EET1122/ET162 Circuit Analysis

Introduction

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on “Introduction to Circuit Analysis” by Robert Boylestad, Prentice Hall, 10th edition.

Acknowledgement

I want to express my gratitude to Prentice Hall giving me the permission to use instructor’s material for developing this module. I would like to thank the Department of Electrical and Telecommunications Engineering Technology of NYCCT for giving me support to commence and complete this module. I hope this module is helpful to enhance our students’ academic performance.

Sunghoon Jang

OUTLINES

- Introduction to Electrical Engineering
- A Brief History
- Units of Measurement
- Systems of Units
- Operation of a Scientific Calculator
- Significant Figures

Key Words: Electrical Engineering, Units, Powers, Calculator

Introduction – The Electrical/Electronics Engineering

The growing sensitivity to the technologies on Wall Street is clear evidence that the electrical/electronics industry is one that will have a sweeping impact on future development in a wide range of areas that affect our life style, general health, and capabilities.

- **Semiconductor Device**
- **Analog & Digital Signal Processing**
- **Telecommunications**
- **Biomedical Engineering**
- **Fiber Optics & Opto-Electronics**
- **Integrated Circuit (IC)**

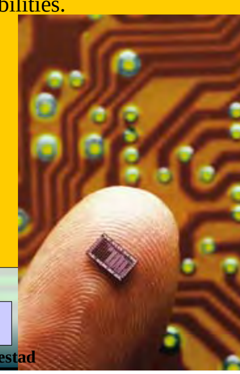


Figure 1.1 Computer chip on finger. (Courtesy of Intel Corp.)

Introduction – A Brief History

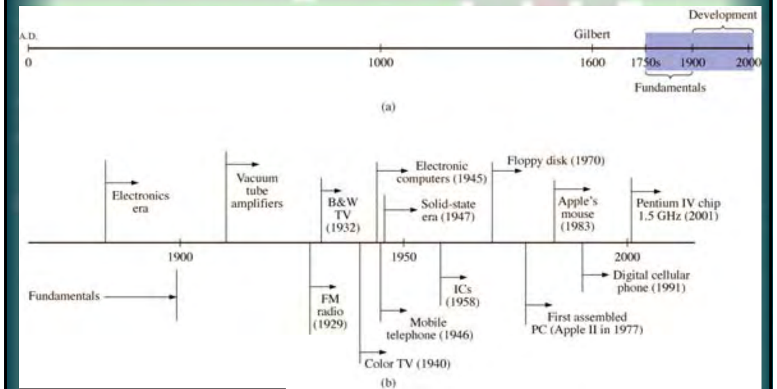


FIGURE 1.2 Time charts: (a) long-range; (b) expanded.

Units of Measurement

The numerical value substituted into an equation must have The unit of measurement specified by the equation

Examples

1 mi = 5280 ft
4000 ft = 0.7576 mi
1 min = 0.0167 h

$$v = \frac{d}{t} = \frac{0.7576\,mi}{0.0167\,h} = 45\ 73\,mi/h$$

Systems of Units

The **English** system is based on a single standard, the **metric** is subdivided into two interrelated standards: the **MKS** and the **CGS**.

English	**Metric**
Length: Yard (yd)	Length: Meter (m)
Mass: Slug	Mass: Kilogram (kg)
Force: Pound	Force: Newton (N)
Temperature: Fahrenheit (°F)	Temperature: Kelvin (K)
Energy: Foot-pound (ft-lb)	Energy: Joule (J)
Time: Seconds (s)	Time: Seconds (s)

Systems of Units

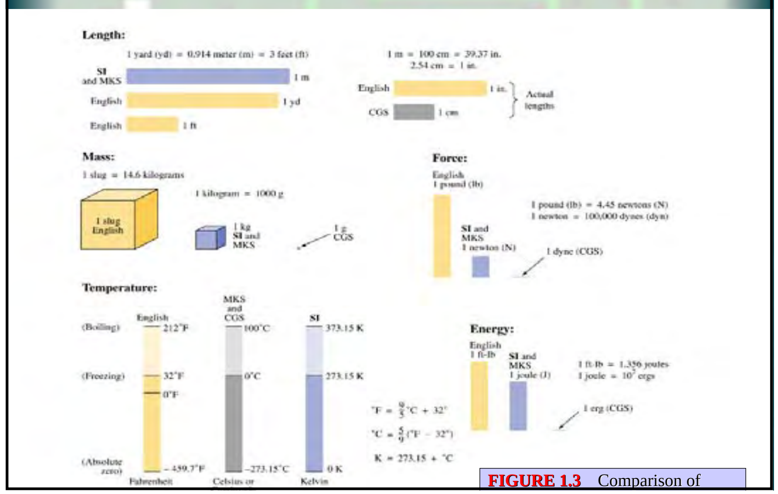


FIGURE 1.3 Comparison of

Significant Figures, Accuracy, and Rounding off

*In the **addition** or **subtraction** of approximate numbers, the entry with the lowest level of accuracy determines the format of the solution.*
*For the **multiplication** and **division** of approximate numbers, the result has the same number of significant figures as the number with the latest number of significant figures.*

Ex. 1-1 Perform the indicated operations with the following approximate numbers and round off to the appropriate level of accuracy.

a. 532.6 + 4.02 + 0.036 = 536.656 ≈ 536.7
b. 0.04 + 0.003 + 0.0064 = 0.0494 ≈ 0.05
c. 4.632 × 2.4 = 11.1168 ≈ 11
d. 3.051 × 802 = 2446.902 ≈ 2450
e. 1402/6.4 = 219.0625 ≈ 220
f. 0.0046/0.05 = 0.0920 ≈ 0.09

Powers of Ten

$1 = 10^0$
$10 = 10^1$
$100 = 10^2$
$1000 = 10^3$

$1/10 = 0.1 = 10^{-1}$
$1/100 = 0.01 = 10^{-2}$
$1/1000 = 0.001 = 10^{-3}$
$1/10{,}000 = 0.0001 = 10^{-4}$

Ex. 1-2

$$a.\ \frac{1}{1000} = \frac{1}{10^{+3}} = 10^{-3}$$

$$b.\ \frac{1}{0.00001} = \frac{1}{10^{-5}} = 10^{+5}$$

Ex. 1-3

a. $(1000)(10{,}000) = (10^3)(10^4) = 10^{(3+4)} = 10^7$
b. $(0.00001)(100) = (10^{-5})(10^2) = 10^{(-5+2)} = 10^{-3}$

Ex. 1-4

$$a.\ \frac{100000}{100} = \frac{10^5}{10^2} = 10^{(5-2)} = 10^3$$

$$b.\ \frac{1000}{0.0001} = \frac{10^3}{10^{-4}} = 10^{(3-(-4))} = 10^{(3+4)} = 10^7$$

Ex. 1-5

a. $(100)^4 = (10^2)^4 = 10^{(2)(4)} = 10^8$
b. $(1000)^{-2} = (10^3)^{-2} = 10^{(3)(-2)} = 10^{-6}$
c. $(0.01)^{-3} = (10^{-2})^{-3} = 10^{(-2)(-3)} = 10^6$

Basic Arithmetic Operations

When adding or subtracting numbers in a powers-of-ten format, be sure that the power of ten is the same for each number. Then separate the multipliers, perform the required operation, and apply the same power of ten to the results.

Ex. 1-6

a. 6300 + 75,000 $= (6.3)(1000) + (75)(1000)$
$= 6.3 \times 10^3 + 75 \times 10^3$
$= (6.3 + 75) \times 10^3$
$= 81.3 \times 10^3$

a. 0.00096 – 0.000086 $= (96)(0.00001) - (8.6)(0.00001)$
$= 96 \times 10^{-5} - 8.6 \times 10^{-5}$
$= (96 - 8.6) \times 10^{-5}$
$= 87.4 \times 10^{-5}$

Ex. 1-7

a. **(0.0002)(0.000007)** $= [(2)(0.0001)]\,[(7)(0.000001)]$
$= (2 \times 10^{-4})(7 \times 10^{-6})$
$= (2)(7) \times (10^{-4})(10^{-6})$
$= 14 \times 10^{-10}$

a. **(340,000)(0.00061)** $= (3.4 \times 10^{5})(61 \times 10^{-5})$
$= (3.4)(61) \times (10^{5})(10^{-5})$
$= 207.4 \times 10^{0}$
$= 207.4$

Ex. 1-8

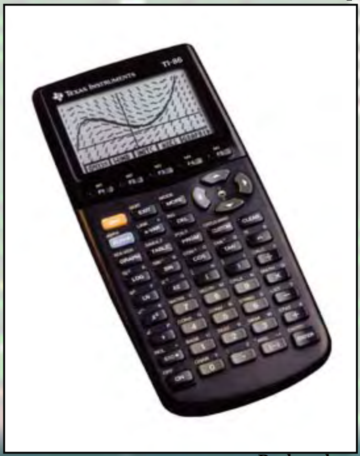
$$a.\ \frac{0.00047}{0.002} = \frac{47\times10^{-5}}{2\times10^{-3}} = \left(\frac{47}{2}\right)\times\left(\frac{10^{-5}}{10^{-3}}\right) = 23.5\times10^{-2}$$

$$b.\ \frac{690{,}000}{0.00000013} = \frac{69\times10^{4}}{13\times10^{-8}} = \left(\frac{69}{13}\right)\times\left(\frac{10^{4}}{10^{-8}}\right) = 5.31\times10^{12}$$

Ex. 1-9

a. **(0.00003)³** $= (3 \times 10^{-5})^3 = (3 \times 10^{-5})^3 = (3)^3 \times 10^{-15}$
b. **(90,800,000)²** $= (9.08 \times 10^{7})^2 = (9.08)^2 \times (10^{7})^2$
$= 82.4464 \times 10^{14}$

Calculators and Order of Operation



HW 1-26 Perform the following conversions:

a. 1.5 min to seconds
b. 0.04 h to seconds
c. 0.05 s to microseconds
d. 0.16 m to millimeters
e. 0.00000012 s to nanoseconds
f. 3,620,000 s to days

Homework 1: 12, 14, 24, 26, 41, 42, 43

a. $1.5\ \cancel{\text{min}}\left[\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right] = \mathbf{90\ s}$

b. $0.04\ \cancel{\text{h}}\left[\frac{60\ \cancel{\text{min}}}{1\ \cancel{\text{h}}}\right]\left[\frac{60\ \text{s}}{1\ \cancel{\text{min}}}\right] = \mathbf{144\ s}$

c. $0.05\ \cancel{\text{s}}\left[\frac{1\ \mu\text{s}}{10^{-6}\ \cancel{\text{s}}}\right] = \mathbf{0.05 \times 10^{6}\ \mu s = 50 \times 10^{3}\ \mu s}$

d. $0.16\ \cancel{\text{m}}\left[\frac{1\ \text{mm}}{10^{-3}\ \cancel{\text{m}}}\right] = 0.16 \times 10^{3}\ \text{mm} = \mathbf{160\ mm}$

e. $1.2 \times 10^{-7}\ \cancel{\text{s}}\left[\frac{1\ \text{ns}}{10^{-9}\ \cancel{\text{s}}}\right] = 1.2 \times 10^{2}\ \text{ns} = \mathbf{120\ ns}$

f. $3.62 \times 10^{6}\ \cancel{\text{s}}\left[\frac{1\ \cancel{\text{min}}}{60\ \cancel{\text{s}}}\right]\left[\frac{1\ \cancel{\text{h}}}{60\ \cancel{\text{min}}}\right]\left[\frac{1\ \text{day}}{24\ \cancel{\text{h}}}\right] = \mathbf{41.90\ days}$

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Current and Voltage

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Engineering Technology Department

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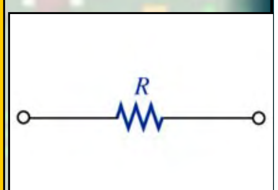
OUTLINES

- Resistance and Conductance
- Ohmmeters
- Current and Voltage
- Ammeters and Voltmeters

Key Words: Resistance, Ohmmeter, Current, Voltage, Ammeter, Voltmeter

Introduction to Resistance

The flow of charge through any material encounters an opposing force similar in many aspect to mechanical friction. This opposition, due to the collisions between electrons and other atoms in the material, *which converts electrical energy into another form of energy such as heat*, is called the **resistance** of the material. The unit of measurement of resistance is the ohm (Ω).



At a fixed temperature of 20°C (room temperature), the resistance is related to the other three factor by

(ohms, Ω)

ρ : resistivity of the sample (CM-ohms/ft at T=20° C)
l : the length of the sample (feet)
A : cross-sectional area of the sample (circular mils (CM))

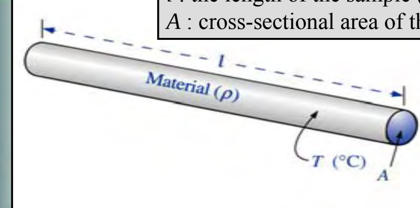


FIGURE 1.2 Factors affecting the resistance of a conductor.

Resistance: Circular Wires

For two wires of the same physical size at the same temperature,

- *the higher the resistivity (ρ), the more the resistance*
- *the longer the length of a conductor, the more the resistance*
- *the smaller the area of a conductor, the more the resistance*
- *the higher the temperature of a conductor, the more the resistance*

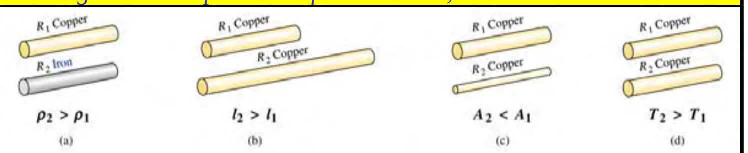


FIGURE 1.3 Cases in which $R_2 > R_1$. For each case, all remaining parameters that control the resistance level are the same.

Types of Resistors – Fixed Resistors

Resistors are made in many forms, but all belong in either of two groups: **fixed** or **variable**. The most common of the low-wattage, fixed-type resistors is the molded carbon composition resistor.

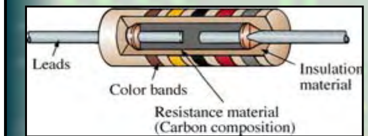


FIGURE 1.3 Fixed composition resistor.

The relative sizes of all fixed and variable resistors change with the **power rating**, increasing in size for increased power ratings in order to withstand the higher currents and dissipation losses.

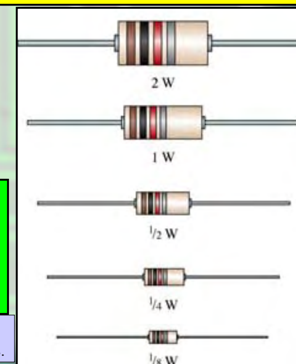


FIGURE 1.4 Fixed composition resistors of different wattage ratings.

Types of Resistors – Variable Resistors

Variable resistors have resistance that can be varied by turning a dial, knob, screw, or whatever seems appropriate for the application.

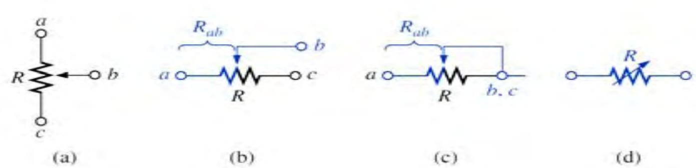


FIGURE 1.5 Potentiometer: (a) symbol: (b) & (c) rheostat connections; (d) rheostat symbol.

Color Coding and Standard Resistor Values

A whole variety of resistors are large enough to have their resistance in ohms printed on the casing. However, some are too small to have numbers printed on them, so a system of **color coding** is used.

FIGURE 1.6 Color coding of fixed molded composition resistor.

The first and second bands represent the first and second digits, respectively. The third band determines the power-of-ten multiplier for the first two digits. The fourth band is the manufacture's tolerance. The fifth band is a reliability factor, which gives the percentage of failure per 1000 hours of use.

Band 1-2	Band 3	Band 4	Band 5
0 Black	10^0	5% Gold	1% Brown
1 Brown	10^1	10% Silver	0.1% Red
2 Red	10^2	20% No band	0.01% Orange
3 Orange	10^3		0.01% Yellow
4 Yellow	10^4		
5 Green	10^5		
6 Blue	10^6		
7 Violet	10^7		
8 Gray	10^8		
9 White	10^9		

Table 1 Resistor color coding

Ex. 1-1 Find the range in which a resistor having the following color bands must exist to satisfy the manufacturer's tolerance:

b.	1st Band	2nd Band	3rd Band	4th Band	5th Band
	Orange	**White**	**Gold**	**Silver**	**No color**
	3	**9**	**$10^{-1} = 0.1$**	**±10%**	

a. 82Ω ± 5% (1% reliability)

Since 5% of 82 = 4.10, the resistor should be within the range of 82Ω ± 4.10Ω, or between 77.90 and 86.10Ω.

b. 3.9Ω ± 10% = 3.9Ω ± 0.39Ω

The resistor should be somewhere between 3.51 and 4.29Ω.

Conductance

The quantity of how well the material will conduct electricity is called **conductance** (S).

$$G = \frac{1}{R} \quad \text{(siemens, S)}$$

$$G = \frac{A}{\rho \cdot l} \quad \text{(S)}$$

Indicating that increasing the area or decreasing either the length or the resistivity will increase the **Conductance**.

Ex. 1-2 What is the relative increase or decrease in conductivity of a conductor if the area is reduced by 30% and the length is increased by 40%? The resistivity is fixed.

$$G = \frac{A_i}{\rho_i l_i} \quad \text{(siemens, S)}$$

with the subscript *i* for the initial value. Using the subscript *n* for new value :

$$G_n = \frac{A_n}{\rho_n l_n} = \frac{0.70\,A_i}{\rho_i(1.4\,l_i)} = \frac{0.70}{1.4}\frac{A_i}{\rho_i l_i} = \frac{0.70}{1.4\,G_i} = 0.5\,G_i$$

Ohmmeters

The ohmmeter is an instrument used to perform the following tasks and several other useful functions.

1. Measure the resistance of individual or combined elements
2. Direct open-circuit (high-resistance) and short-circuit (low-resistance) situations
3. Check continuity of network connections and identify wires of a multi-lead cable
4. Test some semiconductor devices

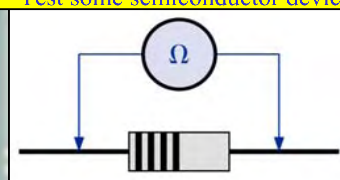


FIGURE 1.7 Measuring the resistance of a single element.

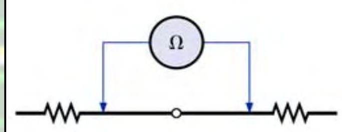
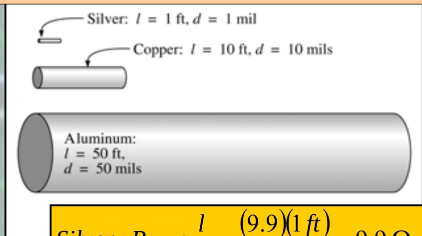


FIGURE 1.8 Checking the continuity of a connection.

Ex 1-3 In Figure, three conductors of different materials are presented.

a. Without working out the numerical solution, determine which section would appear to have the most resistance. Explain.

b. Find the resistance of each section and compare with the result of (a) (T = 20°C)



a. $R_{silver} > R_{copper} > R_{aluminum}$

$$Silver: R = \rho\frac{l}{A} = \frac{(9.9)(1\,ft)}{1CM} = 9.9\,\Omega$$

$$Copper: R = \rho\frac{l}{A} = \frac{(10.37)(10\,ft)}{100CM} = 1.037\,\Omega$$

$$Alu\min um: R = \rho\frac{l}{A} = \frac{(17)(50\,ft)}{2500CM} = 0.34\,\Omega$$

Voltage

The **voltage** across an element is the work (energy) required to move a unit positive charge from the − terminal to the + terminal. The unit of voltage is the volt, **V**.

*A **potential difference** of 1 volt (V) exists between two points if 1 joul (J) of energy is exchanged in moving 1 coulomb (C) of charge between the two points.*

In general, the potential difference between two points is determined by:

$$V = \frac{W}{Q}$$

V = voltage (V)
Q = coulombs (C)
W = potential energy (J)

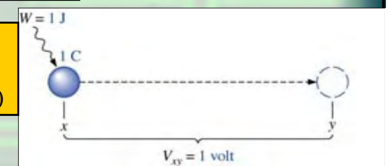


FIGURE 1.9 Defining the unit of measurement for voltage.

Ex. 1-4 Find the potential difference between two points in an electrical system if 60 J of energy are expended by a charge of 20 C between these two points.

$$V = \frac{W}{Q} = \frac{60\ J}{20\ C} = 3\ V$$

Ex. 1-5 Determine the energy expended moving a charge of 50 μC through a potential difference of 6 V.

$$W = Q \cdot V = \left(50 \times 10^{-6}\right)\left(6\ V\right)$$
$$= 300 \times 10^{-6}\ J = 300\ \mu J$$

Fixed (dc) Supplies

The terminology **dc** is an *abbreviation* for **direct current**, which encompasses the various electrical systems in which there is a unidirectional ("one direction") flow of charge.

DC Voltage Sources

Dc voltage sources can be divided into three broad categories: (1) **Batteries** (chemical action), (2) **generators** (electro-mechanical), and (3) **power supplies** (rectification).

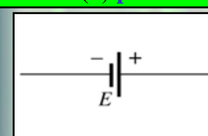


FIGURE 1.10 Symbol for a dc voltage source.

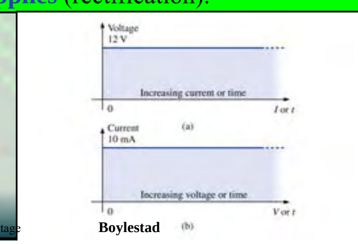


FIGURE 1.11 Terminal characteristics: (a) ideal voltage source; (b) ideal current source.

Current

The electrical effects caused by charges in motion depend on the rate of charge flow. The rate of charge flow is known as the electrical current. *With no external forces applied, the net flow of charge in a conductor in any direction is zero.*

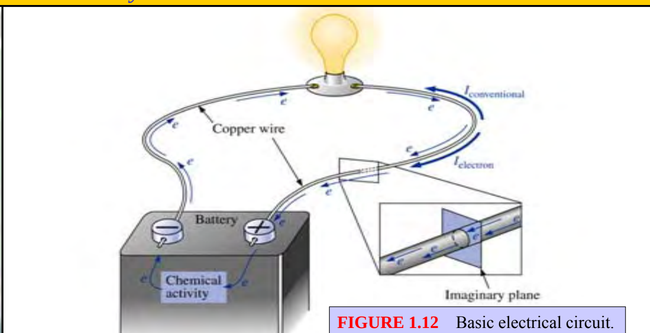


FIGURE 1.12 Basic electrical circuit.

If 6.242×10^{18} electrons (1 coulomb) pass through the imaginary plane in Fig. 2.9 in 1 second, the flow of charge, or current, is said to be 1 ampere (A).

$$Charge / electron = Q_e = \frac{1C}{6.242 \times 10^{18}} = 1.6 \times 10^{-19} C$$

The current in amperes can now be calculated using the following equation:

$$I = \frac{Q}{t}$$

I = amperes (A)
Q = coulombs (C)
t = seconds (s)

$$Q = I \cdot t \quad (coulomb, C)$$

and

$$t = \frac{Q}{I} \quad (seconds, s)$$

Ex. 1-6 The charge flowing through the imaginary surface of Fig. 1-12 is 0.16 C every 64 ms. Determine the current in ampere.

$$I = \frac{Q}{t} = \frac{0.16C}{64 \times 10^{-3} s} = \frac{160 \times 10^{-3} C}{64 \times 10^{-3} s} = 2.50\ A$$

Ex. 1-7 Determine the time required for 4×10^{16} electrons to pass through the imaginary surface of Fig. 1.12 if the current is 5 mA.

$$Q = 4 \times 10^{16} electron\left(\frac{1C}{6.242 \times 10^{18} electrons}\right) = 0.641 \times 10^{-2} C = 0.00641\, C = 6.41\, mC$$

$$t = \frac{Q}{I} = \frac{6.41 \times 10^{-3} C}{5 \times 10^{-3} A} = 1.282\ s$$

Ammeters and Voltmeters

It is important to be able to measure the current and voltage levels of an operating electrical system to check its operation, isolate malfunctions, and investigate effects. **Ammeters** are used to measure current levels while **voltmeters** are used to measure the potential difference between two points.

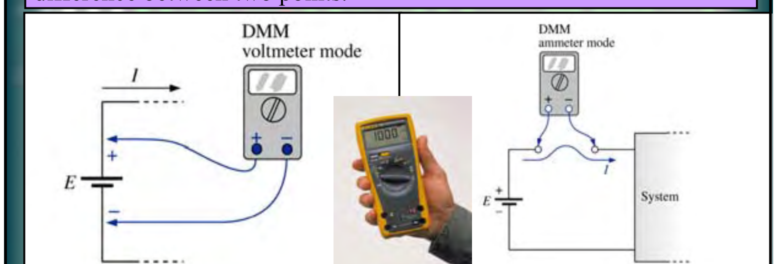


FIGURE 1.13 Voltmeter and ammeter connection for an up-scale (+) reading.

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Ohm's Law

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OUTLINES

- Introduction to Ohm's Law
- Plotting Ohm's Law
- Power

Key Words: Ohm's Law, Current, Voltage, Power

Introduction to Ohm's Law

Ohm's law clearly reveals that a fixed resistance, the greater the voltage across a resistor, the more the current, the more the resistance for the same voltage, the less the current.

$$I = \frac{E}{R} \quad (amperes, A)$$

$$E = I\,R \quad (volts, V)$$

$$R = \frac{E}{I} \quad (ohms, \Omega)$$

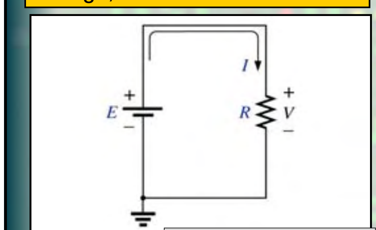
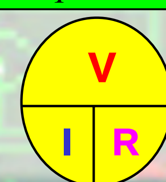


Figure 4.1 Basic Circuit.



Ex. 4-1 Determine the current resulting from the application of a 9-V battery across a network with a resistance of 2.2 Ω.

$$I = \frac{E}{R} = \frac{9\ V}{2.2\ \Omega} = 4.09\ A$$

Ex. 4-2 Calculate the resistance of a 60-W bulb if a current of 500 mA results from an applied voltage of 120 V.

$$R = \frac{E}{I} = \frac{120\ V}{500 \times 10^{-3}\ A} = 240\ \Omega$$

For an isolated resistive element, the polarity of the voltage drop is as shown in Fig. 4.2(a) for the indicated current direction. A reversal in current will reverse the polarity, as shown in Fig. 4.2(b). In general, the flow of charge is from a high (+) to a low (–) potential.

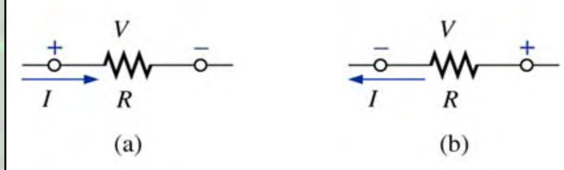


FIGURE 4.2 Defining polarities.

Ex. 4-3 Calculate the current through the 2-kΩ resistor of Fig. 4.3 if the voltage drop across it is 16 V.

$$I = \frac{E}{R} = \frac{16\ V}{2 \times 10^{3}\ \Omega} = 8\ mA$$

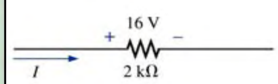


FIGURE 4.3 Example 4.3

Ex. 4-4 Calculate the voltage that must be applied across the soldering iron of Fig. 4.5 to establish a current of 1.5 A through the iron if its internal resistance is 80 Ω.

$$E = I \cdot R = (1.5\ A)(80\ \Omega) = 120\ V$$

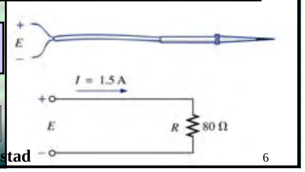


FIGURE 4.4 Example 4.4

Plotting Ohm's Law

Graph, characteristics, plots play an important role in every technical field as a mode through which the broad picture of the behavior or response of a system can be conveniently displayed. It is therefore critical to develop the skills necessary both to read data and to plot them in such a manner that they can be interpreted easily.

For most sets of characteristics of electronic devices, the current is represented by the vertical axis, and the voltage by the horizontal axis, as shown in Fig. 4.5.

