

$$V_L = I_L Z_L = (20A)(15\ \Omega) = 300\ V$$

Homework 22: 1-3,4,8,12