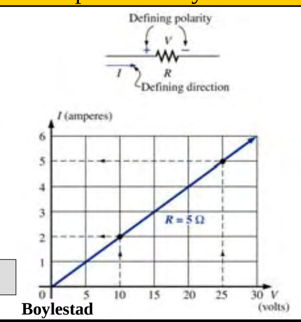


FIGURE 4.5 Plotting Ohm's law

If the resistance of a plot is unknown, it can be determined at any point on the plot since a straight line indicates a fixed resistance. At any point on the plot, find the resulting current and voltage, and simply substitute into following equation:

$$R_{dc} = \frac{V}{I}$$

The equation states that by choosing a particular ΔV, one can obtain the corresponding ΔI from the graph, as shown in Fig. 4.6 and 4.7, and then determine the resistance.

$$R = \frac{\Delta V}{\Delta I}$$

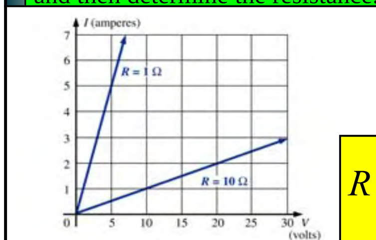


FIGURE 4.6 Demonstrating on an I-V plot that the less the resistance, the steeper is the slope

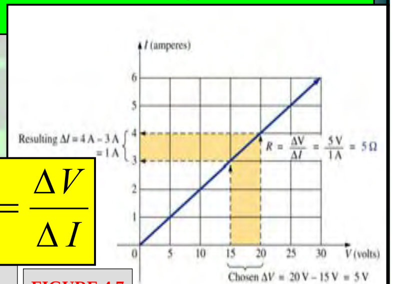


FIGURE 4.7

Ex. 4-5 Determine the resistance associated with the curve of Fig. 4.8 using equations from previous slide, and compare results.

$$At \quad V = 6\,V,\ I = 3\,mA,\ and$$

$$R_{dc} = \frac{V}{I} = \frac{6\,V}{3\,mA} = 2\ k\Omega$$

$$At\ the\ interval\ between\ 6\,V\ and\ 8\,V,$$

$$R = \frac{\Delta V}{\Delta I} = \frac{2\,V}{1\,mA} = 2\ k\Omega$$

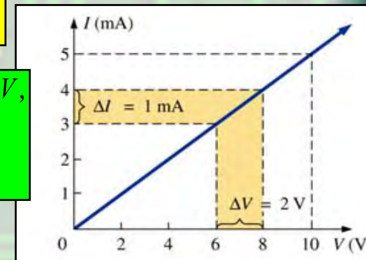


FIGURE 4.8 Example 4.5.

Power

Power is an indication of how much work can be done in a specified amount of time, that is, a *rate* of doing work. Since converted energy is measured in *joules* (J) and time in seconds (s), power is measured in joules/second (J/s). The electrical unit of measurement for power is the watt (W), defined by

1 watt (W) = 1 joules/second (J/s)

$$P = \frac{W}{t} \quad (watts,\ W,\ or\ joules/second,\ J/s)$$

1 horsepower ≈ 746 watts

$$P = \frac{W}{t} = \frac{Q \cdot V}{t} = V\frac{Q}{t} = V\,I\ (watts) \quad where\ I = \frac{Q}{t}$$

Eq. 1

Eq. 2

$$P = V \cdot I = V\left(\frac{V}{R}\right) = \frac{V^2}{R} \quad (watts)$$

Eq. 3

$$P = V \cdot I = (I \cdot R)I = I^2 R \quad (watts)$$

The result is that the power absorbed by the resistor of Fig. 4.9 can be found directly depends on the information available.

Power can be delivered or absorbed as defined by the polarity of the voltage and direction of the current. For all dc voltage sources, power is being delivered by the source if the current has the direction appearing in Fig. 4.10 (a).

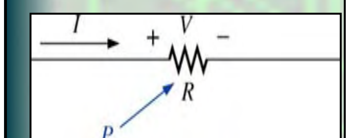


FIGURE 4.9 Defining the power to a resistive element.

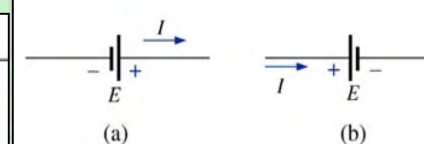


FIGURE 4.10 Battery power: (a) supplied; (b) absorbed.

Ex. 4-6 Find the power delivered to the dc motor of Fig. 4.11.

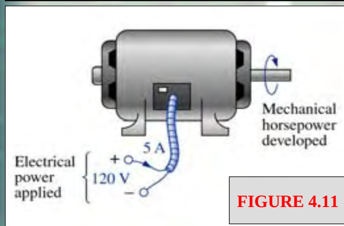


FIGURE 4.11 Example 4.6.

$$P = V\,I = (120\,V)(5\,A) = 600\,W = 0.6\,kW$$

Ex. 4-7 What is the power dissipated by a 5-Ω resistor if the current is 4 A?

$$P = I^2R = (4\ A)^2(5\ \Omega) = 80\ W$$

Ex. 4-8 The I-V characteristics of a light bulb are powered in Fig. 4.12. Note the nonlinearity of the curve, indicating a wide range in resistance of the bulb with applied voltage as defined by the earlier discussion. If the rated voltage is 120 V, find the wattage rating of the bulb. Also calculate the resistance of the bulb under rated conditions.

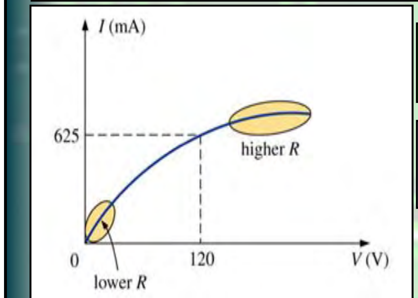


FIGURE 4.12 The nonlinear I-V characteristics of a 75-W light bulb.

$$At\ 120V, \quad I = 0.625\,A$$
$$P = V\,I = (120V)(0.0625\,A) = 75W$$

$$At\ 120V, \quad R = \frac{V}{I} = \frac{120V}{0.625\,A} = 192\,\Omega$$

Sometimes the power is given and the current or voltage must be determined.

$$P = I^2R \Rightarrow I^2 = \frac{P}{R} \quad or \quad I = \sqrt{\frac{P}{R}} \quad (ampere)$$

$$P = \frac{V^2}{R} \Rightarrow V^2 = PR \quad or \quad V = \sqrt{PR} \quad (volts)$$

Ex. 4-9 Determine the current through a 5-kΩ resistor when the power dissipated by the element is 20 mW.

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{20\times10^{-3}\,W}{5\times10^{3}\,\Omega}} = \sqrt{4\times10^{-6}} = 2\times10^{-3}\,A = 2\ mA$$

HW 4-52 A stereo system draws 2.4 A at 120 V. The audio output power is 50 W.

a. How much power is lost in the form of heat in the system?
b. What is the efficiency of the system?

$$a.\quad P_i = EI = (120\,V)(2.4\,A) = 288\,W$$
$$P_i = P_o + P_{lost}, \quad P_{lost} = P_i - P_0 = 288\,W - 50\,W = 238\,W$$
$$b.\quad \eta\% = \frac{P_0}{P_i} = 100\% = \frac{50\,W}{288\,W}\times 100\% = 17.36\%$$

Homework 4: 2, 4, 6, 8, 20, 24, 25, 26, 49, 52

EET1122/ET162 Circuit Analysis

Series Circuits

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 10th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Series Circuits
- Kirchhoff's Voltage Law
- Voltage Divider Rule
- Interchanging Series Elements
- Series Circuits – Notation
- Ideal dc Voltage Sources vs. Non-ideal Sources
- Voltage Regulation

Key Words: Series Circuit, Kirchhoff's Voltage Law, Voltage Divider Rule

Series Circuits - Introduction

Two types of current are available to the consumer today. One is **direct current (dc),** in which ideally the flow of charge (current) does not change in magnitude with time. The other is sinusoidal **alternating current (ac),** in which the flow of charge is continually changing in magnitude with time.

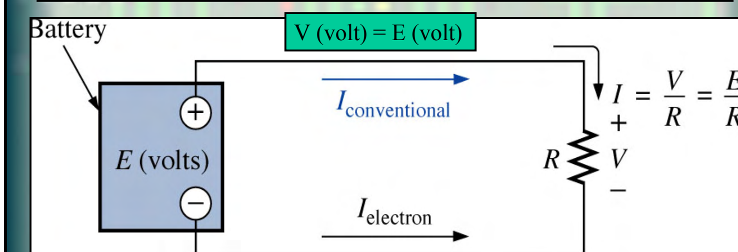


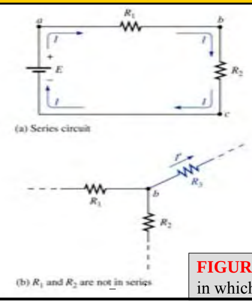
FIGURE 5.1 Introducing the basic components of an electric circuit.

Series Circuits

A circuit consists of any number of elements joined at terminal points, providing at least one closed path through which charge can flow.

Two elements are in series if

1. They have only one terminal in common
2. The common point between the two points is not connected to another current-carrying element.



In Fig. 5.2(a), the resistors R_1 and R_2 are in series because they have only point b in common.

The current is the same through series elements.

The total resistance of a series circuit is the sum of the resistance levels

FIGURE 5.2 (a) Series circuit; (b) situation in which R_1 and R_2 are not in series.

The total resistance of a series circuit is the sum of the resistance levels. In general, to find the total resistance of N resistors in series, the following equation is applied:

$R_T = R_1 + R_2 + R_3 + \cdots + R_N$ (ohms, Ω)

$I_s = \dfrac{E}{R_T}$ (amperes, A)

$V_1 = IR_1, V_2 = IR_2, V_3 = IR_3, \cdots V_N = IR_N$ (volts, V)

$P_1 = V_1 I_1 = I_1^2 R_1 = \dfrac{V_1^2}{R_1}$ (watts, W)

$P_{del} = EI$ (watts, W)

$P_{del} = P_1 + P_2 + P_3 + \cdots + P_N$

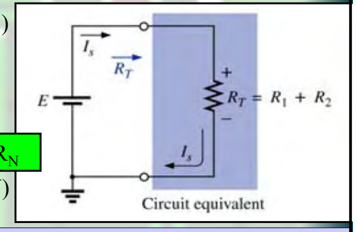


FIGURE 5.3 Replacing the series resistors R_1 and R_2 of Fig. 5.2 (a) with the total resistance.

The total power delivered to a resistive circuit is equal to the total power dissipated by resistive elements.

Ex. 5-1 a. Find the total resistance for the series circuit in Figure 5.4.
b. Calculate the source current I_s.
c. Calculate the voltages V_1, V_2, and V_3.
d. Calculate the power dissipated by R_1, R_2, and R_3.
e. Determine the power delivered by the source, and compare it to the sum of the power levels of part (b).

(a) $R_T = R_1 + R_2 + R_3 = 2\Omega + 1\Omega + 5\Omega = 8\Omega$

$I_s = \dfrac{E}{R_T} = \dfrac{20V}{8\Omega} = 2.5A$

(c) $V_1 = IR_1 = (2.5A)(2\Omega) = 5V$
$V_2 = IR_2 = (2.5A)(1\Omega) = 2.5V$
$V_3 = IR_3 = (2.5A)(5\Omega) = 12.5V$

(d) $P_1 = V_1 I_s = (5V)(2.5A) = 12.5W$
$P_2 = V_2 I_s = (2.5V)(2.5A) = 6.25\ W$
$P_3 = V_3 I_s = (12.5V)(2.5A) = 31.25\ W$

(e) $P_{del} = EI = (20V)(2.5A) = 50W$
$P_{del} = P_1 + P_2 + P_3$
$50W = 12.5W + 6.25W + 31.25W$

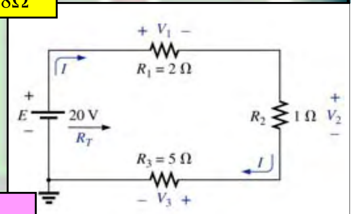


FIGURE 5.4

Ex. 5-2 Determine R_T, I_s, and V_2 for the circuit of Figure 5.5.

$R_T = R_1 + R_2 + R_3 + R_3$
$= 7\Omega + 4\Omega + 7\Omega + 7\Omega$
$= 25\Omega$

$I_s = \dfrac{E}{R_T} = \dfrac{50V}{25\Omega} = 2A$

$V_2 = I_s R_2 = (2A)(4\Omega) = 8V$

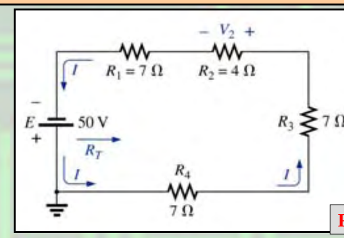


Figure 5.5

Ex. 5-3 Given R_T and I, calculate R_1 and E for the circuit of Figure 5.6.

$R_T = R_1 + R_2 + R_3$
$12\,k\Omega = R_1 + 4\,k\Omega + 6\,k\Omega$
$R_1 = 12\,k\Omega - 10\,k\Omega = 2\,k\Omega$
$E = IR_T = (6\times10^{-3}A)(12\times10^3\Omega) = 72\,V$

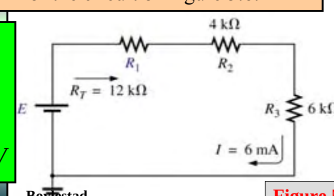


Figure 5.6

Voltage Sources in Series

Voltage sources can be connected in series, as shown in Fig. 5.7, to increase or decrease the total voltage applied to a system. The net voltage is determined simply by summing the sources with the same polarity and subtracting the total of the sources with the opposite polarity.

$E_T = E_1 + E_2 + E_3 = 10V + 6V + 2V = 18V$

$E_T = E_2 + E_3 - E_1 = 9V + 3V - 4V = 8V$

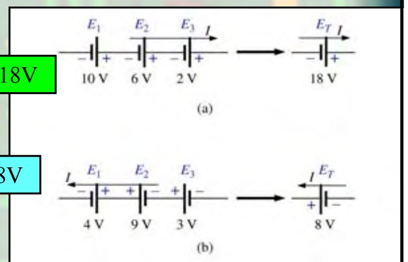


FIGURE 5.7 Reducing series dc voltage sources to a single source.

Kirchhoff's Voltage Law

Kirchhoff's voltage law (KVL) states that the algebraic sum of the potential rises and drops around a closed loop (or path) is zero.

A **closed loop** is any continuous path that leaves a point in one direction and returns to that same point from another direction without leaving the circuit.

$\sum V = 0$

(Kirchhoff's voltage law in symbolic form)

$E - V_1 - V_2 = 0$
or $E = V_1 + V_2$

$\sum V_{rises} = \sum V_{drops}$

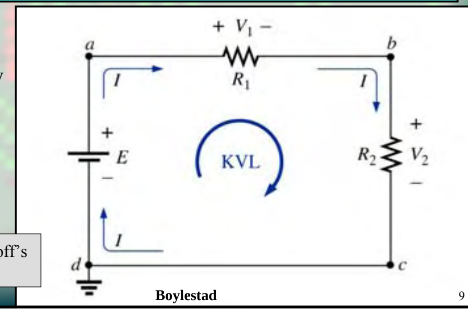


FIGURE 5.8 Applying Kirchhoff's voltage law to a series dc circuit.

Ex. 5-4 For the circuit of Figure 5.9:
a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_2.

a. Kirchhoff's voltage law (clockwise direction):

$-E + V_3 + V_2 + V_1 = 0$
or $E = V_1 + V_2 + V_3$
and $V_2 = E - V_1 - V_3$
$= 54V - 18V - 15V = 21V$

b. $I = \frac{V_2}{R_2} = \frac{21V}{7\Omega} = 3A$

c. $R_1 = \frac{V_1}{I} = \frac{18V}{3A} = 6\Omega$

$R_3 = \frac{V_3}{I} = \frac{15V}{3A} = 5\Omega$

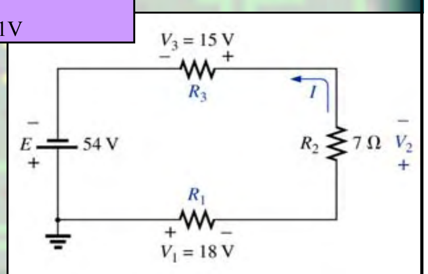


FIGURE 5.9

Ex. 5-5 Find V_1 and V_2 for the network of Fig. 5.10.

For path 1, starting at point a in a clockwise direction:

$-25V + V_1 - 15V = 0$
and $V_1 = 40V$

For path 2, starting at point a in a clockwise direction:

$V_2 + 20V = 0$
and $V_2 = -20V$

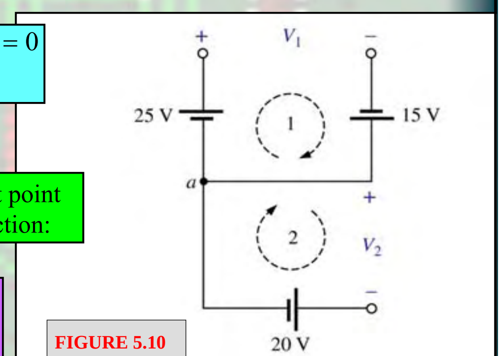


FIGURE 5.10

Ex. 5-6 Using Kirchhoff's voltage law, determine the unknown voltage for the network of Fig. 5.11.

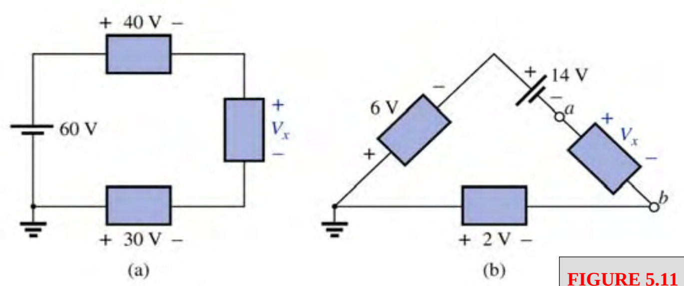


FIGURE 5.11

$$-60V + 40V + V_x - 30V = 0$$
$$\text{and} \quad V_x = 50\,V$$

$$6V + 14V + V_x - 2V = 0$$
$$\text{and} \quad V_x = -18\,V$$

Ex. 5-8 For the circuit of Fig. 5.12.

a. Determine V_2 using Kirchhoff's voltage law.
b. Determine I.
c. Find R_1 and R_3.

a. Kirchhoff's voltage law (clockwise direction):

$$54V - 15V - V_2 - 18V = 0$$
$$\text{or} \quad V_2 = 21\,V$$

$$b. \quad I = \frac{V_2}{R_2} = \frac{21\,V}{7\,\Omega} = 3\,A$$

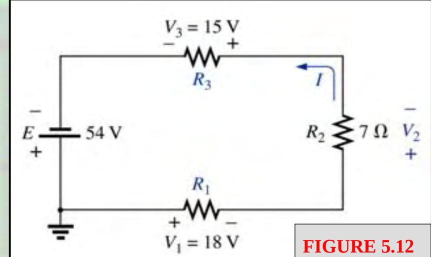


FIGURE 5.12

$$c. \quad R_1 = \frac{V_1}{I} = \frac{18\,V}{3\,A} = 6\,\Omega \qquad R_3 = \frac{V_3}{I} = \frac{15\,V}{3\,A} = 5\,\Omega$$

Voltage Divider Rule (VDR)

The voltage across the resistive elements will divide as the magnitude of the resistance levels.

The voltages across the resistive elements of Fig. 5.13 are provided. Since the resistance level of R_1 is 6 times that of R_3, the voltage across R_1 is 6 times that of R_3. The fact that the resistance level of R_2 is 3 times that of R_1 results in three times the voltage across R_2. Finally, since R_1 is twice R_2, the voltage across R_1 is twice that of R_2. If the resistance levels of all resistors of Fig. 5.13 are increased by the same amount, as shown in Fig. 5.14, the voltage levels will all remain the same.

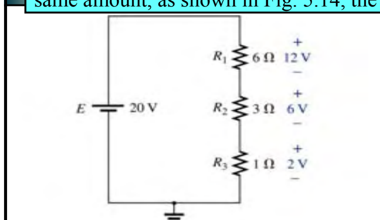


FIGURE 5.13 Revealing how the voltage will divide across series resistive elements.

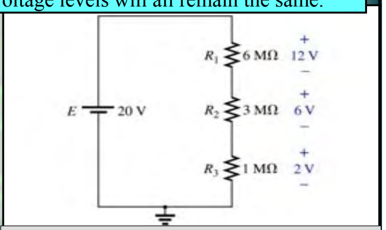


FIGURE 5.14 The ratio of the resistive values determines the voltage division of a series dc circuit.

The voltage divider rule (VDR) can be derived by analyzing the network of Fig. 5.15.

$R_T = R_1 + R_2$

and $I = E/R_T$

Applying Ohm's law:

$$V_1 = IR_1 = \left(\frac{E}{R_T}\right)R_1 = \frac{R_1 E}{R_T}$$

$$V_2 = IR_2 = \left(\frac{E}{R_T}\right)R_2 = \frac{R_2 E}{R_T}$$

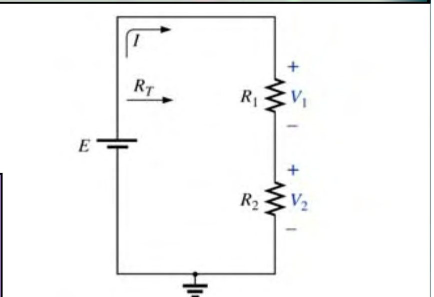


FIGURE 5.15 Developing the voltage divider rule.

$$V_x = \frac{R_x E}{R_T} \quad \text{(voltage divider rule)}$$

Ex. 5-9 Using the voltage divider rule, determine the voltages V_1 and V_3 for the series circuit of Figure 5.16.

$$V_1 = \frac{R_1E}{R_T} = \frac{R_1E}{R_1 + R_2 + R_3} = \frac{(2k\Omega)(45V)}{2k\Omega + 5k\Omega + 8k\Omega} = \frac{(2\times10^3\Omega)(45V)}{(15\times10^3\Omega)} = \frac{90V}{15} = 6V$$

$$V_3 = \frac{R_3E}{R_T} = \frac{(8k\Omega)(45V)}{15k\Omega} = \frac{(8\times10^3\Omega)(45V)}{15\times10^3\Omega} = \frac{360V}{15} = 24V$$

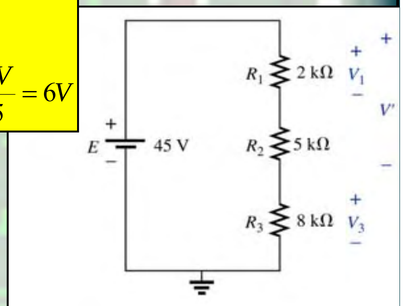


FIGURE 5.16

Notation-Voltage Sources and Ground

Notation will play an increasingly important role on the analysis to follow. Due to its importance we begin to examine the notation used throughout the industry.

Except for a few special cases, electrical and electronic systems are grounded for reference and safety purposes. The symbol for the **ground** connection appears in Fig. 5.25 with its defined potential level-zero volts.

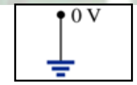


FIGURE 5.25 Ground potential.

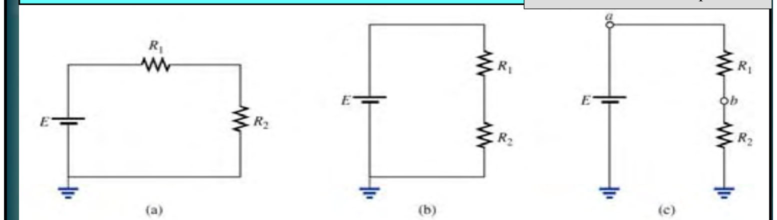


FIGURE 5.26 Three ways to sketch the same series dc circuit.

On large schematics where space is at a premium and clarity is important, voltage sources may be indicated as shown in Figs. 5.27(a) and 5.28(a) rather than as illustrated in Figs. 5.27(b) and 5.28(b).

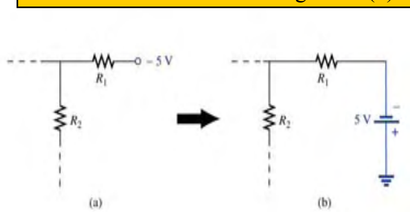


FIGURE 5.27 Replacing the special notation for dc voltage source with the standard symbol.

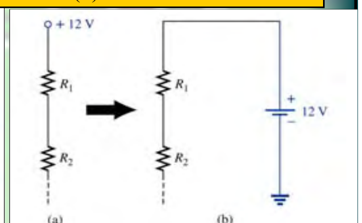


FIGURE 5.28 Replacing the notation for a negative dc supply with the standard notation.

In addition, potential levels may be indicated in Fig. 5.29, to permit a rapid check of the potential levels at various points in a network with respect to ground to ensure that the System is operating properly.

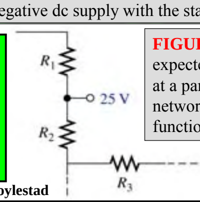


FIGURE 5.29 The expected voltage level at a particular point in a network of the system is functioning properly.

Double-Subscript Notation

The fact that voltage is an across variable and exists between two points has resulted in a double-script notation that defined the first subscript as the higher potential.

In Fig. 5.30(a), the two points that define the voltage across the resistor R are denoted by *a* and *b*. Since *a* is the first subscript for V_{ab}, point a must have higher potential than point *b* if V_{ab} is to have a positive value. If point *b* is at a higher potential than point *a*, V_{ab} will have a negative value, as indicated in Fig. 5.30(b). ***The voltage V_{ab} is the voltage at point a with respect to point b.***

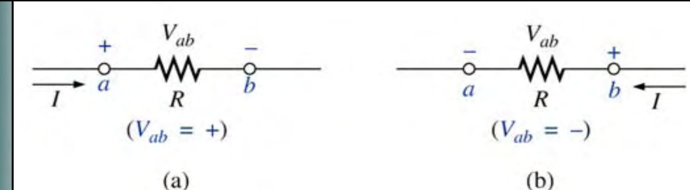


FIGURE 5.30 Defining the sign for double-subscript notation.

Single-Subscript Notation

A single-subscript notation can be employed that provides the voltage at a point with respect to ground.

In Fig. 5.31, V_a is the voltage from point a to ground. In this case it is obviously 10V since it is right across the source voltage E. The voltage V_b is the voltage from point b to ground. Because it is directly across the 4-Ω resistor, V_b = 4V.

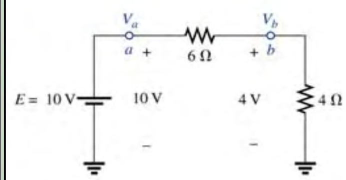


FIGURE 5.31 Defining the use of single-subscript notation for voltage levels.

The single-subscript notation V_a specifies the voltage at point a with respect to ground (zero volts). If the voltage is less than zero volts, a negative sign must be associated with the magnitude of V_a.

General Comments

A particularly useful relationship can now be established that will have extensive applications in the analysis of electronic circuits. For the above notational standards, the following relationship exists:

$$V_{ab} = V_a - V_b$$

Ex. 5-14 Find the voltage V_{ab} for the conditions of Fig. 5.32.

$$\begin{aligned} V_{ab} &= V_a - V_b \\ &= 16V - 20V \\ &= -4V \end{aligned}$$

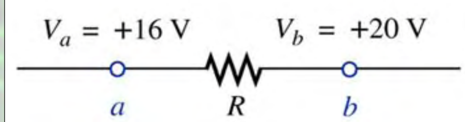


FIGURE 5.32 Example 5.14.

Ex. 5-15 Find the voltage V_a for the configuration of Fig. 5.33.

$$\begin{aligned} V_{ab} &= V_a - V_b \\ V_a &= V_{ab} + V_b = 5V + 4V \\ &= 9V \end{aligned}$$

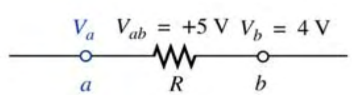


FIGURE 5.33

Ex. 5-16 Find the voltage V_{ab} for the configuration of Fig. 5.34.

$$\begin{aligned} V_{ab} &= V_a - V_b = 20V - (-15V) \\ &= 20V + 15V = 35V \end{aligned}$$

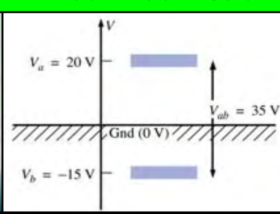


FIGURE 5.35 The impact of positive and negative voltages on the total voltage drop.

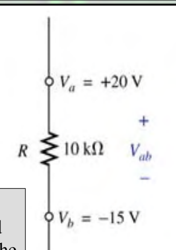


FIGURE 5.34

Ex. 5-17 Find the voltage V_b, V_c and V_{ac} for the network of Fig. 5.36.

Starting at Ground, we proceed through a rise of 10 V to reach point a and then pass through a drop in potential of 4 V to point b. The result is that the meter will read

V_b = +10V – 4V = 6V

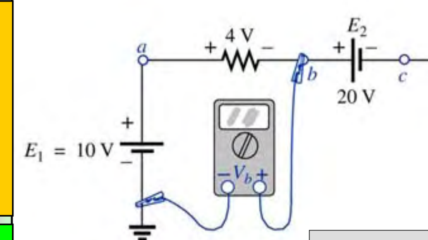


FIGURE 5.36

If we then proceed to point c, there is an additional drop of 20V, result in

$V_c = V_b - 20V - 6V$

$= 6V - 20V = -14V$

The voltage V_{ac} can be obtained

$V_{ac} = V_a - V_c$

$= 10V - (-14V)$

$= 24V$

Ex. 5-18 Determine V_{ab}, V_{cb} and V_c for the network of Fig. 5.37.

There are two ways to approach this problem. The first is that there is a 54-V drop across the series resistors R_1 and R_2.

$$I = \frac{54V}{45\Omega} = 1.2\ A$$

$$V_{ab} = I\,R_2 = (1.2\ A)(25\Omega) = 30\,V$$

$$V_{cb} = -I\,R_1 = -(1.2\ A)(20\Omega) = -24\,V$$

$$V_c = E_1 = -19\,V$$

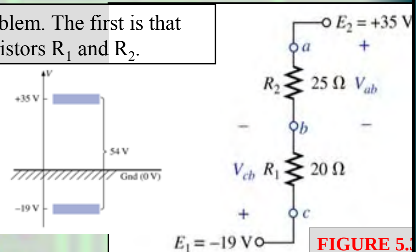


FIGURE 5.37

The other approach is to redraw the network as shown in Fig. 5.37 to clearly establish the aiding effect of E_1 and E_2 and then solve the resulting series circuit.

$$I = \frac{E_1 + E_2}{R_T} = \frac{19V + 35V}{45\Omega} = \frac{54V}{45\Omega} = 1.2\ A$$

$$and\quad V_{ab} = 30\,V \quad V_{cb} = -24\,V \quad V_c = -19\,V$$

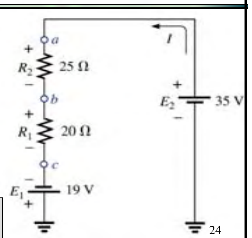


FIGURE 5.38 Redrawing the circuit of Fig. 5.37 using dc voltage supply symbols.

Ex. 5-19 Using the voltage divider rule, determine the voltages V_1 and V_2 for of Fig. 5.39.

Redrawing the network with standard battery symbol will result in the network of Fig.5.40. Applying the voltage divider rule,

$$V_1 = \frac{R_1\,E}{R_1 + R_2} = \frac{(4\Omega)(24V)}{4\Omega + 2\Omega} = 16\,V$$

$$V_2 = \frac{R_2\,E}{R_1 + R_2} = \frac{(2\Omega)(24V)}{4\Omega + 2\Omega} = 8\,V$$

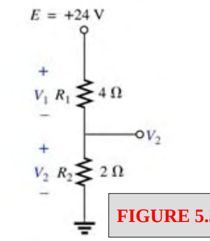


FIGURE 5.39

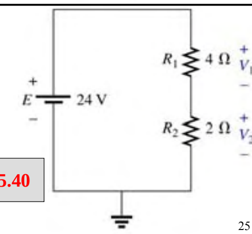


FIGURE 5.40

Ex. 5-20 For the network of Fig. 5.40:

a. Calculate V_{ab}.
b. Determine V_b.
c. Calculate V_c.

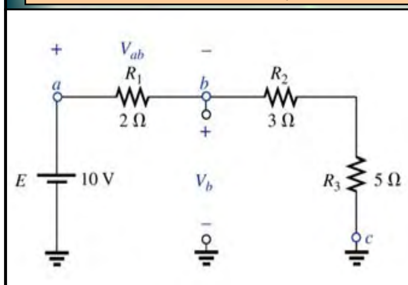


FIGURE 5.40

a. *Voltage divider rule:*

$$V_{ab} = \frac{R_1\,E}{R_T} = \frac{(2\Omega)(10V)}{2\Omega + 3\Omega + 5\Omega} = +2V$$

b. *Voltage divider rule:*

$$V_b = V_{R_2} + V_{R_3} = \frac{(R_2 + R_3)E}{R_T}$$

$$= \frac{(3\Omega + 5\Omega)(10V)}{10\Omega} = 8\,V$$

$$or\quad V_b = V_a - V_{ab} = E - V_{ab}$$

$$= 10V - 2V = 8\,V$$

c. V_c = ground potential = 0V

Ideal Voltage Sources vs. Non-ideal Voltage Sources

Every source of voltage, whether a generator, battery, or laboratory supply as shown in Fig. 5.41(a), will have some **internal resistance** **(know as the non-ideal voltage source)**. The equivalent circuit of any source of voltage will therefore appear as shown in Fig. 5.41(b).

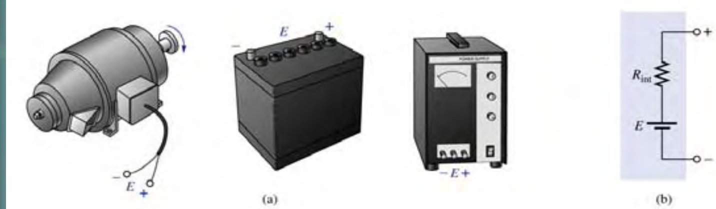


FIGURE 5.41 (a) Sources of dc voltage; (b) equivalent circuit.

In all the circuit analyses to this point, **the ideal voltage source** (no internal resistance) was used shown in Fig. 5.42(a). The ideal voltage source has no internal resistance and an output voltage of *E* volts with no load or full load. In the practical case [Fig. 5.42(b)], where we consider the effects of the internal resistance, the output voltage will be *E* volts only when no-load (I_L = *0*) conditions exist. When a load is connected [Fig. 5.42(c)], the output voltage of the voltage source will decrease due to the voltage drop across the internal resistance.

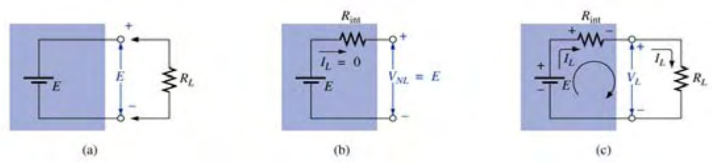


FIGURE 5.42 Voltage source: (a) ideal, $R_{int} = 0\ \Omega$; (b) Determining VNL; (c) determining R_{int}.

Voltage Regulation

For any supply, ideal conditions dictate that for the range of load demand (I_L), the terminal voltage remain fixed in magnitude. By definition, the **voltage regulation (VR)** of a supply between the limits of full-load and no-load conditions (Fig. 5.43) is given by the following:

$$\textit{Voltage regulation } (VR)\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$

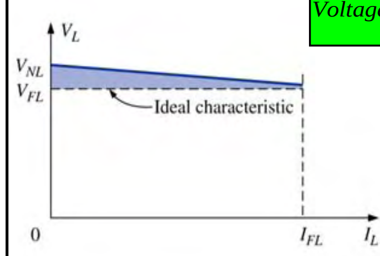


FIGURE 5.43 Defining voltage regulation.

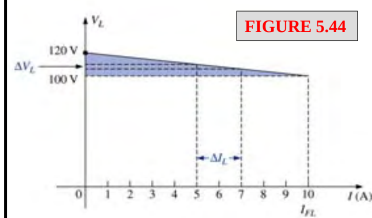
For ideal conditions, VR% = V_{NL} and VR% = 0. Therefore, the smaller the voltage regulation, the less the variation in terminal voltage with change in load.

It can be shown with a short derivation that the voltage regulation is also given by

$$VR\% = \frac{R_{int}}{R_L} \times 100\%$$

Ex. 5-21 Calculate the voltage regulation of a supply having the characteristics of Fig. 5.44.

FIGURE 5.44



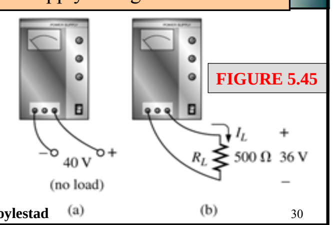
$$VR\% = \frac{V_{NL} - V_{FL}}{V_{FL}} \times 100\%$$
$$= \frac{120V - 100V}{100V} \times 100\%$$
$$= \frac{20}{100} \times 100\% = 20\ \%$$

Ex. 5-22 Determine the voltage regulation of the supply of Fig. 5.45.

$$VR\% = \frac{R_{int}}{R_L} \times 100\%$$
$$= \frac{19.48\Omega}{500\Omega} \times 100\%$$
$$= 3.9\%$$

R_{int} = *19.48 Ω*

FIGURE 5.45



HW 5-24 Determine the values of the unknown resistors in Fig. 5.108 using the provided voltage levels.

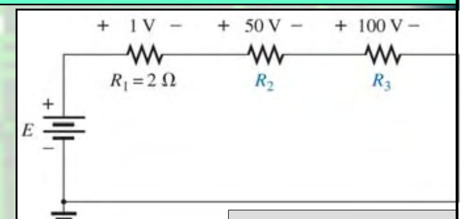


Figure 5.108 Problem 24.

$$\frac{1\text{ V}}{2\ \Omega} = \frac{50\text{ V}}{R_2},\ R_2 = \frac{(50\text{ V})(2\ \Omega)}{1\text{ V}} = \mathbf{100\ \Omega}$$
$$\frac{1\text{ V}}{2\ \Omega} = \frac{100\text{ V}}{R_3},\ R_3 = \frac{(100\text{ V})(2\ \Omega)}{1\text{ V}} = \mathbf{200\ \Omega}$$

Homework 5: 1, 2, 4, 5,7, 10, 11, 15, 16, 22, 23, 24, 26, 30, 41, 43

EET1122/ET162 Circuit Analysis

Parallel Circuits

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 10th edition.

OUTLINES

Key Words: Parallel Circuit, Kirchhoff's Current Law, Current Divider Rule, Voltage Source

Parallel Circuits – Introduction & Elements

A circuit configuration in which the elements have two points in common

Two elements, branches, or networks are in parallel if they have two points in common.

In *Fig. 6.1*, for example, elements 1 and 2 have terminals a and b in common; they are therefore in parallel.

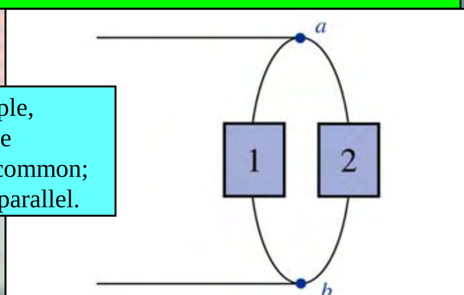


FIGURE 6.1 Parallel elements

Parallel Circuits – Parallel Elements

In *Fig. 6.2*, all the elements are in parallel because they satisfy the previous criterion. Three configurations provided to demonstrate how the parallel networks can be drawn.

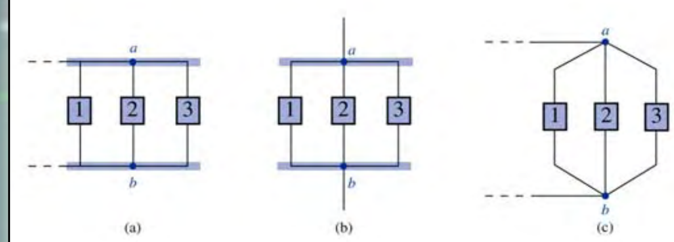


Figure 6.2 Different ways in which three parallel elements may appear.

Parallel Circuits – Parallel Elements

In *Fig. 6.3*, elements 1 and 2 are in parallel because they have terminals a and b in common. The parallel combination of 1 and 2 is then in series with element 3 due to the common terminal point b.

In *Fig. 6.4*, elements 1 and 2 are in series due to the common point a, but the series combination of 1 and 2 is in parallel with elements 3 as defined by the common terminal connections at a and b.

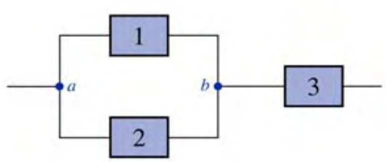


Figure 6.3 Network in which 1 and 2 are in parallel and 3 is in series with the parallel combination of 1 and 2.

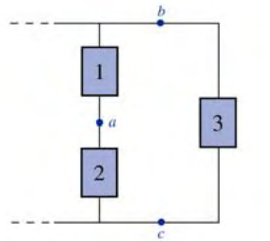


Figure 6.4 Network in which 1 and 2 are in series and 3 is in parallel.

Parallel Circuits – Total Conductance

For parallel elements, the total conductance is the sum of the individual conductances.

$$\mathbf{G_T = G_1 + G_2 + G_3 + \cdots + G_N}$$

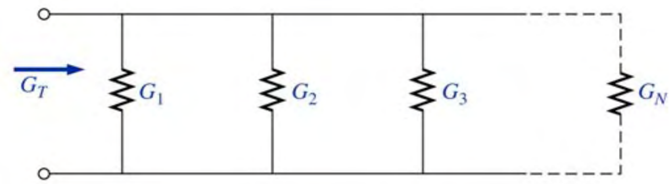


Figure 6.5 Determining the total conductance of parallel conductances.

Parallel Circuits – Total Resistance

Since **G = 1/R**, the total resistance for the network can be determined by direct substitution into following equation.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_N}$$

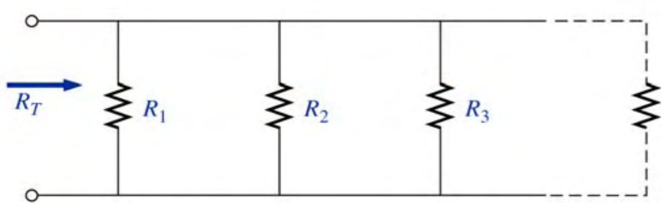


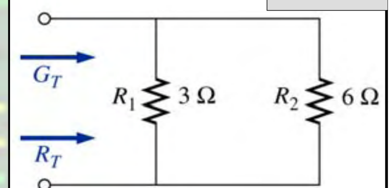
Figure 6.6 Determining the total resistance of parallel resistors.

Ex. 6-1 Determine the total conductance and resistance for the parallel network of *Figure 6.7*.

$$G_T = G_1 + G_2 = \frac{1}{3\Omega} + \frac{1}{6\Omega}$$
$$= 0.333S + 0.167S = 0.5S$$

and $R_T = \dfrac{1}{G_T} = \dfrac{1}{0.5S} = 2\Omega$

FIGURE 6.7



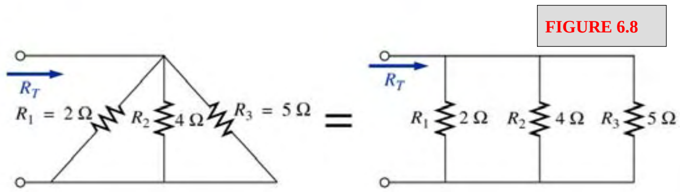
Ex. 6-2 Determine the effect on the total conductance and resistance and resistance of the network of Fig. 6.7 if another resistor of 10 Ω were added in parallel with the other element.

$$G_T = 0.5S + \frac{1}{10\Omega} = 0.5S + 0.1S = 0.6\,S$$

$$R_T = \frac{1}{G_T} = \frac{1}{0.6S} \cong 1.667\,\Omega$$

Ex. 6-3 Determine the total resistance for the network of *Fig. 6.8*.

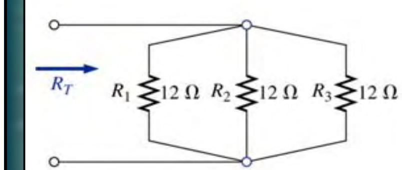
FIGURE 6.8



$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_2}$$
$$= \frac{1}{2\Omega} + \frac{1}{4\Omega} + \frac{1}{5\Omega}$$
$$= 0.5\,S + 0.25\,S + 0.2\,S$$
$$= 0.95\,S$$

$$R_T = \frac{1}{0.95\,S} = 1.053\,\Omega$$

Ex. 6-4 a. Find the resistance of the network of *Fig. 6.9*.
b. Calculate the total resistance for the network of *Fig. 6.10*.



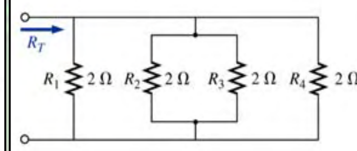
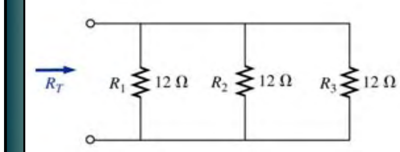


Figure 6.10 Example 6-4: four parallel resistors of equal value.

Figure 6.9 Example 6-4: three parallel resistors



$$(a)\quad R_T = \frac{R}{N} = \frac{12\,\Omega}{3} = 4\,\Omega$$

$$(b)\quad R_T = \frac{R}{N} = \frac{2\,\Omega}{4} = 0.5\,\Omega$$

The total resistance of parallel resistors is always less than the value of the smallest resistor.

Parallel Circuits – Total Resistance

For two parallel resistors, we write

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_2 + R_1}{R_1 R_2} \qquad (1)$$

and

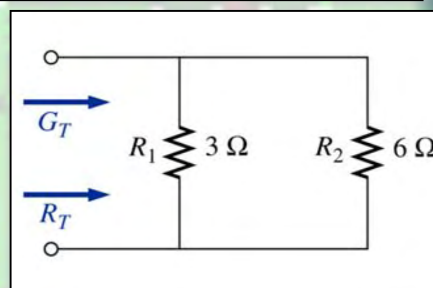
$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

The total resistance of two parallel resistors is the product of the two divided by their sum.

For three parallel resistors, the equation for R_T becomes

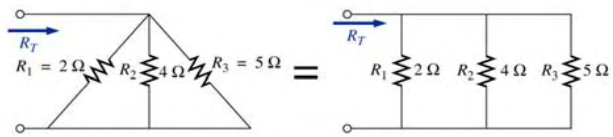
$$R_T = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} = \frac{R_1 R_2 R_3}{R_1 R_2 + R_1 R_3 + R_2 R_3} \qquad (2)$$

Ex. 6-5 Repeat Example 6.1 using Eq.(1).



$$R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = \frac{18\Omega}{9} = 2\,\Omega$$

Ex. 6-6 Repeat Example 6.3 using Eq.(2).



$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{\frac{1}{2\Omega}+\frac{1}{4\Omega}+\frac{1}{5\Omega}}$$

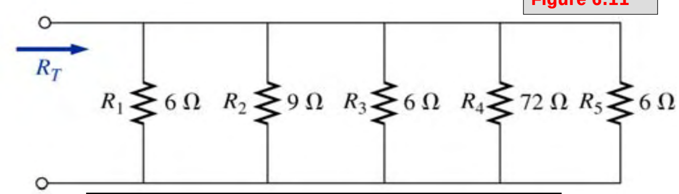
$$= \frac{1}{0.5S+0.25S+0.2S} = \frac{1}{0.95S} = 1.053\Omega$$

$$R_T' = 5\Omega\,//\,4\Omega = \frac{(5\Omega)(4\Omega)}{5\Omega+4\Omega} = 2.222\ \Omega$$

$$R_T = R_T'\,//\,2\Omega = \frac{(2.22\Omega)(2\Omega)}{2.222\Omega+2\Omega} = 1.053\ \Omega$$

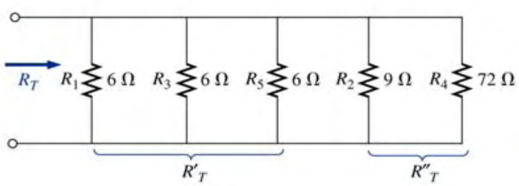
Ex. 6-7 Calculate the total resistance of the parallel network of Fig. 6.11.

Figure 6.11



$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}+\frac{1}{R_5}}$$

$$= \frac{1}{\frac{1}{6\Omega}+\frac{1}{9\Omega}+\frac{1}{6\Omega}+\frac{1}{72\Omega}+\frac{1}{6\Omega}} = 1.6\ \Omega$$



$$R_T' = \frac{R}{N} = \frac{6\Omega}{3} = 2\ \Omega$$

$$R_T'' = \frac{R_2 R_4}{R_2+R_4} = \frac{(9\Omega)(72\Omega)}{9\Omega+72\Omega} = \frac{648\Omega}{81} = 8\ \Omega$$

$$R_T = R_T'\,//\,R_T'' = \frac{R_T' R_T''}{R_T'+R_T''}$$

$$= \frac{(2\Omega)(8\Omega)}{2\Omega+8\Omega} = \frac{16\Omega}{10} = 1.6\ \Omega$$

Ex. 6-8 Determine the value of R_2 in *Fig. 6.12* to establish a total resistance of 9 kΩ.

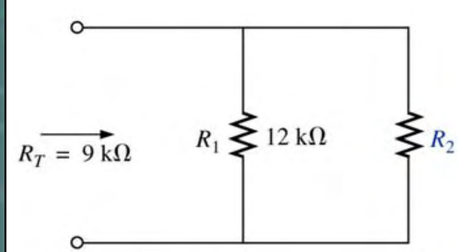


Figure 6.12

$$R_T = \frac{1}{\frac{1}{R_1}+\frac{1}{R_2}}$$

$$\frac{1}{R_1}+\frac{1}{R_2} = \frac{1}{R_T}$$

$$\frac{1}{R_2} = \frac{1}{R_T} - \frac{1}{R_1}$$

$$= \frac{1}{9\,k\Omega} - \frac{1}{12\,k\Omega}$$

$$= 0.028\times10^{-3}\ S$$

$$R_2 = 35.7\ k\Omega \cong 36\ k\Omega$$

Ex. 6-9 Determine the values of R_1, R_2, and R_3 in *Fig. 6.13* if $R_2 = 2R_1$ and $R_3 = 2R_2$ and the total resistance is 16 kΩ.

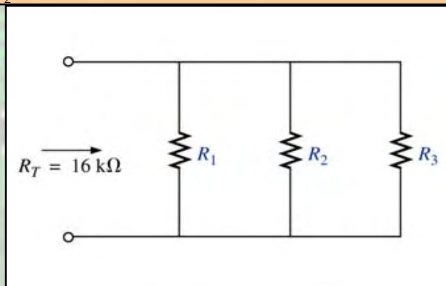


Figure 6.13

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{16\,k\Omega} = \frac{1}{R_1} + \frac{1}{2R_1} + \frac{1}{4R_1} = \frac{4+2+1}{4R_1} = \frac{7}{4R_1}$$

$$R_1 = \frac{7 \times 16\,k\Omega}{4} = 28\,k\Omega$$

Ex. 6-10 a. Determine the total resistance of the network of *Fig. 6.14*.
b. What is the effect on the total resistance of the network of Fig.*6.14* if additional resistor of the same value is added, as shown in *Fig.6.15*?
c. What is the effect on the total resistance of the network of Fig.*6.14* if very large resistance is added in parallel, as shown in *Fig.6.16*?
d. What is the effect on the total resistance of the network of *Fig.6.14* if very small resistance is added in parallel, as shown in *Fig.6.17*?

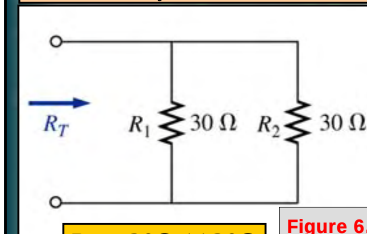


Figure 6.14

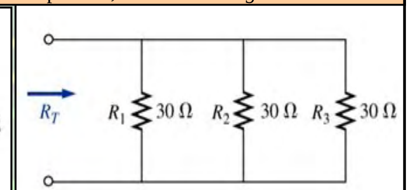


Figure 6.15

$$R_T = 30\Omega \,//\, 30\Omega = \frac{30\Omega}{2} = 15\,\Omega$$

$$R_T = 30\Omega \,//\, 30\Omega \,//\, 30\Omega = \frac{30\Omega}{3} = 10\,\Omega < 15\,\Omega$$

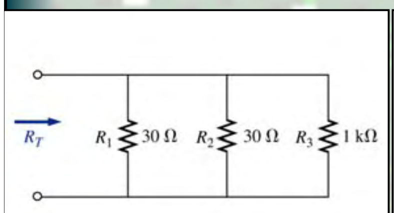


Figure 6.16

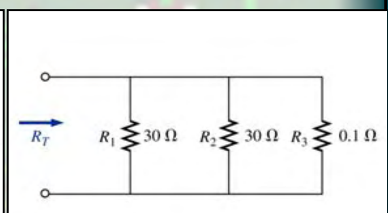


Figure 6.17

$$R_T = 30\Omega \,//\, 30\Omega \,//\, 1k\Omega = 15\Omega \,//\, 1\,k\Omega = \frac{(15\Omega)(1000\Omega)}{15\Omega + 1000\Omega} = 14.778\,\Omega < 15\,\Omega$$

Small decrease in R_T

$$R_T = 30\Omega \,//\, 30\Omega \,//\, 0.1\Omega = 15\Omega \,//\, 0.1\,\Omega = \frac{(15\Omega)(0.1\Omega)}{15\Omega + 0.1\Omega} = 0.099\,\Omega \ll 15\,\Omega$$

Significant decrease in R_T

Parallel Circuits Analysis and Measurement

The network of Fig.6.18 is the simplest of parallel circuits. All the elements have terminals a and b in common.

The voltage across parallel elements is the same.

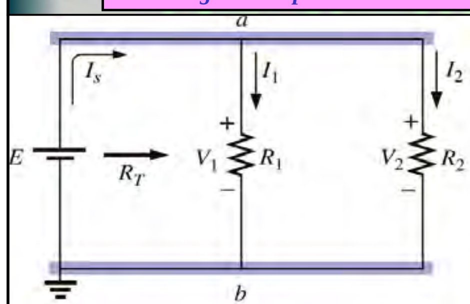


FIGURE 6.18 Parallel network.

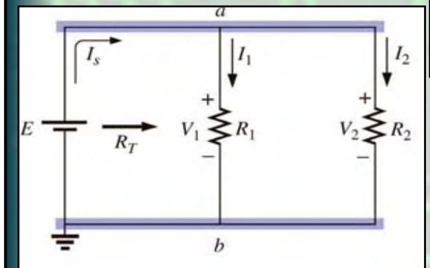
$$V_1 = V_2 = E$$

$$I_1 = \frac{V_1}{R_1} = \frac{E}{R_1}$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2}$$

$$\mathbf{I_s = I_1 + I_2}$$

For single–source parallel networks, the source current (I_s) is equal to the sum of the individual branch current.



The power dissipated by the resistors and delivered by the source can be determined from.

$$P_1 = V_1 I_1 = I_1^2 R_1 = \frac{V_1^2}{R_1}$$

$$P_1 = V_2 I_2 = I_2^2 R_2 = \frac{V_2^2}{R_2}$$

$$P_s = E I_s = I_s^2 R_T = \frac{E^2}{R_T}$$

Ex. 6-11 For the parallel network of *Fig. 6.19*.

a. Calculate R_T.
b. Determine I_s.
c. Calculate I_1 and I_2, and demonstrate that $I_s = I_1 + I_2$.
d. Determine the power to each resistive load.
e. Determine the power delivered by the source, and compare it to the total power dissipated by the resistive elements.

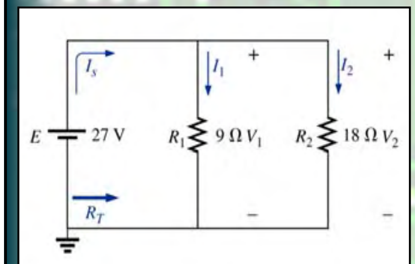
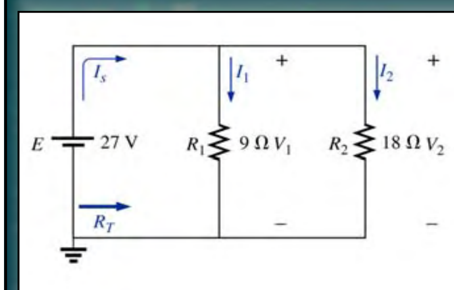


FIGURE 6.19

$$a.\quad R_T = \frac{R_1 R_2}{R_1 + R_2} = \frac{(9\Omega)(18\Omega)}{9\Omega + 18\Omega} = \frac{162\,\Omega}{27} = 6\Omega$$

$$b.\quad I_s = \frac{E}{R_T} = \frac{27V}{6\Omega} = 4.5\ A$$



$$c.\quad I_1 = \frac{V_1}{R_1} = \frac{E}{R_1} = \frac{27V}{9\Omega} = 3\ A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{27V}{18\Omega} = 1.5\ A$$

$$I_s = I_1 + I_2$$

$$4.5\ A = 3\ A + 1.5\ A = 4.5\ A\ (checks)$$

$$d.\quad P_1 = V_1 I_1 = E\,I_1 = (27V)(3A) = 81\ W$$

$$P_2 = V_2 I_2 = E\,I_2 = (27V)(1.5A) = 40.5\ W$$

$$e.\quad P_s = E\,I_s = (27V)(4.5A) = 121.5\ W$$

$$= P_2 + P_2 = (81W)(40.5W) = 121.5\ W$$

Ex. 6-12 Given the information provided in *Fig.6.20*.

a. Determine R_3.
b. Calculate E.
c. Find I_s & I_2.
d. Determine P_2.

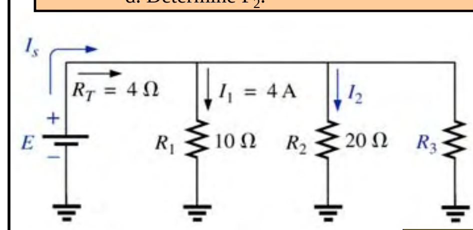


FIGURE 6.20

$$(a)\ \frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{4\Omega} = \frac{1}{10\Omega} + \frac{1}{20\Omega} + \frac{1}{R_3}$$

R_3 = 10 Ω

(b) E = $V_1 = I_1R_1$ = (4A)(10Ω) = 40V

$$(c)\ I_s = \frac{E}{R_T} = \frac{40V}{4\Omega} = 10\ A$$

$$I_2 = \frac{V_2}{R_2} = \frac{E}{R_2} = \frac{40V}{20\Omega} = 2\ A$$

$$(d)\ P_2 = I_2^2 R_2 = (2\ A)^2 (20\,\Omega) = 80\ W$$

Kirchhoff's Current Law

Kirchhoff's current law (KCL) states that the algebraic sum of the current entering and leaving an area, system, or junction is zero.

The sum of the currents entering an area, system, or junction must equal to the sum of the currents leaving the area, system, or junction.

$$\sum I_{entering} = \sum I_{leaving}$$

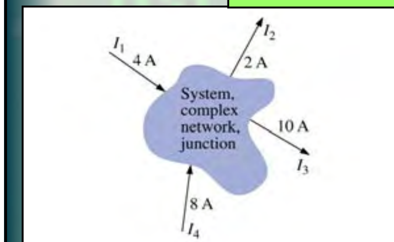


FIGURE 6.21 Introduction to KCL:

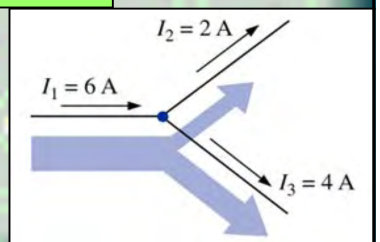


FIGURE 6.22 Demonstrating KCL:

Ex. 6-13 Determine the currents I_3 and I_4 of *Fig. 6.23* using Kirchhoff's current law.

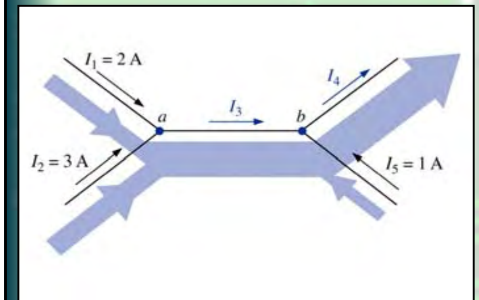


FIGURE 6.23

At a: $\sum I_{entering} = \sum I_{leaving}$

$$I_1 + I_2 = I_3$$
$$2\ A + 3\ A = I_3$$
$$I_3 = 5\ A$$

At b: $\sum I_{entering} = \sum I_{leaving}$

$$I_3 + I_5 = I_4$$
$$5\ A + 1\ A = I_3$$
$$I_4 = 6\ A$$

Ex. 6-14 Determine the currents I_1, I_3, I_4, and I_5 for the network of *Fig. 6.24.*

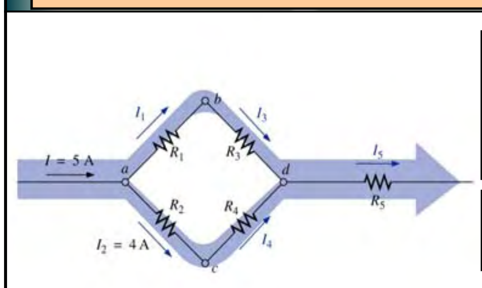


FIGURE 6.24

At a: $\sum I_{entering} = \sum I_{leaving}$

$$I = I_1 + I_2$$
$$5\ A = I_1 + 4\ A$$
$$I_1 = 1\ A$$

At b: $\sum I_{entering} = \sum I_{leaving}$

$$I_1 = I_3 = 1\ A$$

At c: $\sum I_{entering} = \sum I_{leaving}$

$$I_2 = I_4 = 4\ A$$

At d: $\sum I_{entering} = \sum I_{leaving}$

$$I_3 + I_4 = I_5$$
$$1\ A + 4\ A = I_5$$
$$I_5 = 5\ A$$

Ex. 6-15 Determine the currents I_3 and I_5 of *Fig. 6.25* through applications of Kirchhoff's current law.

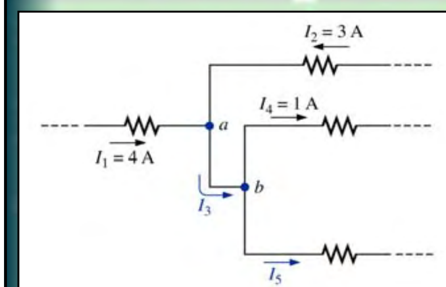


FIGURE 6.25

At node a,

$$I_1 + I_2 = I_3$$
$$4\ A + 3\ A = I_3$$
$$I_3 = 7\ A$$

At node b,

$$I_3 = I_4 + I_5$$
$$7\ A = 1\ A + I_5$$
$$I_5 = 7\ A - 1\ A = 6\ A$$

Ex. 6-16 Find the magnitude and direction of the currents I_3, I_4, I_6, and I_7 for the network of Fig.6.26.

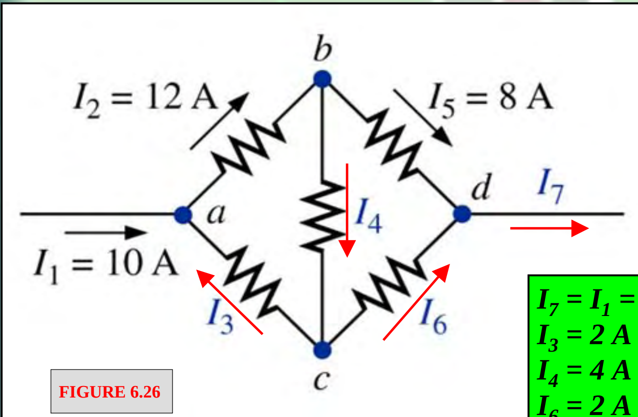


FIGURE 6.26

$I_7 = I_1 = 10$ A
$I_3 = 2$ A
$I_4 = 4$ A
$I_6 = 2$ A

Current Divider Rule (CDR)

For two parallel elements of equal value, the current will divide equally.

For parallel elements with different values, the smaller the resistance, the greater the share of input current.

For parallel elements with different values, the current will split with a ratio equal to the inverse of their resistor values.

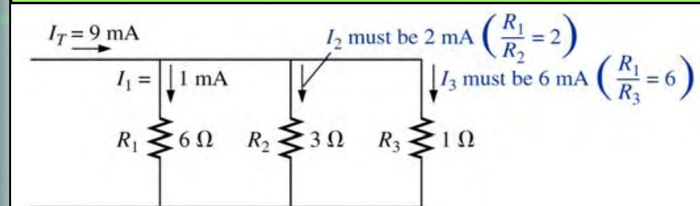
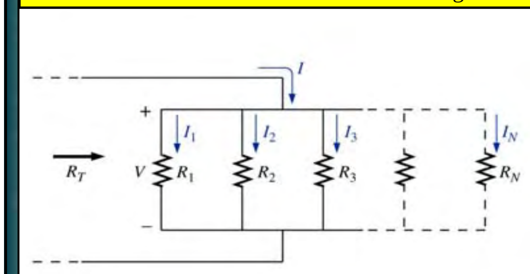


FIGURE 6.27 Demonstrating how current will divide between unequal resistors.

For networks in which only the resistor values are given along with the input current, the current divider rule should be applied to determine the various branch currents. It can be derived using the network of *Fig. 6.28*.



For the particular case of two parallel resistors, as shown in *Fig. 6.29*.

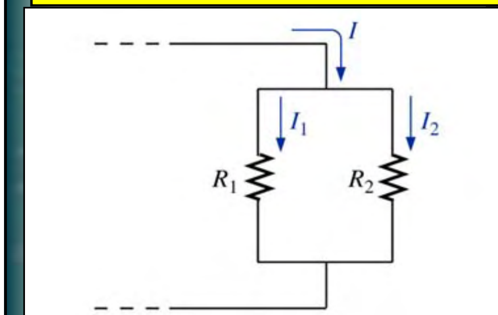


FIGURE 6.29 Developing an equation for current division between two parallel resistors.

$$R_T = \frac{R_1 R_2}{R_1 + R_2}$$

nd

$$I_1 = \frac{R_T}{R_1} I = \frac{\frac{R_1 R_2}{R_1 + R_2}}{R_1} I = \frac{R_2 I}{R_1 + R_2}$$

$$I_2 = \frac{R_1 I}{R_1 + R_2}$$

In words, for two parallel branches, the current through either branch is equal to the product of the other parallel resistor and the input current divided by the sum of the two parallel resistances.

Ex. 6-17 Determine the current I_2 for the network of *Fig.6.30* using the current divider rule.

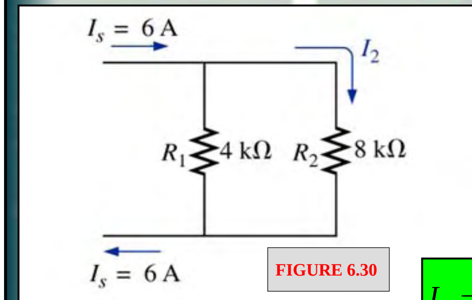


FIGURE 6.30

$$I_2 = \frac{R_1\, I_s}{R_1 + R_2} = \frac{(4\,k\Omega)(6\,A)}{4\,k\Omega + 8\,k\Omega}$$

$$= \frac{4}{12}(6\,A) = 2\,A$$

Ex. 6-18 Find the current I_1 for the network of *Fig.6.31*.

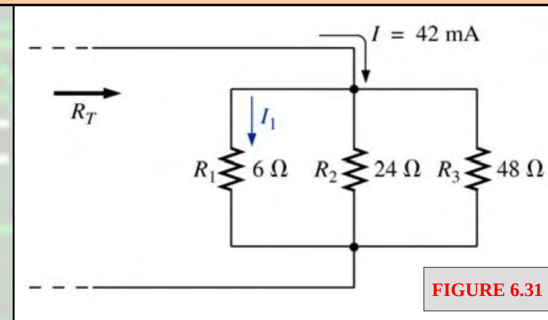


FIGURE 6.31

$$I_1 = \frac{\frac{1}{R_1}}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}} I_T = \frac{\frac{1}{6\Omega}}{\frac{1}{6\Omega} + \frac{1}{24\Omega} + \frac{1}{48\Omega}} 42\,mA = 30.54\,mA$$

Ex. 6-19 Determine the magnitude of the currents I_1, I_2, and I_3 for network of *Fig. 6.32*.

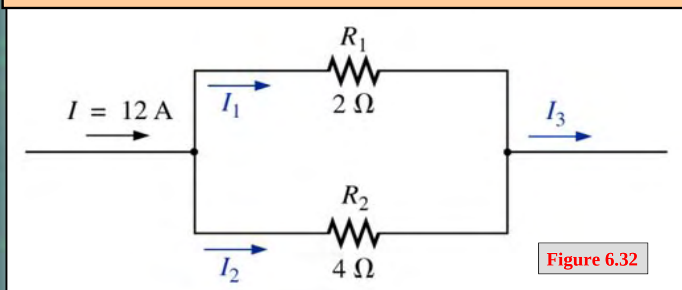


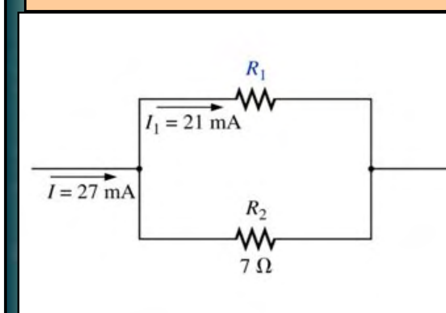
Figure 6.32

$$I_1 = \frac{R_2\, I}{R_1 + R_2} = \frac{(4\,\Omega)(12\,A)}{2\Omega + 4\Omega} = 8\,A$$

$$I_2 = \frac{R_1\, I}{R_1 + R_2} = \frac{(2\,\Omega)(12\,A)}{2\Omega + 4\Omega} = 4\,A$$

$I = I_3 = 12\,A$

Ex. 6-19 Determine the resistance R_1 to effect the division of current in *Fig. 6.33*.



$$I_1 = \frac{R_2\, I}{R_1 + R_2}$$

$$(R_1 + R_2) I_1 = R_2\, I$$

$$R_1\, I_1 + R_2\, I_1 = R_2\, I$$

$$R_1\, I_1 = R_2\, I - R_2\, I_1$$

$$R_1 = \frac{R_2\, I - R_2\, I_1}{I_1} = \frac{R_2 (I - I_1)}{I_1}$$

Figure 6.32

$$R_1 = \frac{7\,\Omega\,(27\,mA - 21\,mA)}{21\,mA}$$

$$= 7\,\Omega\left(\frac{6}{21}\right) = 2\,\Omega$$

Current seeks the path of least resistance.

1. More current passes through the smaller of two parallel resistors.
2. The current entering any number of parallel resistors divides into these resistors as the inverse ratio of their ohmic values. This relationship is depicted in *Fig.6.33*.

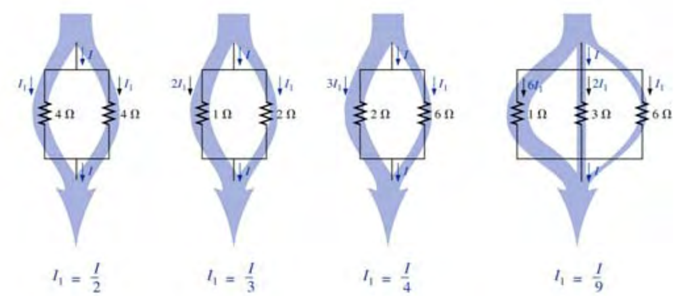


Figure 6.33 Current division through parallel branches.

Voltage Sources in Parallel

Voltage sources are placed in parallel as shown in *Fig. 6.34* only if they have same voltage rating.

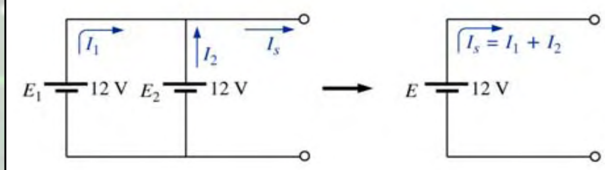
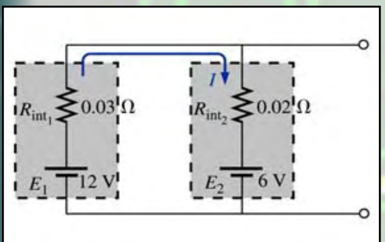


Figure 6.34 Parallel voltage sources.

If two batteries of different terminal voltages were placed in parallel both would be left ineffective or damaged because the terminal voltage of the larger battery would try to drop rapidly to that of the lower supply.

Consider two lead-acid car batteries of different terminal voltage placed in parallel, as shown in *Fig. 6. 35*.



$$I = \frac{E_1 - E_2}{R_{int1} + R_{int2}} = \frac{12V - 6V}{0.03\Omega + 0.02\,\Omega} = \frac{6V}{0.05\Omega} = 120\ A$$

Figure 6.35 Parallel batteries of different terminal voltages.

HW 6-29 Based solely on the resistor values, determine all the currents for the configuration in Fig. 6.99. Do not use Ohm's law.

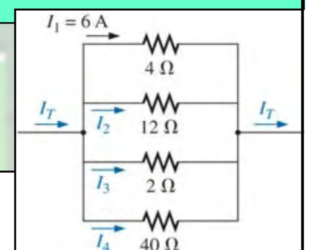


Figure 6.99 Problem 29.

$$I_2 = \frac{4\ \Omega}{12\ \Omega} I_1 = \frac{1}{3} I_1 = \mathbf{2\ A}$$

$$I_3 = \frac{4\ \Omega}{2\ \Omega} I_1 = 2 I_1 = \mathbf{12\ A}$$

$$I_4 = \frac{4\ \Omega}{40\ \Omega} I_1 = \frac{1}{10} I_1 = \mathbf{0.6\ A}$$

$$I_T = I_1 + I_2 + I_3 + I_4 = 6\ A + 2\ A + 12\ A + 0.6\ A = \mathbf{20.6\ A}$$

Homework 6: 1, 4, 7, 10, 18, 20, 23, 28, 29

EET1122/ET162 Circuit Analysis

Series and Parallel Networks

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 10th edition.

Acknowledgement

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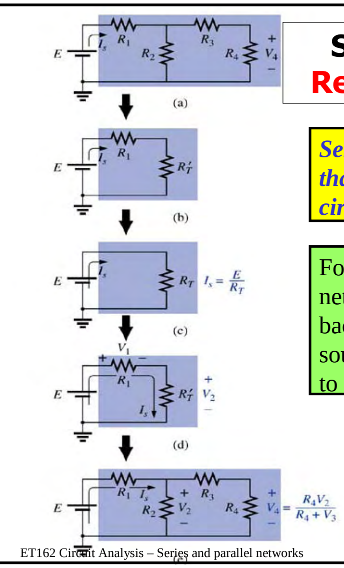
Sunghoon Jang

OUTLINES

- Introduction to Series-Parallel Networks
- Reduce and Return Approach
- Block Diagram Approach
- Descriptive Examples
- Ladder Networks

Key Words: Series-Parallel Network, Block Diagram, Ladder Network

Series-Parallel Networks – Reduce and Return Approach



Series-parallel networks are networks that contain both series and parallel circuit configurations

For many single-source, series-parallel networks, the analysis is one that works back to the source, determines the source current, and then finds its way to the desired unknown.

FIGURE 7.1 Introducing the reduce and return approach.

Series-Parallel Networks
Block Diagram Approach

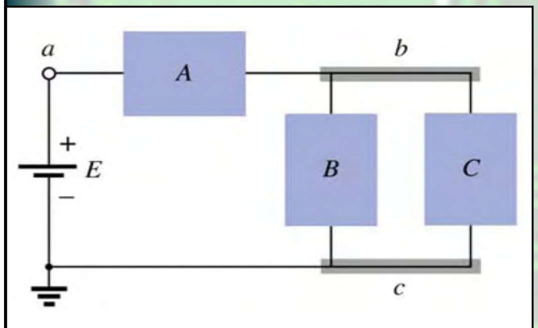


FIGURE 7.2 Introducing the block diagram approach.

The block diagram approach will be employed throughout to emphasize the fact that combinations of elements, not simply single resistive elements, can be in series or parallel.

Ex. 7-1 If each block of *Fig.7.3* were a single resistive element, the network of *Fig. 7.4* might result.

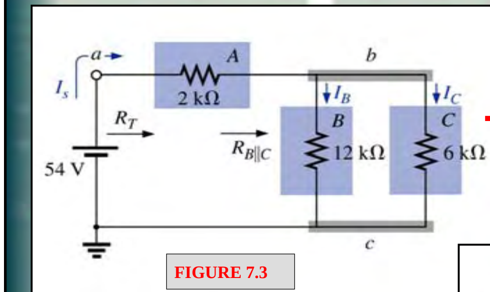


FIGURE 7.3

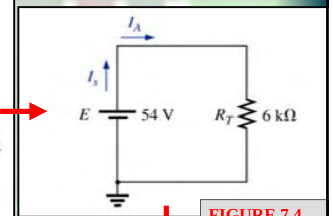
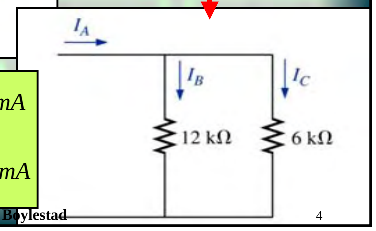


FIGURE 7.4



$$I_B = \frac{6k\Omega\,(I_s)}{6k\Omega + 12k\Omega} = \frac{1}{3}I_s = \frac{1}{3}(9mA) = 3\,mA$$

$$I_C = \frac{12k\Omega\,(I_s)}{12k\Omega + 6k\Omega} = \frac{2}{3}I_s = \frac{2}{3}(9mA) = 6\,mA$$

Ex. 7-2 It is also possible that the blocks A, B, and C of *Fig. 7.2* contain the elements and configurations in *Fig. 7.5*. Working with each region:

$$A: \quad R_A = 4\,\Omega$$

$$B: \quad R_B = R_2 \,//\, R_3 = R_{2//3} = \frac{R}{N} = \frac{4\,\Omega}{2} = 2\,\Omega$$

$$C: \quad R_C = R_4 + R_5 = R_{4,5} = 2\,\Omega$$

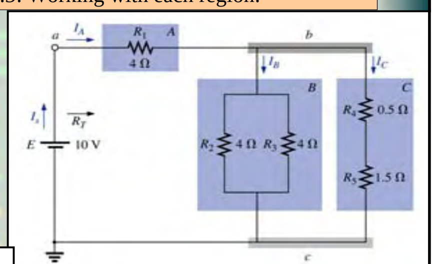


FIGURE 7.5

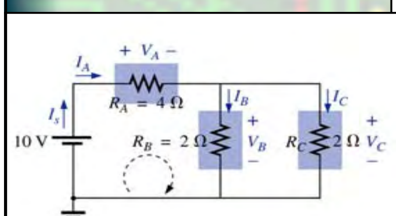


FIGURE 7.6

$$R_{B//C} = \frac{R}{N} = \frac{2\,\Omega}{2} = 1\,\Omega$$

$$R_T = R_A + R_{B//C} = 4\,\Omega + 1\,\Omega = 5\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{10V}{5\,\Omega} = 2\,A$$

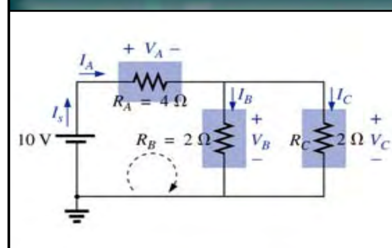


FIGURE 7.6

$$I_A = I_s = 2\,A$$

$$I_B = I_C = \frac{I_A}{2} = \frac{I_s}{2} = \frac{2\,A}{2} = 1\,A$$

$$I_{R_2} = I_{R_3} = \frac{I_B}{2} = 0.5\,A$$

$$V_A = I_A R_A = (2\,A)(4\,\Omega) = 8\,V$$

$$V_B = I_B R_B = (1\,A)(2\,\Omega) = 2\,V$$

$$V_C = V_B = 2\,V$$

Ex. 7-3 Another possible variation of *Fig. 7.2* appears in *Fig. 7.7*.

$$R_A = R_{1//2} = \frac{(9\,\Omega)(6\,\Omega)}{9\,\Omega+6\,\Omega} = \frac{54\,\Omega}{15} = 3.6\,\Omega$$

$$R_B = R_3 + R_{4//5} = 4\,\Omega + \frac{(9\,\Omega)(3\,\Omega)}{9\,\Omega+3\,\Omega} = 4\,\Omega+2\,\Omega = 6\,\Omega$$

$$R_C = 3\,\Omega$$

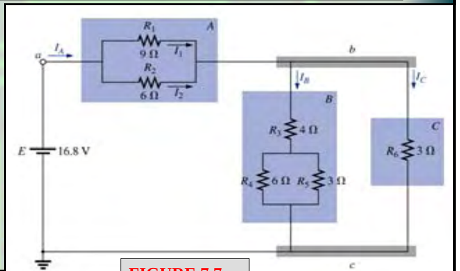


FIGURE 7.7

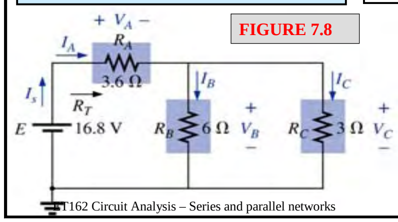
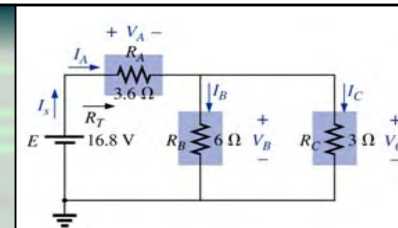


FIGURE 7.8

$$R_T = R_A + R_{B//C} = 3.6\,\Omega + \frac{(6\,\Omega)(3\,\Omega)}{6\,\Omega+3\,\Omega} = 3.6\,\Omega+2\,\Omega = 5.6\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{16.8\,V}{5.6\,\Omega} = 3\,A$$

$$I_A = I_s = 3\,A$$



Applying the current divider rule yields

$$I_B = \frac{R_C\, I_A}{R_C + R_B} = \frac{(3\,\Omega)(3\,A)}{3\,\Omega+6\,\Omega} = 1\,A$$

By Kirchhoff's current law,

$$I_C = I_A - I_B = 3\,A - 1\,A = 2\,A$$

By Ohm's law,

$$V_A = I_A\, R_A = (3\,A)(3.6\,\Omega) = 10.8\,V$$

$$V_B = I_B\, R_B = V_C = I_C\, R_C = (2\,A)(3\,\Omega) = 6\,V$$

$$I_1 = \frac{R_2\, I_A}{R_2 + R_1} = \frac{(6\,\Omega)(3\,A)}{6\,\Omega + 9\,\Omega} = 1.2\,A$$

$$I_2 = I_A - I_1 = 3\,A - 1.2\,A = 1.8\,A$$

Series-Parallel Networks - Descriptive Examples

Ex. 7-4 Find the current I_4 and the voltage V_2 for the network of *Fig. 7.2*.

$$I_4 = \frac{E}{R_B} = \frac{E}{R_4} = \frac{12\,V}{8\,\Omega} = 1.5\,A$$

$$R_D = R_2 // R_3 = 3\,\Omega // 6\,\Omega = 2\,\Omega$$

$$V_2 = \frac{R_D\, E}{R_D + R_C} = \frac{(2\,\Omega)(12\,V)}{2\,\Omega+4\,\Omega} = 4\,V$$

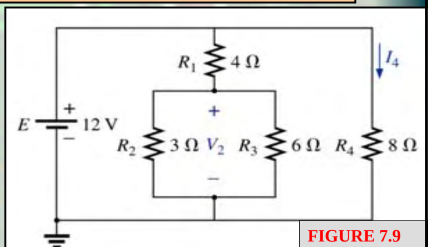


FIGURE 7.9

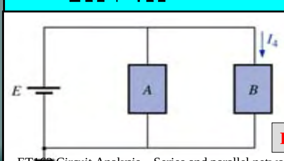


FIGURE 7.10

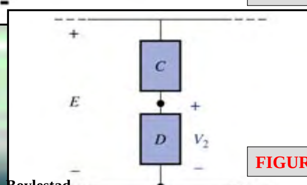


FIGURE 7.11

Ex. 7-5 Find the indicated currents and the voltages for the network of *Fig. 7.12*.

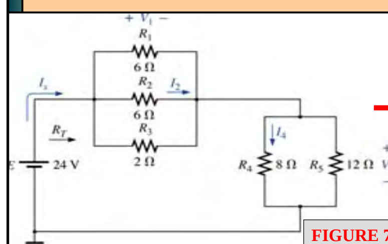


FIGURE 7.12

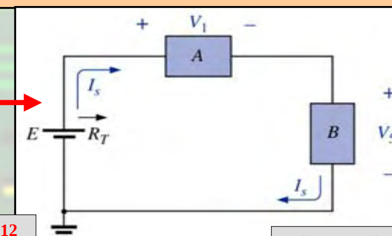


FIGURE 7.13

$$R_{1//2} = \frac{R}{N} = \frac{6\,\Omega}{2} = 3\,\Omega$$

$$R_A = R_{1//2//3} = \frac{(3\,\Omega)(2\,\Omega)}{3\,\Omega+2\,\Omega} = \frac{6\,\Omega}{5} = 1.2\,\Omega$$

$$R_B = R_{4//5} = \frac{(8\,\Omega)(12\,\Omega)}{8\,\Omega+12\,\Omega} = \frac{96\,\Omega}{20} = 4.8\,\Omega$$

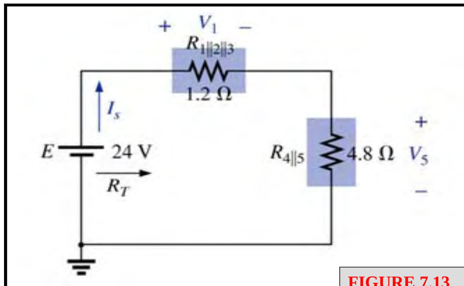


FIGURE 7.13

$$R_T = R_{1//2//3} + R_{4//5} = 1.2\,\Omega + 4.8\,\Omega = 6\,\Omega$$

$$I_s = \frac{E}{R_T} = \frac{24\,V}{6\,\Omega} = 4\,A$$

$$V_1 = I_s\,R_{1//2//3} = (4\,A)(1.2\,\Omega) = 4.8V$$
$$V_2 = I_s\,R_{4//5} = (4\,A)(4.8\,\Omega) = 19.2V$$

$$I_4 = \frac{V_5}{R_4} = \frac{19.2V}{8\Omega} = 2.4\,A$$

$$I_2 = \frac{V_2}{R_2} = \frac{V_1}{R_2} = \frac{4.8V}{6\Omega} = 0.8\,A$$

Ex. 7-6 **a.** Find the voltages V_1, V_2, and V_{ab} for the network of *Fig. 7.14*.
b. Calculate the source current I_s.

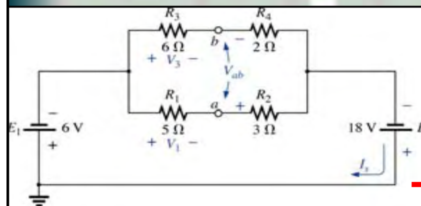


FIGURE 7.14

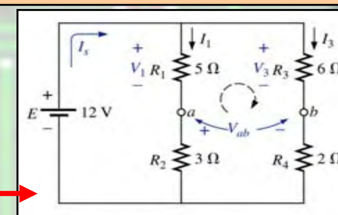


FIGURE 7.15

a.

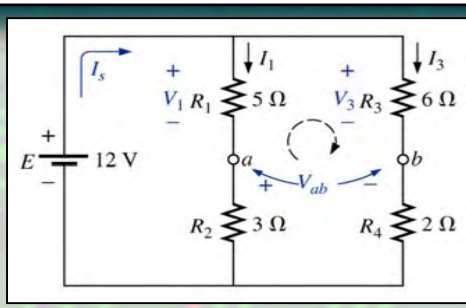
Applying the voltage divider rule yields

$$V_1 = \frac{R_1 E}{R_1 + R_2} = \frac{(5\Omega)(12V)}{5\Omega + 3\Omega} = 7.5\,V$$

$$V_3 = \frac{R_3 E}{R_3 + R_4} = \frac{(6\Omega)(12V)}{6\Omega + 2\Omega} = 9\,V$$

Applying Kirchhoff's voltage law around the indicated loop of Fig.

$$-V_1 + V_3 - V_{ab} = 0$$
$$V_{ab} = V_3 - V_1 = 9V - 7.5V = 1.5V$$



b.

By Ohm's law,

$$I_1 = \frac{V_1}{R_1} = \frac{7.5V}{5\Omega} = 1.5\,A$$

$$I_3 = \frac{V_3}{R_3} = \frac{9V}{6\Omega} = 1.5\,A$$

Applying Kirchhoff's current law,
$I_s = I_1 + I_3 = 1.5A + 1.5A = 3A$

Ex. 7-7 For the network of *Fig. 7.16*, determine the voltages V_1 and V_2 and current I.

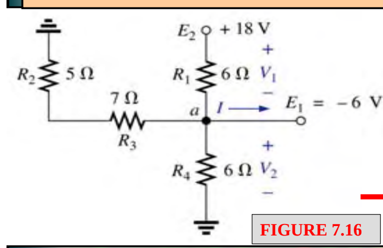


FIGURE 7.16

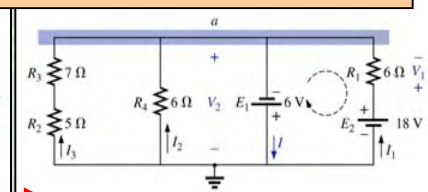


FIGURE 7.17

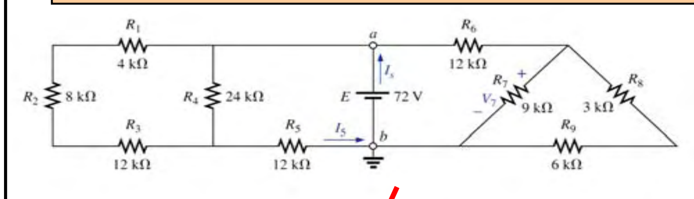
$V_2 = -E_1 = -6V$

Applying KVL to the loop
$E_1 - V_1 + E_2 = 0$
$V_1 = E_2 + E_1 = 18V + 6V = 24V$

Applying KCL to node a yields

$$I = I_1 + I_2 + I_3 = \frac{V_1}{R_1} + \frac{E_1}{R_4} + \frac{E_1}{R_2 + R_3} = \frac{24V}{6\Omega} + \frac{6V}{6\Omega} + \frac{6V}{12\Omega} = 4\,A + 1\,A + 0.5\,A = 5.5\,A$$

Ex. 7-9 Calculate the indicated currents and voltage of *Fig.7.17*.



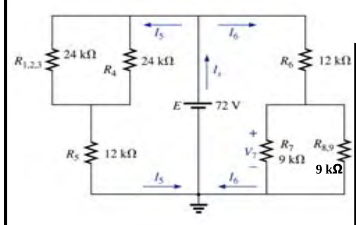


FIGURE 7.17.

$$I_5 = \frac{E}{R_{(1,2,3)//4} + R_5} = \frac{72V}{12k\Omega + 12k\Omega} = \frac{72V}{24k\Omega} = 3mA$$

$$V_7 = \frac{R_{7//(8,9)}E}{R_{7//(8,9)} + R_6} = \frac{(4.5k\Omega)(72V)}{4.5k\Omega + 12k\Omega} = \frac{324V}{16.5} = 19.6V$$

$$I_6 = \frac{V_7}{R_{7//(8,9)}} = \frac{19.6V}{4.5k\Omega} = 4.35mA$$

$I_s = I_5 + I_6 = 3\ mA + 4.35\ mA = 7.35\ mA$

Ex. 7-10 This example demonstrates the power of Kirchhoff's voltage law by determining the voltages V_1, V_2, and V_3 for the network of *Fig.7.18*.

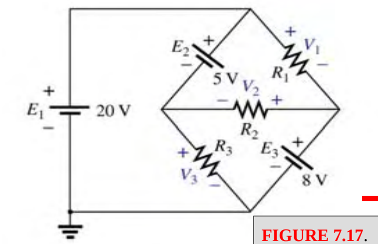


FIGURE 7.17.

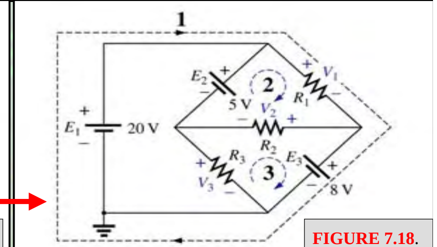


FIGURE 7.18.

For the path 1, $-E_1 + V_1 + E_3 = 0$

$V_1 = E_1 - E_3 = 20V - 8V = 12\ V$

For the path 2, $-E_2 + V_1 + V_2 = 0$

$V_2 = E_2 - V_1 = 5V - 12V = -7\ V$

For the path 3, $-V_3 - V_2 + E_3 = 0$

$V_3 = E_3 - V_2 = 8V - (-7V) = 15\ V$

Series-Parallel Networks – Ladder Networks

A three-section ladder appears in *Fig. 7.19*.

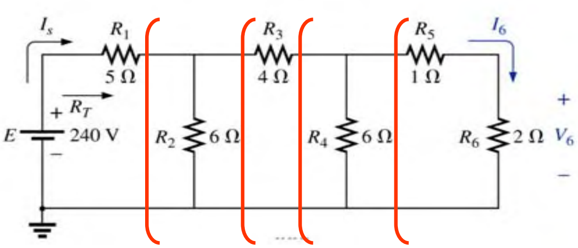


FIGURE 7.19. Ladder network.

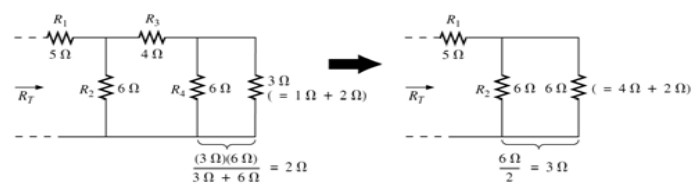
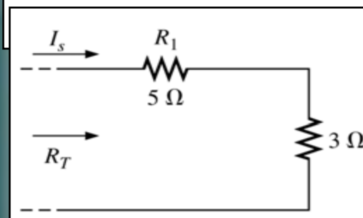


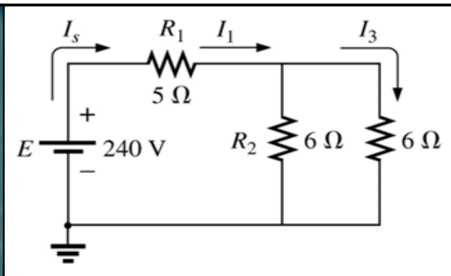
FIGURE 7.20.



$R_T = 5\Omega + 3\Omega = 8\Omega$

$$I_s = \frac{E}{R_T} = \frac{240V}{8\Omega} = 30\ A$$

FIGURE 7.21.

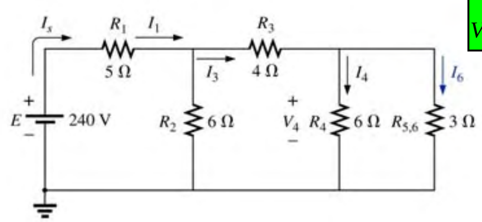


$$I_1 = I_s$$

$$I_3 = \frac{I_s}{2} = \frac{30\,A}{2} = 15\,A$$

$$I_6 = \frac{(6\Omega)\,I_3}{6\Omega + 3\Omega} = \frac{6}{9}(15\,A) = 10\,A$$

$$V_6 = I_6\,R_6 = (10\,A)(2\,\Omega) = 20\,V$$



HW 7-26 For the ladder network in Fig. 7.86:

a Determine R_T.

b. Calculate I.

c. Find I_8.

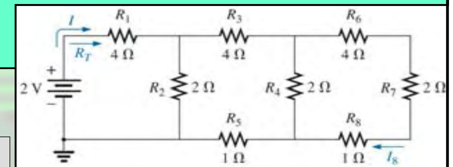


Figure 7.86 Problem 26.

a. $R'_T = R_4 \parallel (R_6 + R_7 + R_8) = 2\ \Omega \parallel 7\ \Omega = 1.56\ \Omega$

$R''_T = R_2 \parallel (R_3 + R_5 + R'_T) = 2\ \Omega \parallel (4\ \Omega + 1\ \Omega + 1.56\ \Omega) = 1.53\ \Omega$

$R_T = R_1 + R''_T = 4\ \Omega + 1.53\ \Omega = \mathbf{5.53\ \Omega}$

b. $I = 2\text{ V}/5.53\ \Omega = \mathbf{361.66\text{ mA}}$

c. $$I_3 = \frac{2\,\Omega(I)}{2\,\Omega + 6.56\,\Omega} = \frac{2\,\Omega(361.66\text{ mA})}{2\,\Omega + 6.56\,\Omega} = \mathbf{84.50\text{ mA}}$$

$$I_8 = \frac{2\,\Omega(84.5\text{ mA})}{2\,\Omega + 7\,\Omega} = \mathbf{18.78\text{ mA}}$$

Homework 7: 2, 4, 7, 11, 15, 25, 26

EET/1122/ET162 Circuit Analysis

Methods of Analysis

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 10th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Method Analysis
- Current Sources
- Source Conversions
- Current Sources in Series
- Branch Current Analysis
- Mesh & Super Mesh Analysis
- Nodal & Super Nodal Analysis

Key Words: Current Source, Source Conversion, Branch Current, Mesh Analysis, Nodal Analysis

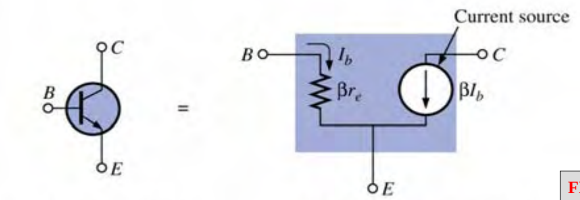
Introduction to Methods of Analysis

The circuits described in the previous chapters had only one source or two or more sources in series or parallel present. The step-by-step procedure outlined in those chapters cannot be applied if the sources are not in series or parallel.

Methods of analysis have been developed that allow us to approach, in a systematic manner, a network with any number of sources in any arrangement. **Branch-current analysis**, **mesh analysis**, and **nodal analysis** will be discussed in detail in this chapter.

Current Sources

The interest in the current sources is due primarily to semiconductor devices such as the transistor. In the physical model (equivalent circuit) of a transistor used in the analysis of transistor networks, there appears a current source as indicated in Fig. 8.1.



(a) Transistor symbol (b) Transistor equivalent circuit

FIGURE 8.1 Current source within the transistor equivalent circuit.

***A current source determines the current in the branch in which it is located and the magnitude* and *polarity of the voltage across a current source are a function of the network to which it is applied*.**

Ex. 8-1 Find the source voltage V_s and the current I_1 for the circuit of *Fig. 7.2*.

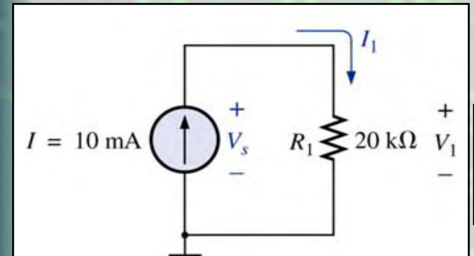


FIGURE 8.2

$I_1 = I = 10 \text{ mA}$

$V_s = V_1 = I_1R_1$

$= (10 \text{ mA})(20 \text{ k}\Omega)$

$= 200 \text{ V}$

Ex. 8-2 Find the voltage V_s and the currents I_1 and I_2 for the network of *Fig. 8.3*.

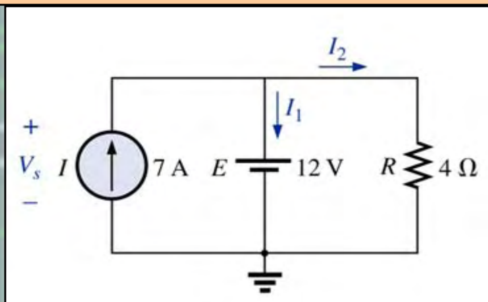


FIGURE 8.3

$$V_s = E = 12V$$

$$I_2 = \frac{V_R}{R} = \frac{E}{R} = \frac{12V}{4\Omega} = 3\,A$$

Applying Kirchhoff's current law:

$$I = I_1 + I_2$$

$$I_1 = I - I_2 = 7\,A - 3\,A = 4\,A$$

Ex. 8-3 Determine the current I_1 and voltage V_s for the network of *Fig. 8.4*.

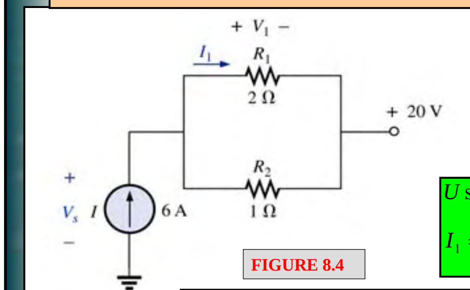


FIGURE 8.4

Using the current divider rule:

$$I_1 = \frac{R_2\,I}{R_2 + R_1} = \frac{(1\Omega)(6\,A)}{(1\Omega) + (2\Omega)} = 2\,A$$

The voltage V_1 is

$$V_1 = I_1R_1 = (2\,A)(2\,\Omega) = 4V$$

and, applying Kirchhoff's voltage law,

$$-V_s + V_1 + 20V = 0$$

$$V_s = V_1 + 20V = 4V + 20V = 24V$$

Source Conversions

All sources-whether they are voltage or current-have some internal resistance in the relative positions shown in *Fig. 8.5* and *8.6*. For the voltage source, if $R_s = 0\ \Omega$ or is so small compared to any series resistor that it can be ignored, then we have an "ideal" voltage source. For the current source, if $R_s = \infty\ \Omega$ or is large enough compared to other parallel elements that it can be ignored, then we have an "ideal" current source.

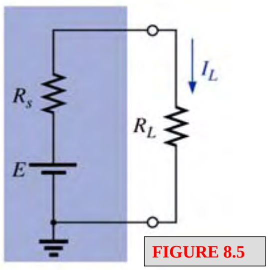


FIGURE 8.5

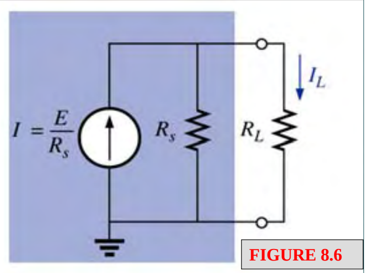


FIGURE 8.6

The equivalent sources, as far as terminals **a** and **b** are concerned, are repeated in *Fig. 8.7* with the equations for converting in either direction. Note, as just indicated, that the resistor R_s is the same in each source; only its position changes. The current of the current source or the voltage of the voltage source is determined using Ohm's law and the parameters of the other configuration.

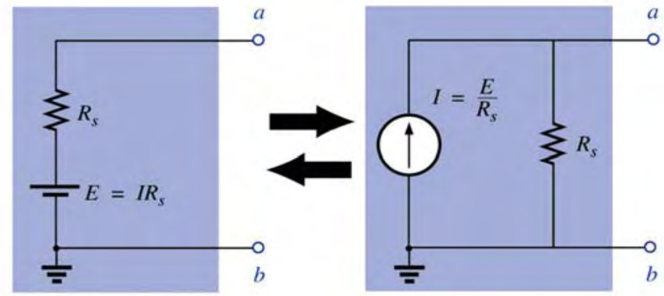
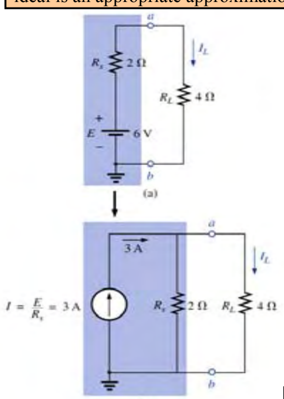


FIGURE 8.6 Source conversion

Ex. 8-4 **a**. Convert the voltage source of Fig. 8.8 (a) to a current source, and calculate the current through the 4-Ω load for each source.
b. Replace the 4-Ω load with a 1-kΩ load, and calculate the current I_L for the voltage source.
c. Replace the calculation of part (b) assuming that the voltage source is ideal ($R_s = 0\ \Omega$) because R_L is so much larger than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?



$$a.\ \ Fig.\,8.8\,(a):\quad I_L = \frac{E}{R_s + R_L} = \frac{6V}{2\Omega + 4\Omega} = 1\,A$$

$$Fig.\,8.8\,(b):\quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(2\ \Omega)(3\,A)}{2\Omega + 4\Omega} = 1\,A$$

$$b.\ \ I_L = \frac{E}{R_s + R_L} = \frac{6V}{2\Omega + 1k\Omega} \cong 5.99\ A$$

$$c.\ \ I_L = \frac{E}{R_L} = \frac{6V}{1k\Omega} = 6\ mA \cong 5.99\ mA$$

$$Yes,\ R_L >> R_s\ (voltage\ source)$$

FIGURE 8.8

Ex. 8-5 **a**. Convert the current source of *Fig. 8.9(a)* to a voltage source, and find the load current for each source.
b. Replace the 6-kΩ load with a 10-kΩ load, and calculate the current I_L for the current source.
c. Replace the calculation of part (b) assuming that the vcurrent source is ideal ($R_s = \infty\ \Omega$) because R_L is so much smaller than R_s. Is this one of those situations where assuming that the source is ideal is an appropriate approximation?

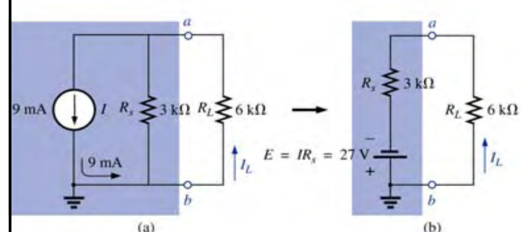


FIGURE 8.9

$$a.\ \ Fig.\,8.9\,(a):\quad I_L = \frac{R_s I}{R_s + R_L} = \frac{(3\ k\Omega)(9\,mA)}{3\,k\Omega + 6\,k\Omega} = 3\,mA$$

$$Fig.\,8.9\,(b):\quad I_L = \frac{E}{R_s + R_L} = \frac{27V}{3\,k\Omega + 6\,k\Omega} = 3\,mA$$

$$b.\ \ I_L = \frac{R_s I}{R_s + R_L} = \frac{(3\,k\Omega)(9\,mA)}{3\,k\Omega + 10\Omega} \cong 8.97\ A$$

$$c.\ \ I_L = I = 9\ mA \cong 8.97\ mA$$

$$Yes,\ R_s >> R_L\ (current\ source)$$

Ex. 8-6 Replace the parallel current sources of *Fig. 8.10* and *8.11* to a single current source.

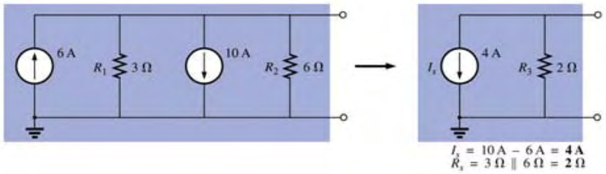


FIGURE 8.10

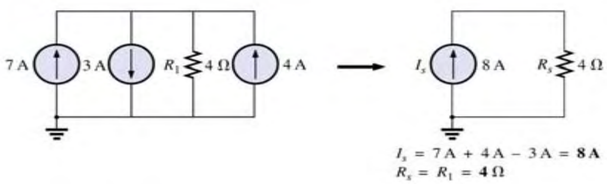


FIGURE 8.11

Ex. 8-7 Reduce the network of *Fig. 8.12* to a single current source, and calculate the current through R_L.

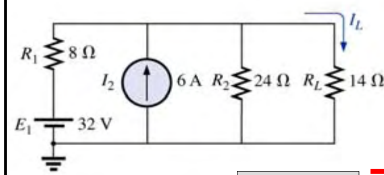
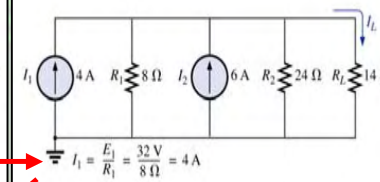
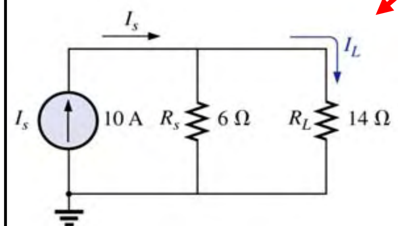


FIGURE 8.12





$$I_s = I_1 + I_2 = 4\,A + 6\,A = 10\,A$$

$$R_s = R_1 \,//\, R_2 = 8\Omega \,//\, 24\Omega = 6\Omega$$

$$I_L = \frac{R_s I_s}{R_s + R_L} = \frac{(6\Omega)(10\,A)}{6\Omega + 14\Omega} = 3\,A$$

Ex. 8-8 Determine the currentI2 in the network of *Fig. 8.13*.

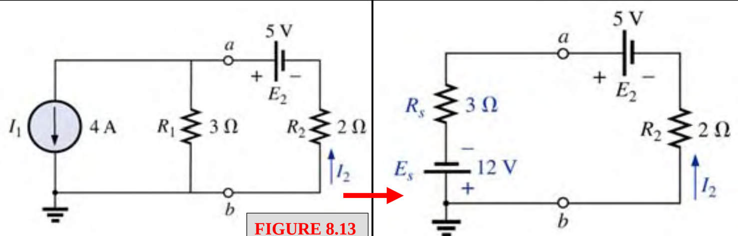


FIGURE 8.13

$$E_s = I_1 R_1 = (4\,A)(3\Omega) = 12V$$

$$R_s = R_1 = 3\Omega$$

$$I_2 = \frac{E_s + E_2}{R_s + R_2} = \frac{12V + 5V}{3\Omega + 2\Omega} = 3.4\,A$$

Current Sources in Series

The current through any branch of a network can be only single-valued. For the situation indicated at point a in Fig. 8.14, we find by application of Kirchoff's current law that the current leaving that point is greater than entering-an impossible situation. Therefore,

Current sources of different current ratings are not connected in parallel.

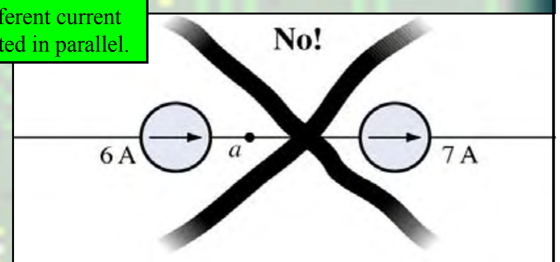


FIGURE 8.14

Branch-Current Analysis

We will now consider the first in a series of methods for solving networks with two or more sources.

1. ***Assign distinct current of arbitrary direction to each branch of the network.***
2. ***Indicate the polarities for each resistor as determined by the assumed current direction.***
3. ***Apply Kirchhoff's voltage law around each closed, independent loop of the network.***
4. ***Apply Kirchhoff's current law at the minimum number of nodes that will include all the branch currents of the network.***
5. ***Solve the resulting simultaneous linear equations for assumed branch currents.***

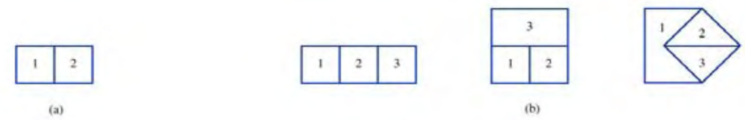


FIGURE 8.15 Determining the number of independent closed loops.

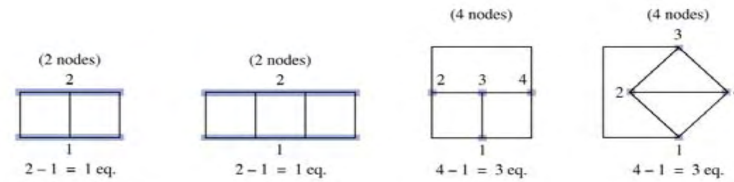


FIGURE 8.16 Determining the number of applications of Kirchhoff's current law required.

Ex. 8-9 Apply the branch-current method to the network of *Fig. 8.17*.

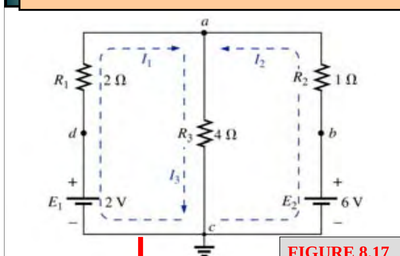
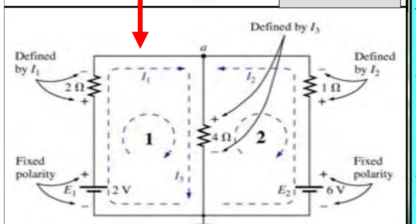


FIGURE 8.17



$$loop\,1:\quad \sum V = -E_1 + V_{R_1} + V_{R_3} = 0$$
$$loop\,2:\quad \sum V = -V_{R_3} - V_{R_2} + E_2 = 0 \quad and$$
$$loop\,1:\quad \sum V = -2V + (2\Omega)I_1 + (4\Omega)I_3 = 0$$
$$loop\,2:\quad \sum V = -(4\Omega)I_3 - (1\Omega)I_2 + 6V = 0$$

Applying $I_1 + I_2 = I_3$

$$loop\,1:\quad \sum V = -2V + (2\Omega)I_1 + (4\Omega)(I_1 + I_2) = 0$$
$$loop\,2:\quad \sum V = -(4\Omega)(I_1 + I_2) - (1\Omega)I_2 + 6V = 0$$

$$6I_1 + 4I_2 = 2$$
$$-4I_1 - 5I_2 = -6$$
$$I_1 = \frac{\begin{vmatrix} 2 & 4 \\ -6 & -5 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ -4 & -5 \end{vmatrix}} = \frac{-10-(-24)}{-30-(-16)} = -1\ A$$
$$I_2 = \frac{\begin{vmatrix} 6 & 2 \\ -4 & -6 \end{vmatrix}}{\begin{vmatrix} 6 & 4 \\ -4 & -5 \end{vmatrix}} = \frac{-36-(-8)}{-30-(-16)} = 2\ A$$
$$I_3 = I_1 + I_2 = -1\,A + 2\,A = 1\,A$$

Mesh Analysis

The second method of analysis to be described is called **mesh analysis**. The term *mesh* is derived from the similarities in appearance between the closed loops of a network and wire mesh fence. The **mesh-analysis** approach simply eliminates the need to substitute the results of Kirchhoff's current law into the equations derived from Kirchhoff's voltage law. The systematic approach outlined below should be followed when applying this method.

1. ***Assign a distinct current in the clockwise direction to each independent, closed loop of the network. It is not absolutely necessary to choose the clockwise direction for each loop current.***
2. ***Indicate the polarities with each loop for each resistor as determined by the assumed current direction of loop current for that loop.***
3. ***Apply Kirchhoff's voltage law around each closed loop in the clockwise direction.***
4. ***Solve the resulting simultaneous linear equations for assumed branch currents.***

Ex. 8-10 Consider the same basic network as in Example 8.9 of the preceding dection, now appearing in *Fig.8.18*.

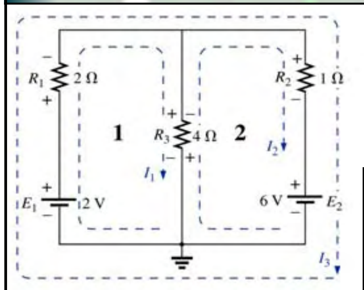


FIGURE 8.18

$loop\,1{:}\quad -2V+(2\Omega)I_1+(4\Omega)(I_1-I_2)=0$

$loop\,2{:}\quad +(4\Omega)(I_2-I_1)+(1\Omega)I_2+6V=0$

and

$loop\,1{:}\quad -2V+6I_1-4I_2=0$

$loop\,2{:}\quad +5I_2-4I_1+6V=0$

$6I_1-4I_2=2$

$-4I_1+5I_2=-6$

$$I_1=\frac{\begin{vmatrix}2 & -4\\ -6 & 5\end{vmatrix}}{\begin{vmatrix}6 & -4\\ -4 & 5\end{vmatrix}}=\frac{10-24}{30-16}=-1\ A$$

$$I_2=\frac{\begin{vmatrix}6 & 2\\ -4 & -6\end{vmatrix}}{\begin{vmatrix}6 & -4\\ -4 & 5\end{vmatrix}}=\frac{-36-(-8)}{30-16}=-2\ A$$

$I_1=-1\ A\quad and\quad I_2=-2\ A$

Ex. 8-11 Find the current through each branch of the network of *Fig.8.19*.

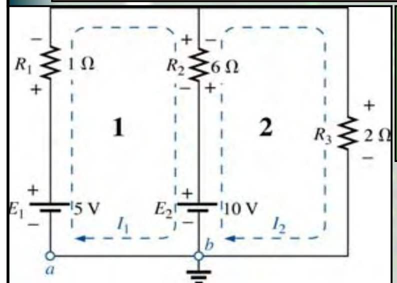


FIGURE 8.19

$loop\,1{:}\quad -5V+(1\Omega)I_1+(6\Omega)(I_1-I_2)+10V=0$

$loop\,2{:}\quad -10V+(6\Omega)(I_2-I_1)+(2\Omega)I_2=0$

and

$loop\,1{:}\quad 7I_1-6I_2=-5$

$loop\,2{:}\quad -6I_1+8I_2=10$

$$I_1=\frac{\begin{vmatrix}-5 & -6\\ 10 & 8\end{vmatrix}}{\begin{vmatrix}7 & -6\\ -6 & 8\end{vmatrix}}=\frac{-40-(-60)}{56-36}=1\ A$$

$$I_2=\frac{\begin{vmatrix}7 & -5\\ -6 & 10\end{vmatrix}}{\begin{vmatrix}7 & -6\\ -6 & 10\end{vmatrix}}=\frac{70-30}{56-36}=2\ A$$

$I_1=1\,A\quad and\quad I_2=2\,A$

The current in the 6Ω resistor and 10V source for loop 1 is

$I_2-I_1=2A-1A=1A$

Ex. 8-12 Find the branch currents of the network of *Fig.8.20*.

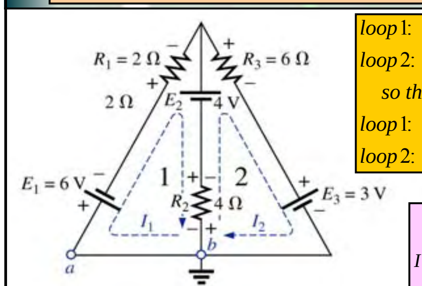


FIGURE 8.20

$loop\,1{:}\quad 6V+(2\Omega)I_1+4V+(4\Omega)(I_1-I_2)=0$

$loop\,2{:}\quad (4\Omega)(I_2-I_1)-4V+(6\Omega)I_2+3V=0$

so that

$loop\,1{:}\quad 6I_1-4I_2=-10$

$loop\,2{:}\quad -4I_1+10I_2=1$

$$I_1=\frac{\begin{matrix}-10 & -4\\ 1 & 10\end{matrix}}{\begin{matrix}6 & -4\\ -4 & 10\end{matrix}}=\frac{-100-(-4)}{60-16}=-2.182\ A$$

$$I_2=\frac{\begin{matrix}6 & -10\\ -4 & 1\end{matrix}}{\begin{matrix}6 & -4\\ -4 & 10\end{matrix}}=\frac{6-40}{60-16}=-0.773\ A$$

$I_1=-2.182\ A\quad and\quad I_2=-0.773\ A$

The current in the 4Ω resistor and 4V source for loop 1 is

$I_1-I_2=-2.182A-(-0.773A)$

$=-1.409A$

Ex. 8-13 Using mesh analysis, determine the currents of the network of *Fig.8.21*.

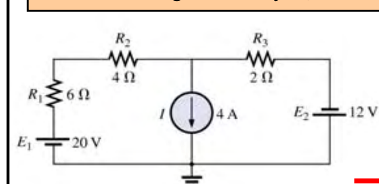
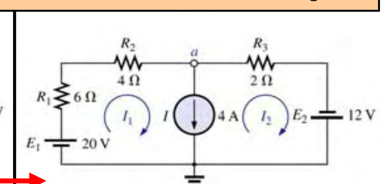
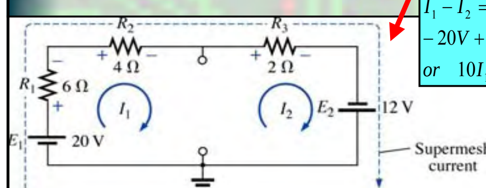


FIGURE 8.21





$I_1-I_2=4\,A$

$-20V+(6\Omega)I_1+(4\Omega)I_1+(2\Omega)I_2-12V=0$

$or\quad 10I_1+2I_2=32$

$Applying\quad I_1=I_2+4\,A$

$10(I_2+4)+2I_2=32$

$12I_2=-8\quad or\quad I_2=-0.67\ A$

$I_1=3.33\ A$

Nodal Analysis

We will employ Kirchhoff's current law to develop a method referred to as **nodal analysis**. A node is defined as a junction of two or more branches. Since a point of zero potential or ground is used as a reference, the remaining nodes of the network will all have a fixed a fixed potential relative to this reference. For a network of N nodes, therefore, there will exist (N – 1) nodes.

1. *Determine the number of nodes within the network.*
2. *Pick a reference node, and label each remaining node with a subscripted value of voltage: V_1, V_2, and so on.*
3. *Apply Kirchhoff's current law at each node except the reference. Assume that all unknown currents leave the node for each application of Kirchhoff's current law.*
4. *Solve the resulting equations for the nodal voltages.*

Ex. 8-14 Apply nodal analysis to the network of *Fig.8.22*.

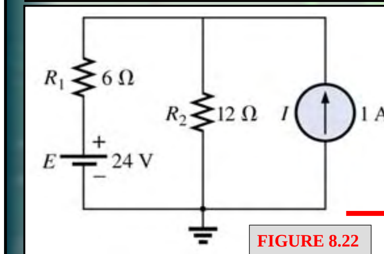
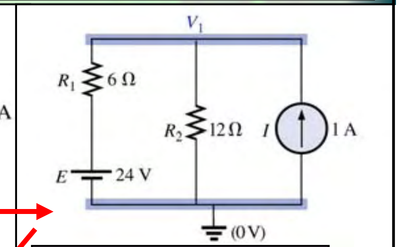
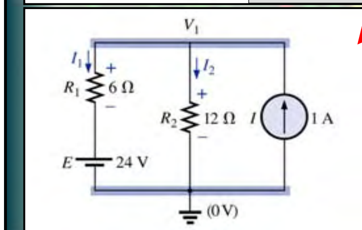


FIGURE 8.22





$$\text{Node 1: } \frac{V_1 - 24}{6\Omega} + \frac{V_1}{12\Omega} - 1\,A = 0$$

$$2V_1 - 48 + V_1 - 12 = 0$$

$$3V_1 = 60 \quad or \quad V_1 = 20V$$

$$I_1 = \frac{V_1 - 24}{6\Omega} = -0.667\,A$$

$$I_2 = \frac{20V}{12\Omega} = 1.667\,A$$

Ex. 8-15 Apply nodal analysis to the network of *Fig.8.23*.

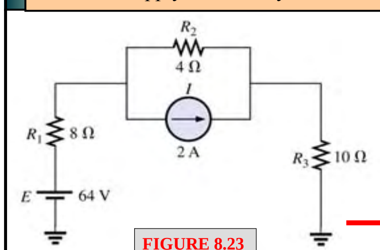
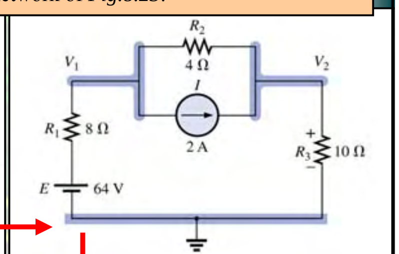
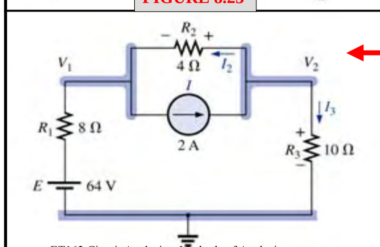
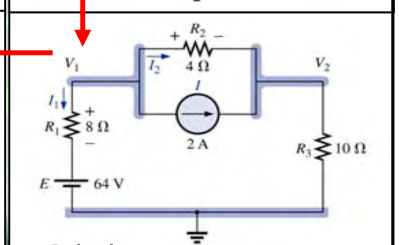
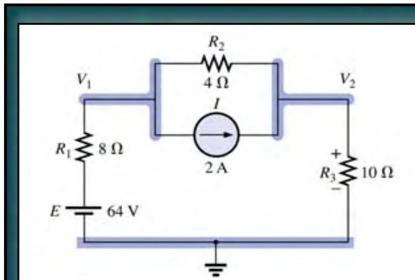


FIGURE 8.23









$$\text{Node 1: } \frac{V_1 - 64V}{8\Omega} + \frac{V_1 - V_2}{4\Omega} + 2\,A = 0$$

$$\text{Node 2: } \frac{V_2 - V_1}{4\Omega} - 2\,A + \frac{V_2}{10\Omega} = 0$$

$$V_1 - 64 + 2V_1 - 2V_2 + 16 = 0$$

$$5V_2 - 5V_1 - 40 + 2V_2 = 0$$

so that

$$3V_1 - 2V_2 = 48$$

$$-5V_1 + 7V_2 = 40$$

$$V_1 = \frac{\begin{vmatrix} 48 & -2 \\ 40 & 7 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -5 & 7 \end{vmatrix}} = \frac{336 - (-80)}{21 - 10} = 37.818\,V$$

$$V_2 = \frac{\begin{vmatrix} 3 & 48 \\ -5 & 40 \end{vmatrix}}{\begin{vmatrix} 3 & -2 \\ -5 & 7 \end{vmatrix}} = \frac{120 - (-240)}{21 - 10} = 32.727\,V$$

$$I_{R_1} = \frac{E - V_1}{R_1} = \frac{64V - 37.818V}{8\Omega} = 3.273\,A$$

$$I_{R_3} = \frac{V_{R_3}}{R_3} = \frac{V_2}{R_3} = \frac{32.727V}{10\Omega} = 3.273\,A$$

$$I_{R_2} = \frac{V_1 - V_2}{R_2} = \frac{37.818V - 32.727V}{4\Omega} = 1.273\,A$$

Ex. 8-16 Determine the nodal voltages for the network of *Fig.8.24*.

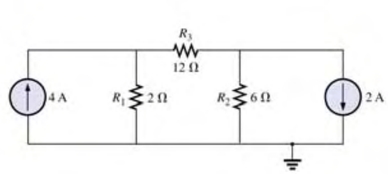
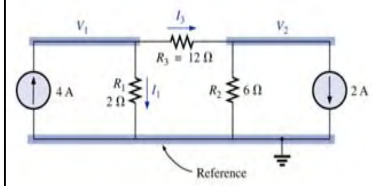


FIGURE 8.24



$$Node\ 1: \quad -4 + \frac{V_1}{2\Omega} + \frac{V_1 - V_2}{12\Omega} = 0$$

$$Node\ 2: \quad \frac{V_2 - V_1}{12\Omega} + \frac{V_2}{6\Omega} + 2 = 0$$

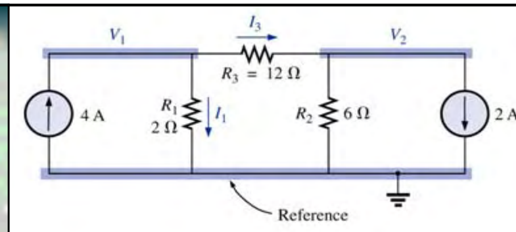
$$48 + 6V_1 + V_1 - 2V_2 = 0$$

$$V_2 - V_1 + 2V_2 + 24 = 0$$

so that

$$7V_1 - V_2 = 48$$

$$-V_1 + 3V_2 = -24$$



$$V_1 = \frac{\begin{vmatrix} 48 & -1 \\ -24 & 3 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{144-24}{21-1} = 6V$$

$$V_2 = \frac{\begin{vmatrix} 7 & 48 \\ -1 & -24 \end{vmatrix}}{\begin{vmatrix} 7 & -1 \\ -1 & 3 \end{vmatrix}} = \frac{-168-(-48)}{21-1} = -6V$$

$$I_{R_3} = \frac{V_1 - V_2}{R_3} = \frac{6V-(-6V)}{12\Omega} = 1A$$

$$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_1}{R_1} = \frac{6V}{2\Omega} = 3A$$

$$I_{R_2} = \frac{V_{R_2}}{R_2} = \frac{V_2}{R_2} = \frac{6V}{6\Omega} = 1A$$

Ex. 8-17 Determine the nodal voltages V_1 and V_2 *Fig.8.25* using the concept of a supernode.

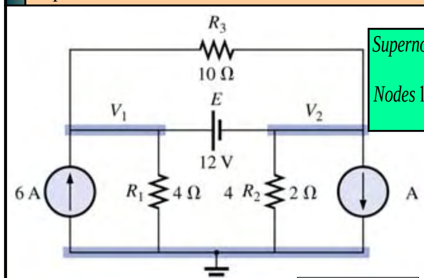


FIGURE 8.25

$$Supernode: \quad V_1 - V_2 = 12V$$

$$Nodes\ 1,2: \quad -6V + \frac{V_1 - V_2}{10\Omega} + \frac{V_1}{4\Omega} + \frac{V_2 - V_1}{10\Omega} + \frac{V_2}{2\Omega} + 4V = 0$$

$$V_1 = \frac{\begin{vmatrix} 12 & -1 \\ 40 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & -1 \\ 5 & 10 \end{vmatrix}} = \frac{120-(-40)}{10-(-5)} = 10.667V$$

$$V_2 = V_1 - 12V = -1.333V$$

$$V_1 - V_2 = 12$$

$$-120 + 2V_1 - 2V_2 + 5V_1 + 2V_2 - 2V_1 + 10V_2 + 80 = 0$$

so that

$$V_1 - V_2 = 12$$

$$5V_1 + 10V_2 = 40$$

HW 8-31 Using mesh analysis, determine the current I_3 for the network in Fig. 8.119. Compare your answer to the solution of Problem 18.

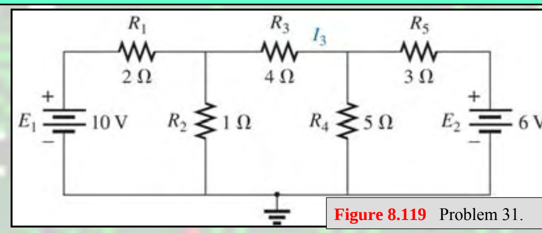


Figure 8.119 Problem 31.

I_1 I_2 I_3

$$I_1(2+1) - 1I_2 = 10$$

$$I_2(1+4+5) - 1I_1 - 5I_3 = 0$$

$$I_3(5+3) - 5I_2 = -6$$

$I_2 = I_{R_3}$ = **−63.69 mA** (exact match with problem 18)

Homework 8: 2, 4, 6, 7, 8, 19, 22, 23, 25, 31

EET1122/ET162 Circuit Analysis

Methods of Analysis

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 10th edition.

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Sunghoon Jang

OUTLINES

- Introduction to Network Theorems
- Superposition
- Thevenin's Theorem
- Norton's Theorem
- Maximum Power Transfer Theorem

Key Words: Network Theorem, Superposition, Thevenin, Norton, Maximum Power

Introduction to Network Theorems

This chapter will introduce the important fundamental theorems of network analysis. Included are the superposition, Thevenin's, Norton's, and maximum power transfer theorems. We will consider a number of areas of application for each. A through understanding of each theorems will be applied repeatedly in the material to follow.

Superposition Theorem

The **superposition theorem** can be used to find the solution to networks with two or more sources that are not in series or parallel. The most advantage of this method is that it does not require the use of a mathematical technique such as determinants to find the required voltages or currents.

The current through, or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the current or voltages produced independently by each source.

Number of networks to be analyzed	=	Number of independent sources

Figure 9.1 reviews the various substitutions required when removing an ideal source, and Figure 9.2 reviews the substitutions with practical sources that have an internal resistance.

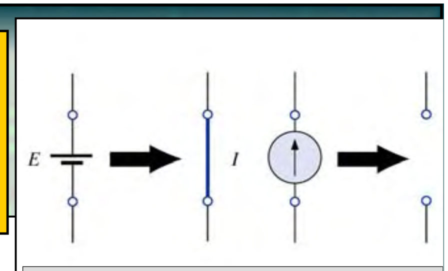


FIGURE 9.1 Removing the effects of practical sources

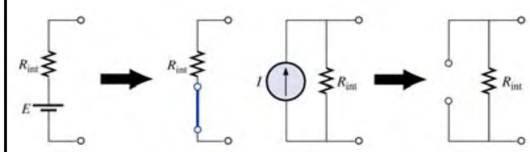


FIGURE 9.2 Removing the effects of ideal sources

Ex. 9-1 Determine I_1 for the network of *Fig. 9.3*.

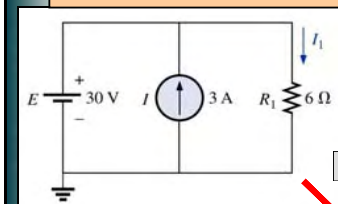


FIGURE 9.3

$$I_1^{'} = 0\,A$$

$$I_1^{''} = \frac{E}{R_1} = \frac{30V}{6\Omega} = 5\,A$$

$$I_1 = I_1^{'} + I_1^{''} = 0\,A + 5\,A = 5\,A$$

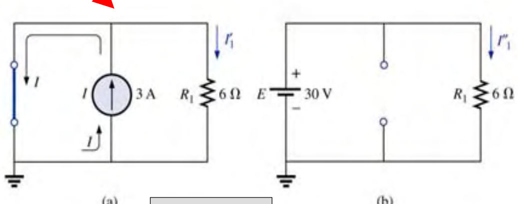


FIGURE 9.4

Ex. 9-2 Using superposition, determine the current through the 4-Ω resistor of *Fig. 9.5*. Note that this is a two-source network of the type considered in chapter 8.

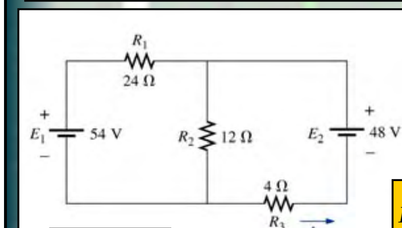


FIGURE 9.5

$$R_T = R_1 + R_2 \,//\, R_3 = 24\Omega + 12\Omega \,//\, 4\Omega = 24\Omega + 3\Omega = 27\Omega$$

$$I = \frac{E_1}{R_T} = \frac{54V}{27\Omega} = 2\,A$$

$$I_3^{'} = \frac{R_2\, I}{R_2 + R_3} = \frac{(12\,A)(2\,A)}{12\Omega + 4\Omega} = 1.5\,A$$

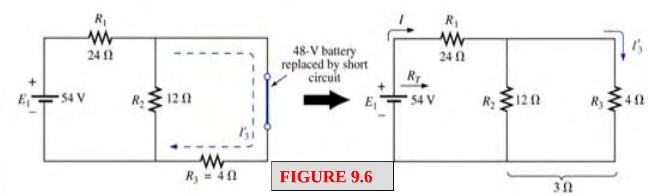


FIGURE 9.6

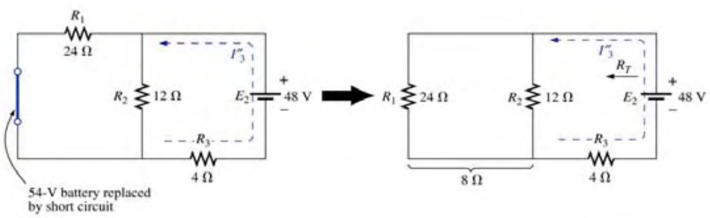


FIGURE 9.7

$$R_T = R_3 + R_1 // R_2 = 4\Omega + 24\Omega // 12\Omega = 4\Omega + 8\Omega = 12\Omega$$

$$I_3^{''} = \frac{E_2}{R_T} = \frac{48V}{12\Omega} = 4\,A$$

$$I_3 = I_3^{''} - I_3^{'} = 4\,A - 1.5\,A = 2.5\,A$$

$(direction\ of\ I_3^{''})$

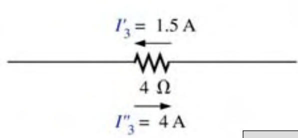


FIGURE 9.8

Ex. 9-3 a. Using superposition, find the current through the 6-Ω resistor of *Fig. 9.9*. b. Determine that superposition is not applicable to power levels.

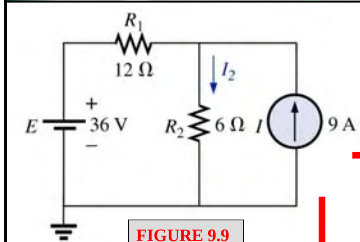


FIGURE 9.9

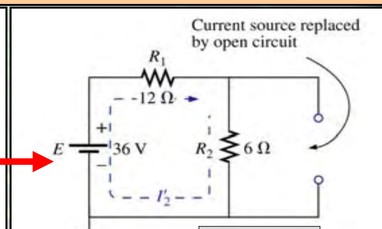


FIGURE 9.10

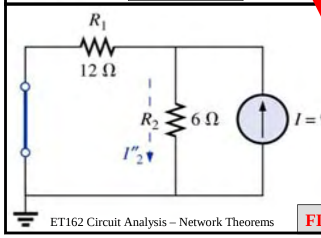


FIGURE 9.11

a. considering that the effect of the 36V source (Fig. 9.10):

$$I_2^{'} = \frac{E}{R_T} = \frac{E}{R_1 + R_2} = \frac{36V}{12\Omega + 6\Omega} = 2\,A$$

considering that the effect of the 9 A source (Fig. 9.11):

$$I_2^{''} = \frac{R_1\,I}{R_1 + R_2} = \frac{(12\Omega)(9\,A)}{12\Omega + 6\Omega} = 6\,A$$

The total current through the 6Ω resistor (Fig 9.12) is

$$I_2 = I_2^{'} + I_2^{''} = 2\,A + 6\,A = 8\,A$$

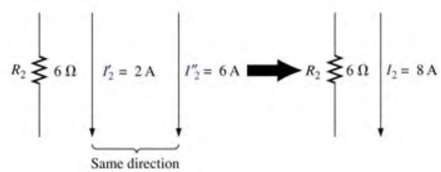


FIGURE 9.12

b. The power to the 6Ω resistor is

$$Power = I^2 R = (8\,A)^2 (6\Omega) = 384\,W$$

The calculated power to the 6Ω resistor due to each source, misusing the principle of superposition, is

$$P_1 = (I_2^{'})^2\,R = (2\,A)^2(6\Omega) = 24\,W$$

$$P_2 = (I_2^{''})^2\,R = (6\,A)^2(6\Omega) = 216\,W$$

$$P_1 + P_2 = 240\,W \neq 384\,W$$

$$because\ (2\,A)^2 + (6\,A)^2 \neq (8\,A)^2$$

Ex. 9-4 Using the principle of superposition, find the current through the 12-kΩ resistor of *Fig. 9.13*.

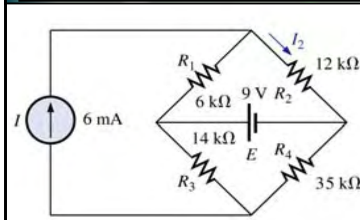


FIGURE 9.13

$$I_2^{'} = \frac{R_1\,I}{R_1 + R_2} = \frac{(6\,k\Omega)(6\,mA)}{6\,k\Omega + 12\,k\Omega} = 2\,mA$$

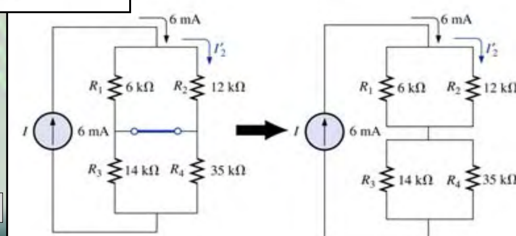


FIGURE 9.14

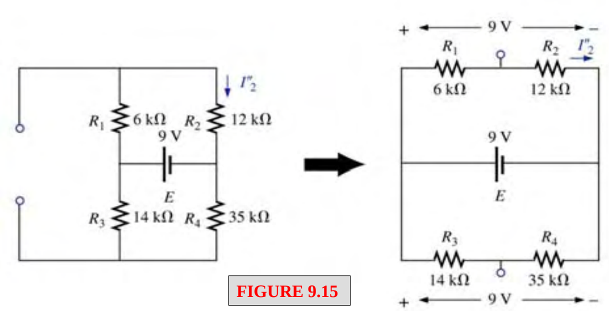


FIGURE 9.15

considering that the effect of the 9V voltage source (Fig. 9.15):

$$I_2^{''} = \frac{E}{R_1 + R_2} = \frac{9V}{6k\Omega + 12k\Omega} = 0.5mA$$

Since $I_2^{'}$ and $I_2^{''}$ have the same direction through R_2, the desired current is the sum of the two:

$$I_2 = I_2^{'} + I_2^{''} = 2mA + 0.5mA = 2.5mA$$

Ex. 9-5 Find the current through the 2-Ω resistor of the network of *Fig. 9.16*. The presence of three sources will result in three different networks to be analyzed.

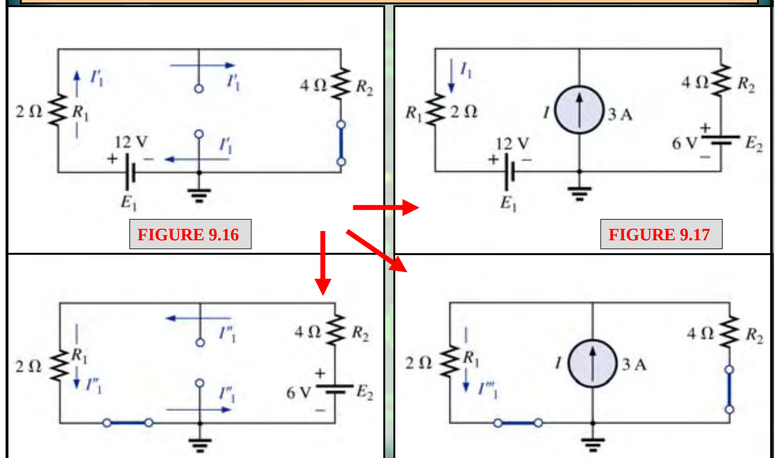


FIGURE 9.16

FIGURE 9.17

FIGURE 9.18

FIGURE 9.19

considering the effect of the 12V source (Fig. 9.17):

$$I_1^{'} = \frac{E_1}{R_1 + R_2} = \frac{12V}{2\Omega + 4\Omega} = 2A$$

considering that the effect of the 6V source (Fig. 9.18):

$$I_1^{''} = \frac{E_2}{R_1 + R_2} = \frac{6V}{2\Omega + 4\Omega} = 1A$$

considering the effect of the 3A source (Fig. 9.19):

$$I_1^{'''} = \frac{R_2 I}{R_1 + R_2} = \frac{(4\Omega)(3A)}{2\Omega + 4\Omega} = 2A$$

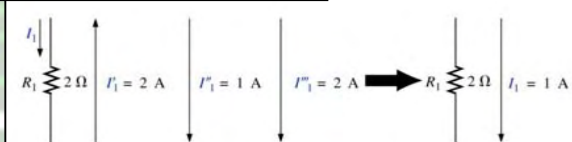


FIGURE 9.20

The total current through the 2Ω resistor appears in Fig.9.20, and

$$I_1 = I_1^{''} + I_1^{'''} - I_1^{'} = 1A + 2A - 2A = 1A$$

Thevenin's Theorem

Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a voltage source and a series resistor, as shown in Fig. 9.21.

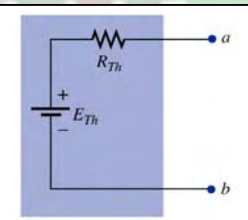


FIGURE 9.21 Thevenin equivalent circuit

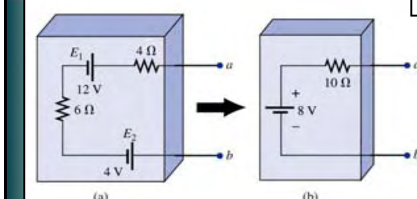


FIGURE 9.22 The effect of applying Thevenin's theorem.

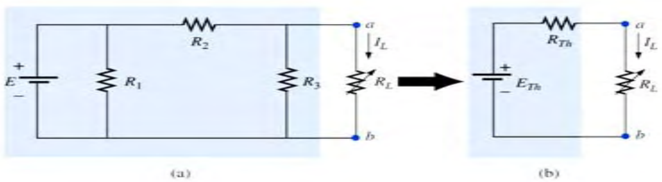


FIGURE 9.23 Substituting the Thevenin equivalent circuit for a complex network.

1. *Remove that portion of the network across which the Thevenin equivalent circuit is to be found. In Fig. 9.23(a), this requires that the road resistor R_L be temporary removed from the network.*
2. *Make the terminals of the remaining two-terminal network.*
3. *Calculate R_{TH} by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuit) and then finding the resultant resistance between the two marked terminals.*
4. *Calculate E_{TH} by first returning all sources to their original position and finding the open-circuit voltage between the marked terminals.*
5. *Draw the Thevenin equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.*

Ex. 9-6 Find the Thevenin equivalent circuit for the network in the shaded area of the network of *Fig. 9.24.* Then find the current through RL for values of 2Ω, 10Ω, and 100Ω.

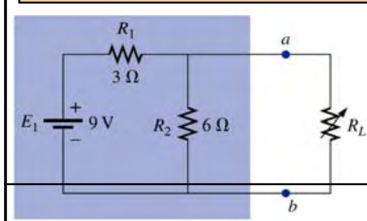


FIGURE 9.24

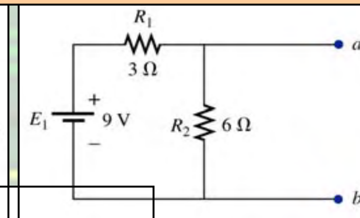


FIGURE 9.25 Identifying the terminals of particular importance when applying Thevenin's theorem.

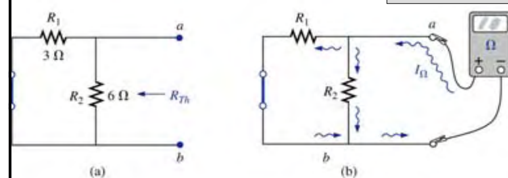


FIGURE 9.26 Determining R_{TH} for the network of Fig. 9.25.

$$R_{TH} = R_1 // R_2 = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = 2\Omega$$

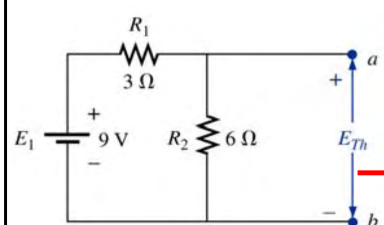


FIGURE 9.27

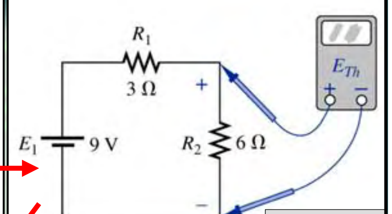


FIGURE 9.28

$$R_{TH} = \frac{R_2 E_1}{R_2 + R_1} = \frac{(6\Omega)(9V)}{6\Omega + 3\Omega} = 6V$$

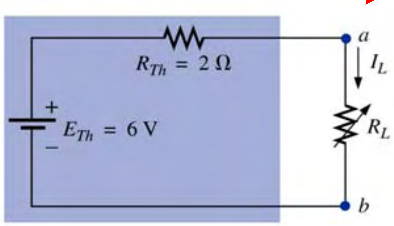


FIGURE 9.29 Substituting the Thevenin equivalent circuit for the network external to R_L in Fig. 9.23.

$$I_L = \frac{E_{TH}}{R_{TH} + R_L}$$

$$R_L = 2\Omega: \quad I_L = \frac{6V}{2\Omega + 2\Omega} = 1.5\ A$$

$$R_L = 10\Omega: \quad I_L = \frac{6V}{2\Omega + 10\Omega} = 0.5\ A$$

$$R_L = 100\Omega: \quad I_L = \frac{6V}{2\Omega + 100\Omega} = 0.059\ A$$

Ex. 9-7 Find the Thevenin equivalent circuit for the network in the shaded area of the network of *Fig. 9.30.*

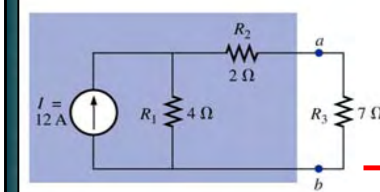


FIGURE 9.30

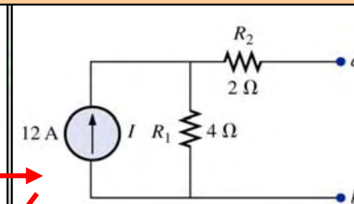


FIGURE 9.31

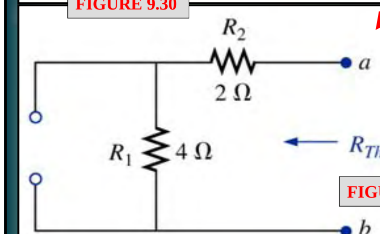


FIGURE 9.32

$$R_{TH} = R_1 + R_2 = 4\Omega + 2\Omega = 6\Omega$$

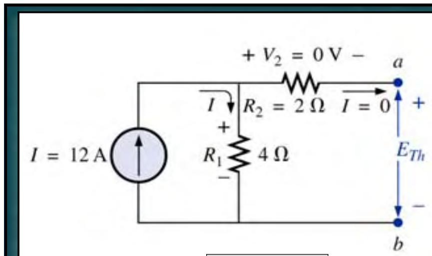


FIGURE 9.33

$$V_2 = I_2\,R_2 = (0)\,R_2 = 0V$$

$$E_{TH} = V_1 = I_1\,R_1 = I\,R_1$$
$$= (12\,A)(4\,\Omega)$$
$$= 48\,\Omega$$

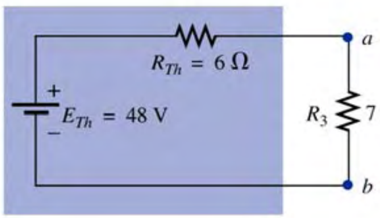


FIGURE 9.34 Substituting the Thevenin equivalent circuit in the network external to the resistor R_3 of Fig. 9.30.

Ex. 9-8 Find the Thevenin equivalent circuit for the network in the shaded area of the network of *Fig. 9.35*. Note in this example that there is no need for the section of the network to be at preserved to be at the "end" of the configuration.

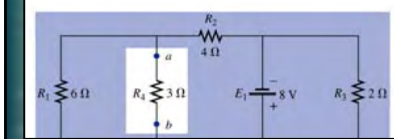


FIGURE 9.35

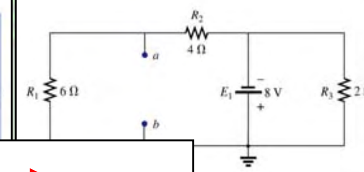
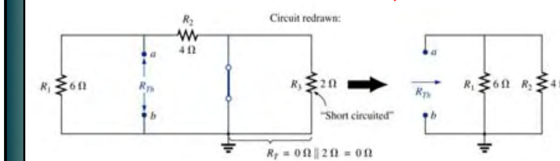


FIGURE 9.36



$$R_{TH} = R_1 // R_2$$
$$= \frac{(6\Omega)(4\Omega)}{6\Omega + 4\Omega}$$
$$= 2.4\,\Omega$$

 FIGURE 9.37

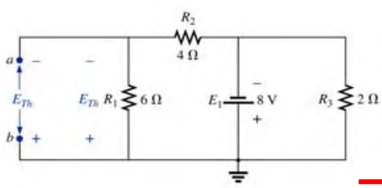


FIGURE 9.38

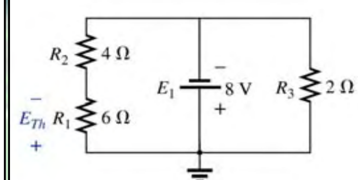
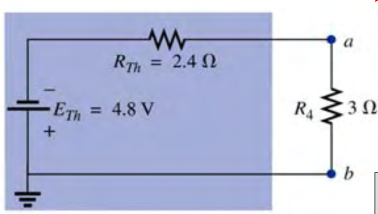


FIGURE 9.39



$$E_{TH} = \frac{R_1\,E_1}{R_1 + R_2}$$
$$= \frac{(6\Omega)(8V)}{6\Omega + 4\Omega}$$
$$= 4.8V$$

FIGURE 9.40 Substituting the Thevenin equivalent circuit in the network external to the resistor R_4 of Fig. 9.35.

Ex. 9-9 Find the Thevenin equivalent circuit for the network in the shaded area of the network of *Fig. 9.41*.

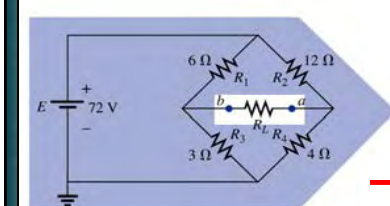


FIGURE 9.41

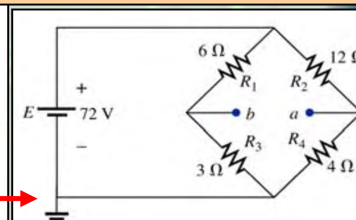
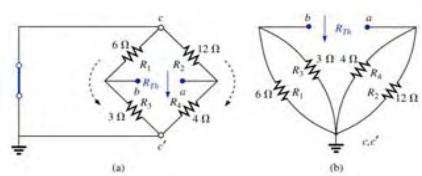


FIGURE 9.42



$$R_{TH} = R_1 // R_3 + R_2 // R_4$$
$$= 6\Omega // 3\Omega + 4\Omega // 12\Omega$$
$$= 2\Omega + 3\Omega = 5\Omega$$

 FIGURE 9.43

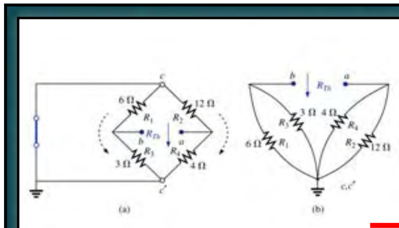


FIGURE 9.44

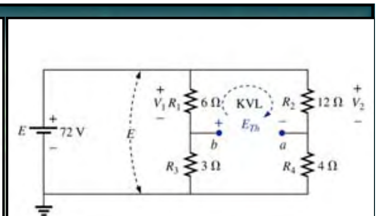


FIGURE 9.45

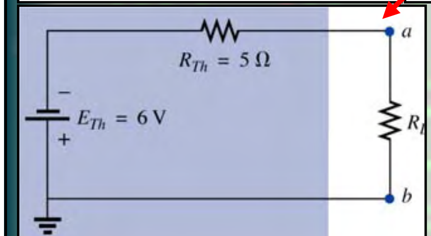


FIGURE 9.46 Substituting the Thevenin equivalent circuit in the network external to the resistor R_L of Fig. 9.41.

$$V_1 = \frac{R_1 E}{R_1 + R_3} = \frac{(6\Omega)(72V)}{6\Omega + 3\Omega} = 48V$$

$$V_2 = \frac{R_2 E}{R_2 + R_4} = \frac{(12\Omega)(72V)}{12\Omega + 4\Omega} = 54V$$

$$\sum V = -E_{TH} - V_1 + V_2 = 0$$

$$E_{TH} = V_2 - V_1 = 54V - 48V = 6V$$

Ex. 9-10 (Two sources) Find the Thevenin equivalent circuit for the network within the shaded area of *Fig. 9.47.*

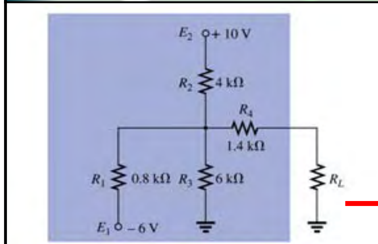


FIGURE 9.47

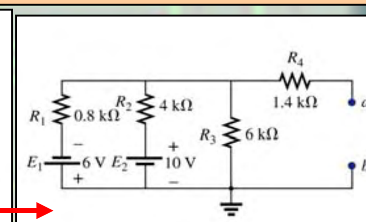


FIGURE 9.48

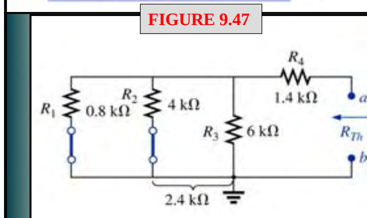


FIGURE 9.49

$$R_{TH} = R_4 + R_1 // R_2 // R_3$$
$$= 1.4k\Omega + 0.8k\Omega // 4k\Omega // 6k\Omega$$
$$= 1.4k\Omega + 0.8k\Omega // 2.4k\Omega$$
$$= 1.4k\Omega + 0.6k\Omega = 2k\Omega$$

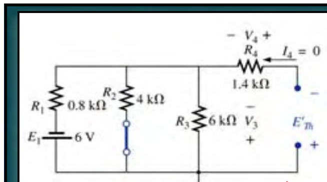


FIGURE 9.50

$$V_4 = I_4 R_4 = (0)R_4 = 0V$$

$$E'_{TH} = V_3$$

$$R'_T = R_2 // R_3 = 4k\Omega // 6k\Omega = 2.4k\Omega$$

$$V_3 = \frac{R'_T E_1}{R'_T + R_1} = \frac{(2.4k\Omega)(6V)}{2.4k\Omega + 0.8k\Omega} = 4.5V$$

$$E'_{TH} = V_3 = 4.5V$$

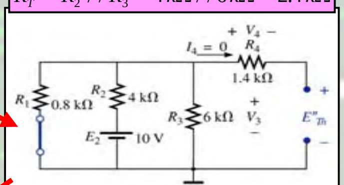


FIGURE 9.51

$$V_4 = I_4 R_4 = (0)R_4 = 0V$$

$$E''_{TH} = V_3$$

$$R'_T = R_1 // R_3 = 0.8k\Omega // 6k\Omega = 0.706k\Omega$$

$$V_3 = \frac{R'_T E_2}{R'_T + R_2} = \frac{(0.706k\Omega)(10V)}{0.706k\Omega + 4k\Omega} = 1.5V$$

$$E''_{TH} = V_3 = 1.5V$$

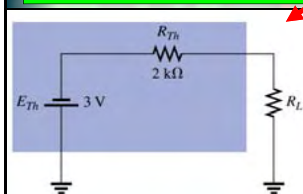


FIGURE 9.52 Substituting the Thevenin equivalent circuit in the network external to the resistor R_L of Fig. 9.47.

$$E_{TH} = E'_{TH} - E''_{TH}$$
$$= 4.5V - 1.5V$$
$$= 3V \ (polarity\ of\ E'_{TH})$$

Experimental Procedures

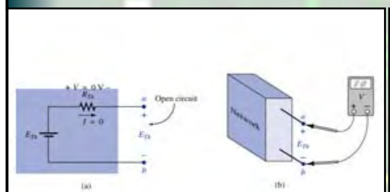


FIGURE 9.53

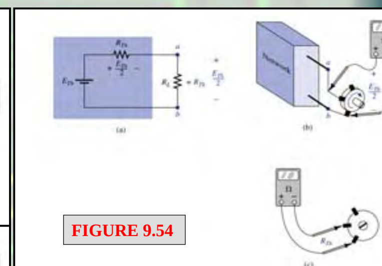


FIGURE 9.54

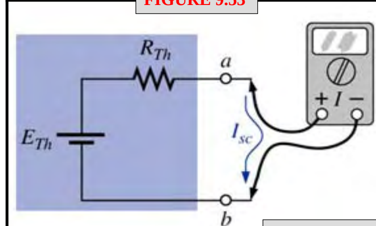


FIGURE 9.55

$$I_{SC} = \frac{E_{TH}}{R_{TH}}$$

$$R_{TH} = \frac{E_{TH}}{I_{SC}}$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} \quad where\ E_{TH} = V_{OC}$$

Norton's Theorem

Any two-terminal, linear bilateral dc network can be replaced by an equivalent circuit consisting of a current source and a parallel resistor, as shown in Fig. 9.56.

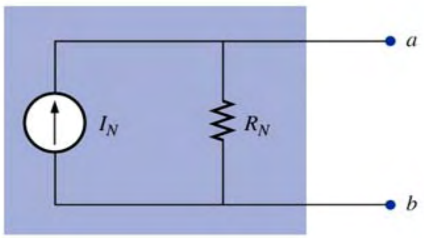


FIGURE 9.56 Norton equivalent circuit

1. *Remove that portion of the network across which the Thevenin equivalent circuit is found.*
2. *Make the terminals of the remaining two-terminal network.*
3. *Calculate R_N by first setting all sources to zero (voltage sources are replaced by short circuits, and current sources by open circuit) and then finding the resultant resistance between the two marked terminals.*
4. *Calculate I_N by first returning all sources to their original position and finding the short-circuit current between the marked terminals.*
5. *Draw the Norton equivalent circuit with the portion of the circuit previously removed replaced between the terminals of the equivalent circuit.*

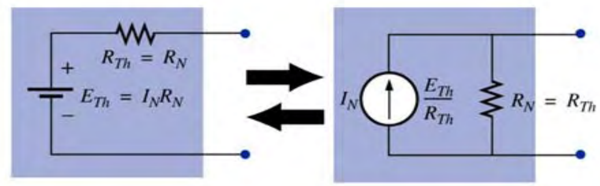


FIGURE 9.57 Substituting the Norton equivalent circuit for a complex network.

Ex. 9-11 Find the Norton equivalent circuit for the network in the shaded area of *Fig. 9.58.*

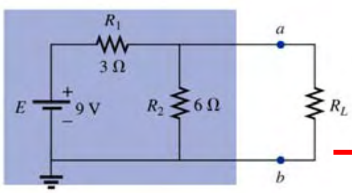


FIGURE 9.58

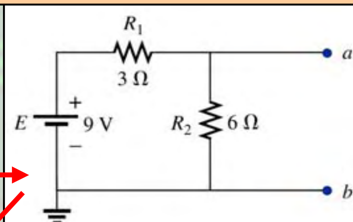


FIGURE 9.59 Identifying the terminals of particular interest for the network of Fig. 9.58.

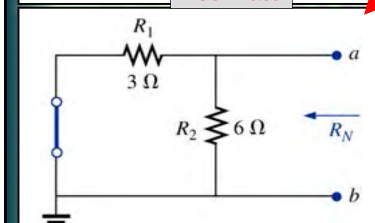


FIGURE 9.60 Determining R_N for the network of Fig. 9.59.

$$R_N = R_1 // R_2 = 3\Omega // 6\Omega = \frac{(3\Omega)(6\Omega)}{3\Omega + 6\Omega} = 2\Omega$$

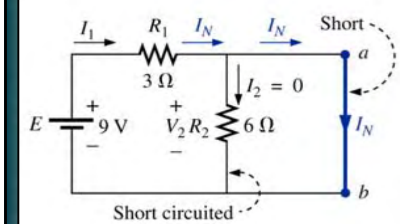


FIGURE 9.61 Determining R_N for the network of Fig. 9.59.

$$V_2 = I_2 R_2 = (0)6\ \Omega = 0\ V$$

$$I_N = \frac{E}{R_1} = \frac{9\ V}{3\ \Omega} = 3\ A$$

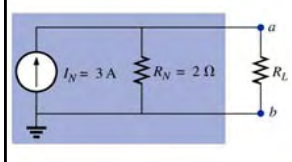


FIGURE 9.62 Substituting the Norton equivalent circuit for the network external to the resistor R_L of Fig. 9.58.

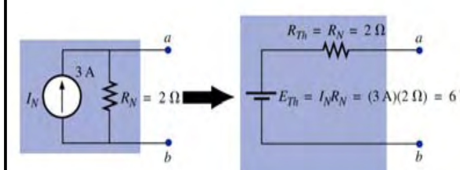


FIGURE 9.63 Converting the Norton equivalent circuit of Fig. 9.62 to a Thevenin's equivalent circuit.

Ex. 9-12 Find the Norton equivalent circuit for the network external to the 9-Ω resistor in *Fig. 9.64.*

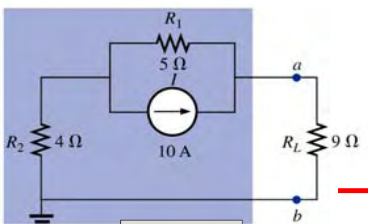


FIGURE 9.64

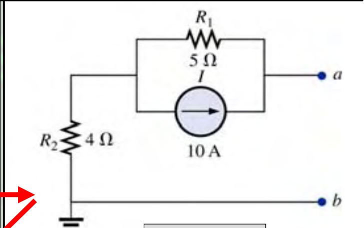


FIGURE 9.65

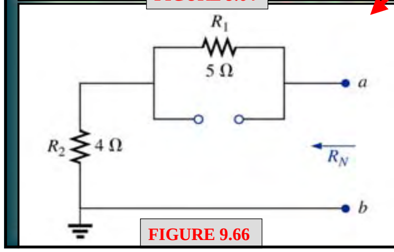


FIGURE 9.66

$$R_N = R_1 + R_2 = 5\,\Omega + 4\,\Omega = 9\,\Omega$$

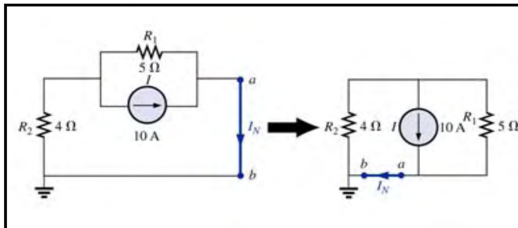


FIGURE 9.67 Determining I_N for the network of Fig. 9.65.

$$I_N = \frac{R_1 I}{R_1 + R_2} = \frac{(5\Omega)(10\Omega)}{5\Omega + 4\Omega} = 5.556\,A$$

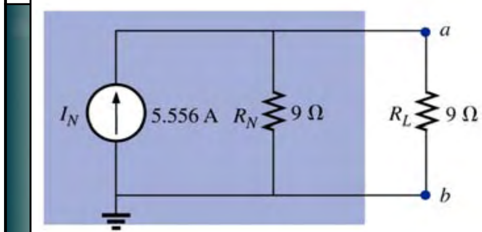


FIGURE 9.68 Substituting the Norton equivalent circuit for the network external to the resistor R_L of Fig. 9.64.

Ex. 9-13 (Two sources) Find the Norton equivalent circuit for the portion of the network external to the left of a-b in *Fig. 9.69.*

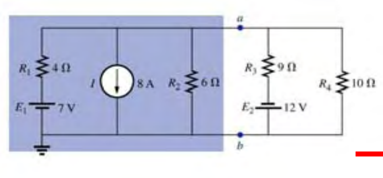


FIGURE 9.69

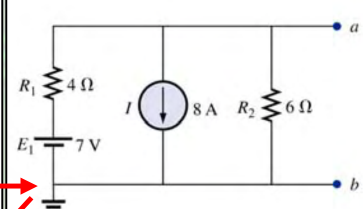


FIGURE 9.70

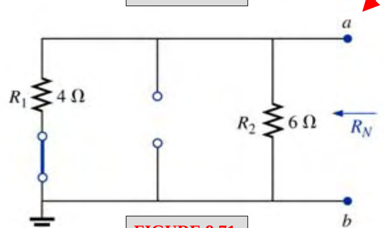


FIGURE 9.71

$$R_N = R_1 // R_2 = 4\Omega // 6\Omega = \frac{(4\Omega)(6\Omega)}{4\Omega + 6\Omega} = 2.4\Omega$$

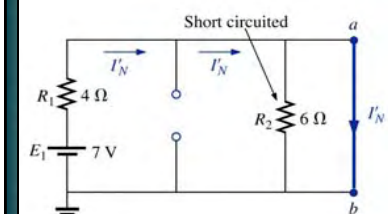


FIGURE 9.72

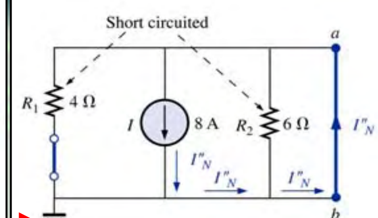


FIGURE 9.73

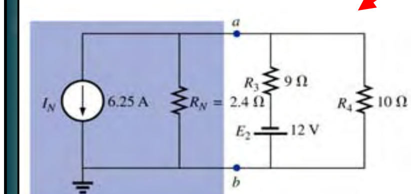


FIGURE 9.74 Substituting the Norton equivalent circuit for the network to the left of terminals a-b in Fig. 9.69.

$$I'_N = \frac{E_1}{R_1} = \frac{7V}{4\Omega} = 1.75\ A$$

$$I''_N = I = 8\ A$$

$$I_N = I''_N - I'_N = 8\,A - 1.75\,A = 6.25\,A$$

Maximum Power Transfer Theorem

A load will receive maximum power from a linear bilateral dc network when its total resistive value is exactly equal to the Thevenin resistance of the network as "seen" by the load.

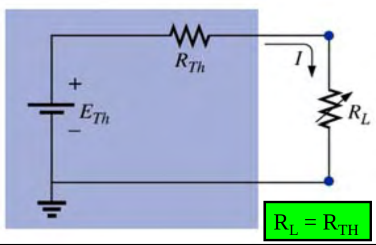


FIGURE 9.74 Defining the conditions for maximum power to a load using the Thevenin equivalent circuit.

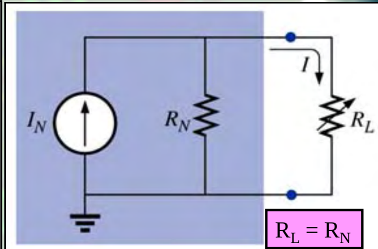
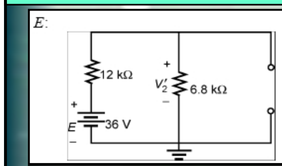
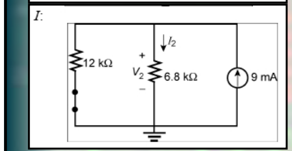


FIGURE 9.75 Defining the conditions for maximum power to a load using the Norton equivalent circuit.

HW 9-6 Using superposition, find the voltage V_2 for the network in Fig. 9.124.



$$V_1' = \frac{6.8k\Omega(36V)}{6.8k\Omega + 12k\Omega} = 13.02\ V$$



$$I_2 = \frac{12k\Omega(9\,mA)}{12k\Omega + 6.8k\Omega} = 5.75\ mA$$

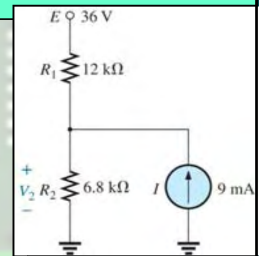


Figure 9.124 Problem 6.

$$V_2'' = I_2 R_2 = (5.75mA)(6.8k\Omega) = 39.10\ V$$

$$V_2 = V_1' + V_2'' = 13.02V + 39.10V = 52.12\ V$$

Homework 9: 2, 4, 6, 13, 14

EET1122/ET162 Circuit Analysis

Capacitors

Electrical and Telecommunications
Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis"
by Robert Boylestad, Prentice Hall, 10th edition.

OUTLINES

- Introduction to Capacitors
- The Electric Field
- Capacitance
- Transients in Capacitive Networks: Charging Phase
- Capacitors in Series and Parallel

Key Words: Capacitor, Electric Field, Capacitance, Transient

Introduction to Capacitors

Thus far, the only passive device appearing in the class has been the resistor. We will now consider two additional passive devices called the **capacitor** and the **inductor**, which are quite different from the resistor in purpose, operation, and construction.

Unlike the resistor, both elements display their total characteristics only when a change in voltage or current is made in the circuit in which they exist. In addition, if we consider the *ideal* situation, they **do not dissipate energy** as does the resistor but **store it** in a form that can be returned to the circuit whenever required by the circuit design.

The Electric Field

The **electric field** is represented by electric flux lines, which are drawn to indicate the strength of the electric field at any point around any charged body; that is, the denser the lines of flux, the stronger the electric field. In *Fig. 10.1*, the electric field strength is stronger at point a than at position b because the flux lines are denser at a than b.

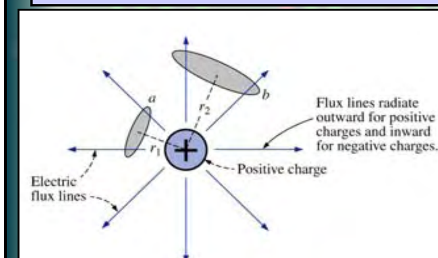


FIGURE 10.1 Flux distribution from an isolated positive charge.

The flux per unit area (flux density) is represented by the capital letter D and is determined by

$$D = \frac{\psi}{A} \quad (\textit{flux / unit area})$$

The Electric Field

The electric field is represented by **electric flux lines**, which are drawn to indicate the strength of the electric field at any point around the charged body. The **electric field** strength at any point distance r from a point charge of Q coulombs is directly proportional to the magnitude of the charge and inversely proportional to the distance squared from the charge.

$$\Psi \equiv Q \quad (coulombs,\ C)$$

$$E = \frac{F}{Q} \quad (networks/coulomb,\ N/C)$$

$$E = \frac{kQ}{r^2} \quad (N/C)$$

Electric flux lines always extend from a positively charged body to a negatively charged body, always extend or terminate perpendicular to the charged surfaces, and never intersect.

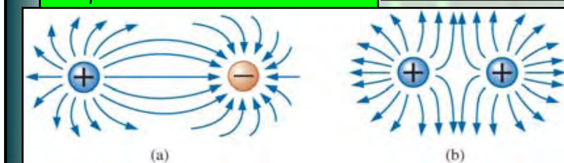


FIGURE 10.2 Electric flux distribution: (a) opposite charges; (b) like charges.

Capacitance

A ***capacitor*** is constructed simply of two parallel conducting plates separated by insulating material (in this case, air). ***Capacitance*** is a measure of a capacitor's ability to store charge on its plates.

A capacitor has a capacitance of 1 farad if 1 coulomb of charge is deposited on the plates by a potential difference of 1 volt cross the plates.

$$C = \frac{Q}{V}$$

C = farad (F)
Q = coulombs (C)
V = volts (V)

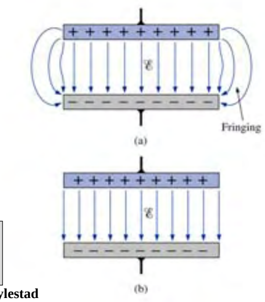
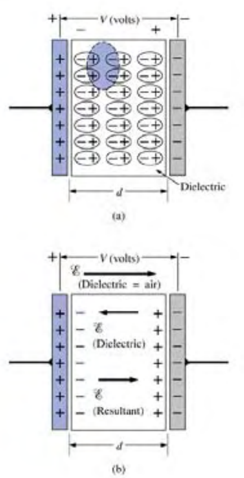


FIGURE 10.3 Electric flux distribution between the plates of a capacitor: (a) including fringing; (b) ideal.



$$\varepsilon = \frac{D}{E} \quad (farads/meter,\ F/M)$$

$$\varepsilon = \varepsilon_r \varepsilon_0$$

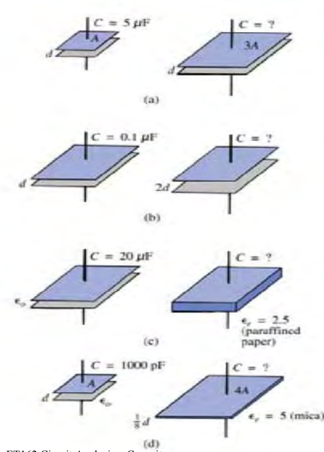
$$\varepsilon = \frac{C\,d}{A}$$

$$C = \varepsilon_r \varepsilon_0 \frac{A}{d} = 8.85 \times 10^{-12} \varepsilon_r \frac{A}{d} \quad (F)$$

E : Electric field (V/m)
D : Flux density
ε : Permittivity (F/m)
C : Capacitance (F)
Q : Charge (C)
A : Area in square meters
d : Distance in meters between the plates

FIGURE 10.4 Effect of a dielectric on the field distribution between the plates of a capacitor: (a) alignment of dipoles in the dielectric; (b) electric field components between the plates of a capacitor with a dielectric present..

Ex. 10-1 Determine the capacitance of each capacitor on the right side of *Fig.10.5*.



$$a.\quad C = 3(5\,\mu F) = 15\,\mu F$$

$$b.\quad C = \frac{1}{2}(0.1\,\mu F) = 0.05\,\mu F$$

$$c.\quad C = 2.5(20\,\mu F) = 50\,\mu F$$

$$d.\quad C = (5)\frac{4}{(1/8)}(1000\,pF) = (160)(1000\,pF) = 0.16\,\mu F$$

FIGURE 10.5

Ex. 10-2 For the capacitor of *Fig. 10.6*:
a. Determine the capacitance.
b. Determine the electric field strength between the plates if 450 V are applied across the plates.
c. Find the resulting charge on each plate.

a. $$C_o = \frac{\varepsilon_o A}{d} = \frac{(8.85\times10^{-12}\,F/m)(0.01m^2)}{1.5\times10^{-3}\,m} = 59.0\times10^{-12}\,F$$

b. $$\varepsilon = \frac{V}{d} = \frac{450V}{1.5\times10^{-3}\,m} \cong 300\times10^{3}\,V/m$$

c. $$C = \frac{Q}{V}$$
$$Q = CV = (59.0\times10^{-12})(450V)$$
$$= 26.550\times10^{-9}\,C = 26.55nC$$

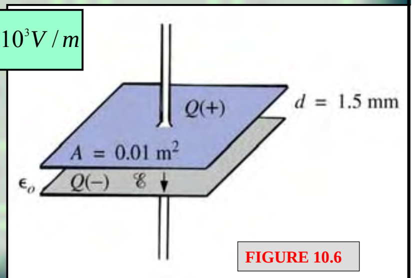


FIGURE 10.6

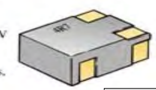
Type: Miniature Axial Electrolytic
Typical Values: 0.1 μF to 15,000 μF
Typical Voltage Range: 5 V to 450 V
Capacitor tolerance: ±20%
Applications: Polarized, used in DC power supplies, bypass filters, DC blocking.

Type: Miniature Radial Electrolyte
Typical Values: 0.1 μF to 15,000 μF
Typical Voltage Range: 5 V to 450 V
Capacitor tolerance: ±20%
Applications: Polarized, used in DC power supplies, bypass filters, DC blocking.

Type: Ceramic Disc
Typical Values: 10 pF to 0.047 μF
Typical Voltage Range: 100 V to 6 kV
Capacitor tolerance: ±5%, ±10%
Applications: Non-polarized, NPO type, stable for a wide range of temperatures. Used in oscillators, noise filters, circuit coupling, tank circuits.

Type: Dipped Tantalum (solid and wet)
Typical Values: 0.047 μF to 470 μF
Typical Voltage Range: 6.3 V to 50 V
Capacitor tolerance: ±10%, ±20%
Applications: Polarized, low leakage current, used in power supplies, high frequency noise filters, bypass filter.

Type: Surface Mount Type (SMT)
Typical Values: 10 pF to 10 μF
Typical Voltage Range: 6.3 V to 16 V
Capacitor tolerance: ±10%
Applications: Polarized and non-polarized, used in all types of circuits, requires a minimum amount of PC board real estate.



Type: Silver Mica
Typical Value: 10 pF to 0.001 μF
Typical Voltage Range: 50 V to 500 V
Capacitor tolerance: ±5%
Applications: Non-polarized, used in oscillators, in circuits that require a stable component over a range of temperatures and voltages.

Type: Mylar Paper
Typical Value: 0.001 μF to 0.68 μF
Typical Voltage Range: 50 V to 600 V
Capacitor tolerance: ±22%
Applications: Non-polarized, used in all types of circuits, moisture resistant.

Type: AC/DC Motor Run
Typical Value: 0.25 μF to 1200 μF
Typical Voltage Range: 240 V to 660 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in motor run-start, high-intensity lighting supplies, AC noise filtering.

Type: Trimmer Variable
Typical Value: 1.5 pF to 600 pF
Typical Voltage Range: 5 V to 100 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in oscillators, tuning circuits, AC filters.

Type: Tuning variable
Typical Value: 10 pF to 600 pF
Typical Voltage Range: 5 V to 100 V
Capacitor tolerance: ±10%
Applications: Non-polarized, used in oscillators, radio tuning circuit.

Transients in Capacitive Networks: Charging Phase

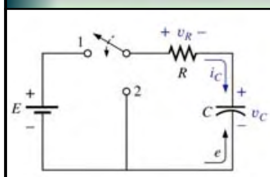


FIG. 10.8 Basic charging network.

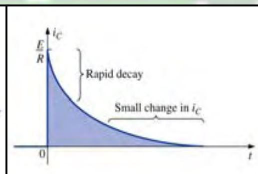


FIG. 10.9 i_c during charging phase.

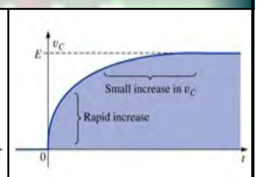


FIG 10.10 V_c during charging phase.

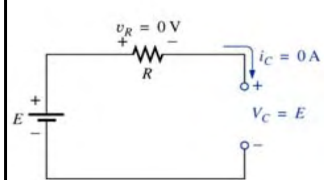


FIG. 10.11 Open-circuit equivalent for a capacitor following the charging phase.

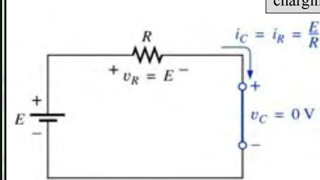
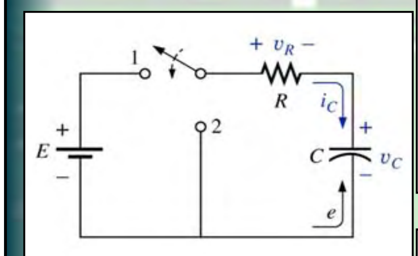


FIG. 10.12 Short-circuit equivalent for a capacitor (switch closed, *t=0*).

A capacitor can be replaced by an open-circuit equivalent once the charging phase in a dc network has passed.



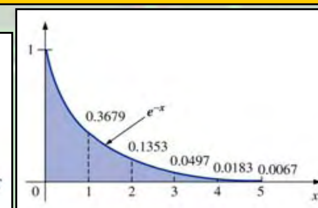


Figure 10.13 The e^{-x} function ($x \geq 0$).

$$i_C = \frac{E}{R}e^{-t/RC}$$

$$t = RC\ (\text{sec ond}, s)$$

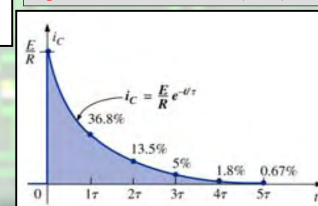
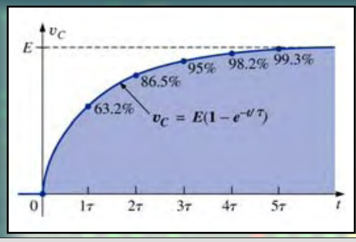
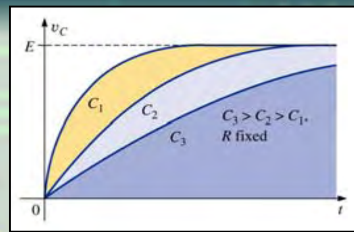
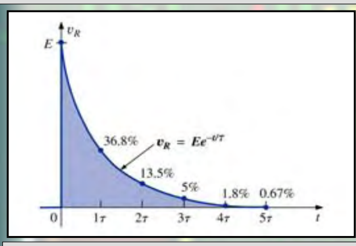


Figure 10.14 i_c versus *t* during the charging phase.







$$v_C = E(1 - e^{-t/RC})$$

$$v_R = Ee^{-t/RC}$$

Ex. 10-3 a. Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R for the circuit of *Fig. 10.18* when the switch is moved to position 1. Plot the curves of v_C, i_C, and v_R.
b. How much time must pass before it can be assumed, for all practical purposes, that $i_C \approx 0$ A and $v_C \approx E$ volt?

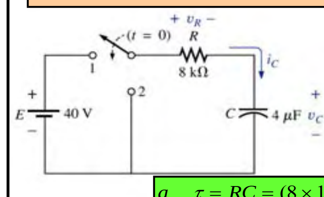


FIGURE 10.18

a. $\tau = RC = (8 \times 10^3 \Omega)(4 \times 10^{-6} F)$
$= 32 \times 10^{-3} s = 32\, ms$

$v_C = E(1 - e^{-t/\tau}) = 40(1 - e^{-t/(32\times10^{-3})})$

$i_C = \dfrac{E}{R} e^{-t/\tau} = \dfrac{80V}{8k\Omega} e^{-t/(32\times10^{-3})}$
$= (5 \times 10^{-3}) e^{-t/(32\times10^{-3})}$

$v_R = Ee^{-t/\tau} = 40 e^{-t/(32\times10^{-3})}$

b. $5\tau = 5(32\, ms) = 160\, ms$

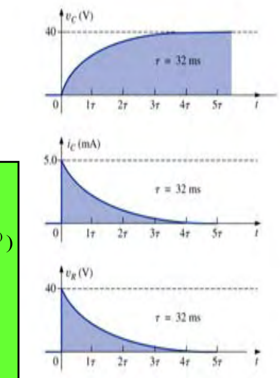


FIGURE 10.19

Transients in Capacitive Networks: Discharging Phase

$v_C = Ee^{-t/RC}$: discharging

$i_C = -\dfrac{E}{R} e^{-t/RC}$ discharging

$v_C = -Ee^{-t/RC}$ discharging

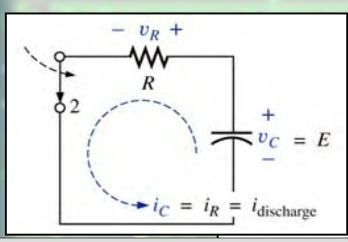


Figure 10.20 Demonstrating the discharge behavior of a capacitive network.

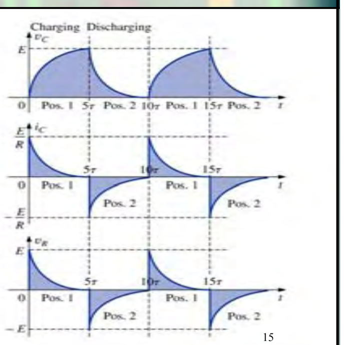


Figure 10.21 The charging and discharging cycles for the network of fig. 10.19.

Ex. 10-4 After v_C in Example 10.3 has reached its final value of 40 V, the switch is shown into position 2, as shown in *Fig. 10.21*. Find the mathematical expressions for the transient behavior of v_C, i_C, and v_R after the closing of the switch. Plot the curves for v_C, i_C, and v_R using the defined directions and polarities of *Fig. 10.18*. Assume that t = 0 when the switch is moved to position 2.

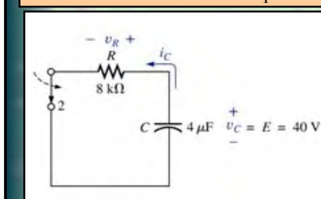


FIGURE 10.21

$\tau = 32\, ms$

$v_C = Ee^{-t/RC} = 40(e^{-t/32\times10^{-3}})$

$i_C = -\dfrac{E}{R} e^{-t/RC} = -(5 \times 10^{-3}) e^{-t/32\times10^{-3}}$

$v_C = -Ee^{-t/RC} = -40 e^{-t/32\times10^{-3}}$

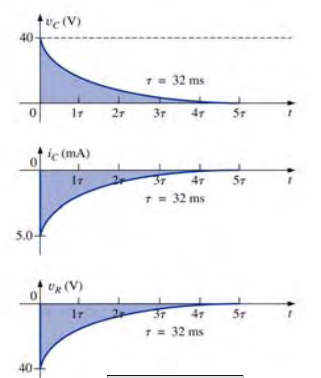
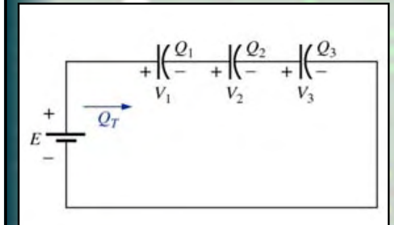
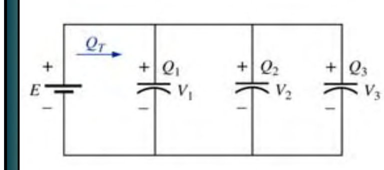


FIGURE 10.23

Capacitors in Series and Parallel





$$Q_T = Q_1 = Q_2 = Q_3$$

$$\frac{1}{C_T} = \frac{1}{C_1} = \frac{1}{C_2} = \frac{1}{C_3}$$

$$C_T = \frac{C_1 C_2}{C_1 + C_2}$$

Ex. 10-5 For the circuit of *Fig. 10.26*:
a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the voltage across each capacitor.

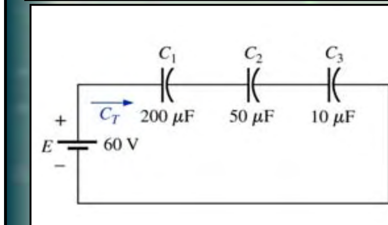


Figure 10.26

$$a.\quad \frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$
$$= \frac{1}{200\times10^{-6}F} + \frac{1}{50\times10^{-6}F} + \frac{1}{10\times10^{-6}F}$$
$$= 0.125\times10^{6}$$
$$C_T = \frac{1}{0.125\times10^{6}} = 8\ \mu F$$

$$b.\quad Q_T = Q_1 = Q_2 = Q_3$$
$$= C_T E = (8\times10^{-6}F)(60V)$$
$$= 480\ \mu C$$

$$c.\quad V_1 = \frac{Q_1}{C_1} = \frac{480\times10^{-6}C}{200\times10^{-6}F} = 2.4\,V$$
$$V_2 = \frac{Q_2}{C_2} = \frac{480\times10^{-6}C}{50\times10^{-6}F} = 9.6\,V$$
$$V_3 = \frac{Q_3}{C_3} = \frac{480\times10^{-6}C}{100\times10^{-6}F} = 48.0\,V$$
$$and\quad E = V_1 + V_2 + V_3 = 60\,V$$

Ex. 10-6 For the circuit of *Fig. 10.27*:
a. Find the total capacitance.
b. Determine the charge on each plate.
c. Find the total charges.

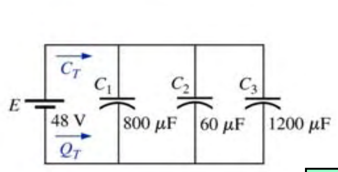


Figure 10.27

$$a.\quad C_T = C_1 + C_2 + C_3$$
$$= 800\ \mu F + 60\ \mu F + 1200\ \mu F$$
$$= 2060\ \mu F$$
$$b.\quad Q_1 = C_1 E = (800\times10^{-6}F)(48V) = 38.4\,mC$$
$$Q_2 = C_2 E = (60\times10^{-6}F)(48V) = 2.88\,mC$$
$$Q_3 = C_1 E = (1200\times10^{-6}F)(48V) = 57.6\,mC$$
$$c.\quad Q_T = Q_1 + Q_2 + Q_3 = 98.88\ mC$$

Ex. 10-7 Find the voltage across and charge on each capacitor for the network of *Fig. 10.28*.

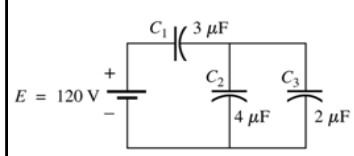


Figure 10.28

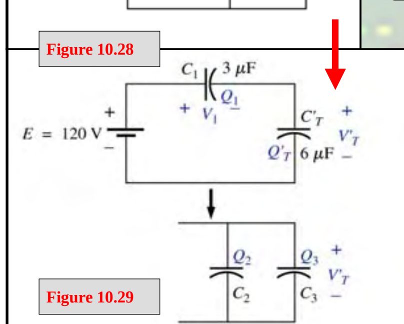


Figure 10.29

$$C_T^{'} = C_2 + C_3 = 4\ \mu F + 2\ \mu F = 6\ \mu F$$
$$C_T = \frac{C_1 C_T^{'}}{C_1 + C_T^{'}} = \frac{(3\ \mu F)(6\ \mu F)}{3\ \mu F + 6\ \mu F} = 2\ \mu F$$
$$Q_T = C_T E = (2\times10^{-6}F)(120V) = 240\ \mu C$$

$$Q_T = Q_1 = Q_T^{'}$$
$$Q_1 = 240\ \mu C$$
$$V_1 = \frac{Q_1}{C_1} = \frac{240\times10^{-6}C}{3\times10^{-6}F} = 80\,V$$
$$Q_T^{'} = 240\ \mu C$$

$$V_T^{'} = \frac{Q_T^{'}}{C_T^{'}} = \frac{240\times10^{-6}C}{6\times10^{-6}F} = 40\,V$$
$$Q_2 = C_2 V_T^{'} = (4\times10^{-6}F)(40V) = 160\ \mu C$$
$$Q_3 = C_3 V_T^{'} = (2\times10^{-6}F)(40V) = 80\ \mu C$$

Ex. 10-8 Find the voltage across and charge on capacitors C_1 of *Fig. 10.30* after it has charged up to its final value.

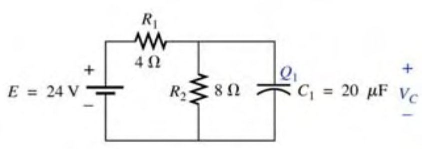


Figure 10.30

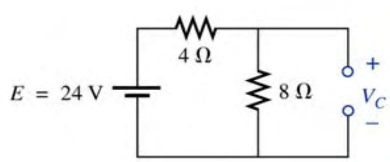


Figure 10.31

$$V_C = \frac{(8\Omega)(24V)}{4\Omega + 8\Omega} = 16V$$

$$Q_1 = C_1 V_C$$

$$= (20 \times 10^{-6} F)(16V)$$

$$= 320\ \mu C$$

Ex. 10-9 Find the voltage across and charge on each capacitor for the network of *Fig. 10.32.* after each has charged up to its final value.

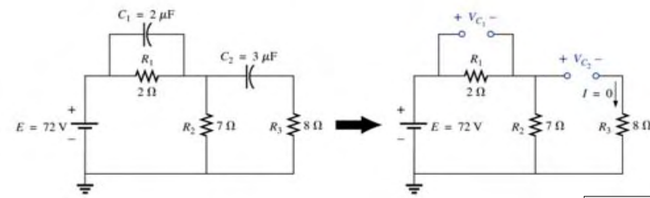


Figure 10.32

$$V_{C_2} = \frac{(7\Omega)(72V)}{7\Omega + 2\Omega} = 76V$$

$$V_{C_1} = \frac{(2\Omega)(72V)}{7\Omega + 2\Omega} = 16V$$

$$Q_1 = C_1 V_{C_1} = (2 \times 10^{-6} F)(16V) = 32\ \mu C$$

$$Q_1 = C_2 V_{C_2} = (3 \times 10^{-6} F)(56V) = 168\ \mu C$$

HW 10-54 Find the voltage across and the charge on each capacitor for the circuit in Fig. 10.115.

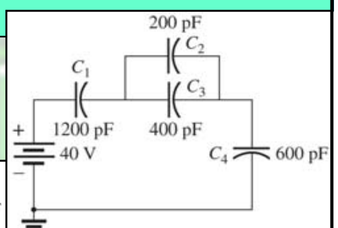


Figure 10.115 Problem 54.

C_T = 1200 pF || (200 pF + 400 pF) || 600 pF
= 1200 pF || 600 pF || 600 pF = 1200 pF || 300 pF
= 240 pF

$Q_T = C_T E$ = (240 pF)(40 V) = 9.60 nC

$Q_1 = Q_4 = Q_T$ = **9.60 nC**

$V_1 = \frac{Q_1}{C_1} = \frac{9.60\text{ nC}}{1200\text{ pF}} = \mathbf{8.00\ V}$, $V_4 = \frac{Q_4}{C_4} = \frac{9.60\text{ nC}}{600\text{ pF}} = \mathbf{16.00\ V}$

$V_2 = V_3 = E - V_1 - V_4$ = 40 V − 8 V − 16 V = **16 V**

$Q_2 = C_2V_2$ = (200 pF)(16 V) = **3.20 nC**, $Q_3 = C_3V_3$ = (400 pF)(16 V) = **6.40 nC**

Homework 10: 3, 4, 14, 51,52, 54, 57

HW 10-22 Repeat Problem 21 for R = 1 MΩ, and compare the results.

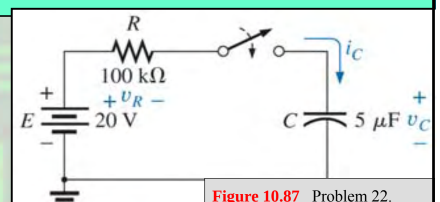


Figure 10.87 Problem 22.

a. $\tau = RC = (10^6\ \Omega)(5.1\ \mu F) = \mathbf{5.1\ s}$

b. $v_C = E(1 - e^{-t/\tau}) = \mathbf{20\ V(1 - e^{-t/5.1s})}$

c. 1τ = **12.64 V**, 3τ = **19 V**, 5τ = **19.87 V**

d. $i_C = \frac{20\text{ V}}{1\text{ M}\Omega} e^{-t/\tau} = \mathbf{20\ \mu A\ e^{-t/5.1s}}$
$v_R = Ee^{-t/\tau} = \mathbf{20V\ e^{-t/5.1s}}$

e. Same as problem 21 with 5τ = 25 s and I_m = 20 μA

Homework 10: 22, 24, 27, 28

EET1122/ET162 Circuit Analysis

Magnetism and Inductors

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

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OUTLINES

- Introduction to Inductors
- The Magnetic Field
- Inductance
- R-L Transients: The Storage Phase
- Thevenin Equivalent

Key Words: Inductor, Magnetic Field, Inductance, Transient, Thevenin

Introduction to Inductors

Three basic components appear in the majority of electrical/electronic systems in use today. They include the resistor and the capacitor, which have already been introduced, and the **inductor**, to be examined in detail in this module. Like the capacitor, ***the inductor exhibits its true characteristics only when a charge in voltage or current is made in the network***.

Recall from previous module that a capacitor can be replaced by an open-circuit equivalent under steady-state conditions. You will see in this module that ***an inductor can be replaced by a short-circuit equivalent under steady-state conditions***. Finally, you will learn that while resistors dissipate the power delivered to them in form of heat, ideal capacitors store the energy delivered to them in the form of an electric field. Inductors are like capacitors in that they also store the energy delivered to them–but in the form of a magnetic field.

Magnetic Field

Magnetism plays an integral part in almost every electrical device used today in industry, research, or the home. The compass relies on a permanent magnet for indicating direction. Michael Faraday, Karl Friedrich Gauss, and James Clerk Maxwell continued to experiment in this area and developed many of basic concepts of **electromagnetism–magnetic effects induced by the flow of charge, or current**.

A magnetic field exists in the region surrounding a permanent magnet, which can be represented by **magnetic flux lines** similar to electric flux lines. Magnetic flux lines, however, do not have origins or terminating points as do electric flux lines but exist in continuous loops, as shown in Fig. 11.1.

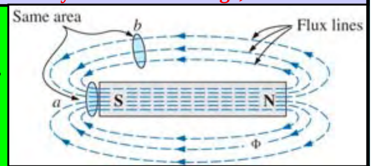


Figure 11.1 Flux distribution for a permanent magnet.

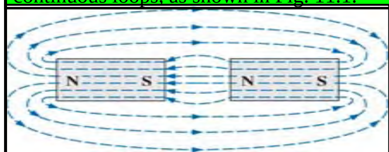


Figure 11.2 Flux distribution for two adjacent, opposite poles

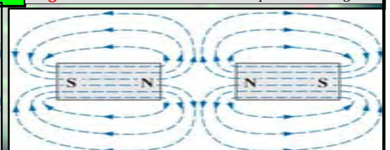


Figure 11.3 Flux distribution for two adjacent, like poles.

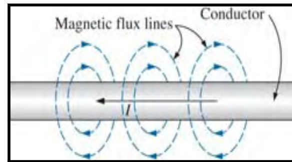


Figure 11.6 Magnetic flux lines around a current-carrying conductor.

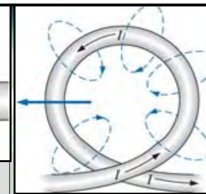


Figure 11.7 Flux distribution of a single-turn coil.

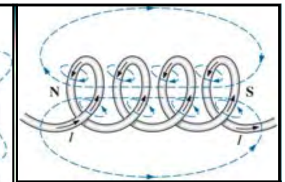


Figure 11.8 Flux distribution of a current carrying coil.

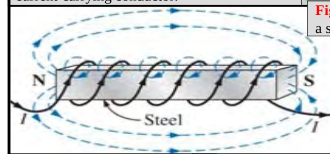


Figure 11.9 Electromagnet.

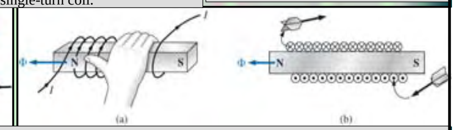


Figure 11.10 Determining the direction of flux for an electromagnet: (a) method; (b) notation.

In the SI system of units, magnetic flux is measured in webers (WB). The applied symbol is Φ. The number of flux lines per unit area, called the flux density, is denoted by B and is measured in teslas (T).

$$B = \frac{\Phi}{A}$$

$B = Wb/m^2 = teslas\,(T)$

$\Phi = webers\,(Wb)$

$A = m^2$

$$1\ tesla = 1T = 1Wb/m^2$$

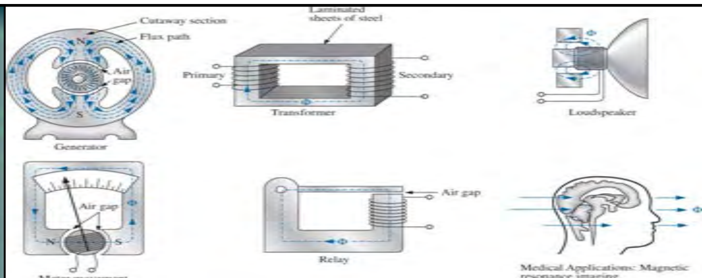


Figure 11.15 Some areas of application of magnetic effects.

Inductance

Sending a current through a coil of wire establishes a magnetic field through and surrounding the unit. This component is called an **inductor**. Its inductance level determines the strength of the magnetic field around the coil due to an applied current.

inductors are designed to set up a strong magnetic field linking the unit, whereas capacitors are designed to set up a strong electric field between the plates.

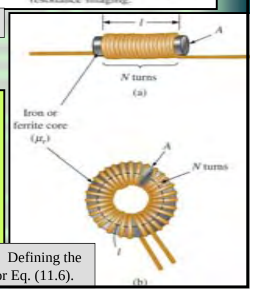


Figure 11.16 Defining the parameters for Eq. (11.6).

Inductance – Inductor Construction

The level of inductance has similar construction sensitivities in that is dependent on the area within the coil, the length of the unit, and the permeability of the core material. It is also sensitive to the number of turns of wire in the coil as depicted by Eq. (11.6) and defined in Fig. 11.16 for two of the most popular shapes:

$$L = \frac{\mu N^2 A}{l} \qquad (11.6)$$

μ : Permeability (Wb/A·m)
N : Number of turns (t)
A : m^2
l : m
L : henries (H)

$$L = \frac{\mu_r \mu_o N^2 A}{l} \quad where\ \mu = \mu_r \mu_o \quad or \quad L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l} \quad (henries, H)$$

$$L = \mu_r\left(\frac{\mu_o N^2 A}{l}\right) = \mu_r L_o$$

The inductance of an inductor with a ferromagnetic core is μ_r times the inductance obtained with an air core.

Ex. 11-1 For the air-core coil in Fig. 11.18:
a. Find the inductance.
b. Find the inductance if a metallic core with $\mu_r = 2000$ is inserted in the coil.

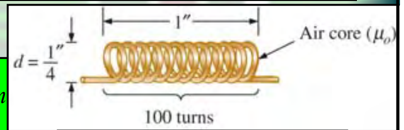


Figure 11.18 Air-core coil for example 11.1.

$$a.\quad d = \frac{1}{4}in.\left(\frac{1\ m}{39.37\ in.}\right) = 6.35\ mm$$

$$A = \frac{\pi d^2}{4} = \frac{\pi (6.35mm)^2}{4} = 31.7\ \mu m^2$$

$$l = 1\,in.\left(\frac{1\,m}{39.37\,in.}\right) = 25.4\ mm$$

$$L = 4\pi \times 10^{-7}\frac{\mu_r N^2 A}{l} = 4\pi \times 10^{-7}\frac{(1)(100t)^2(31.7\mu m^2)}{25.4\,mm} = 15.68\ \mu H$$

$$b.\quad L = \mu_r L_0 = (2000)(15.68\ \mu H) = 31.36\ mH$$

Ex. 11-2 In Fig. 11.19, if each inductor in the left column is changed to the type appearing in the right column, find the new inductance level. For each change, assume that the other factors remain the same.

a. The only change was the number of turns, but it is a squared factor, resulting in L $= (2)^2L_o = (4)(20\,\mu H) = 80\,\mu H$

b. In this case, the area is three times the original size, and the number of turns is ½. Since the area is in the numerator, it increases the inductance by a factor of three. The drop in the number of turns reduces the inductance by a factor of $(½)^2 = ¼$. Therefore, L $= (3)(1/4)L_o = (3/4)(16\,\mu H) = 12\,\mu H$

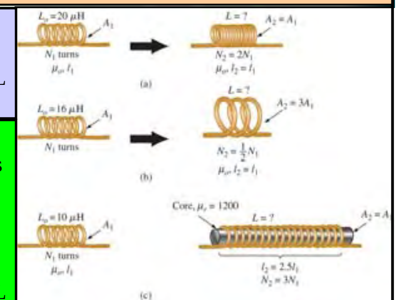


Figure 11.19 Inductors for example 11.2.

c. Both μ and the number of turns have increased, although the increase in the number of turns is squared. The increased length reduce the inductance. Therefore,

$$L = \frac{(3)^2(1200)}{2.5}L_o = (4.32\times 10^3)(10\mu H) = 43.2\,mH$$

Inductance – Types of Inductors

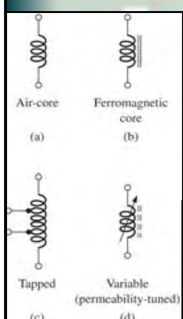


Figure 11.20 Inductor coil symbols.

Inductors, like capacitors and resistors, can be categorized under the general headings **fixed** or **variable**. The symbol for a fixed air-core inductor is provided in Fig. 11.20(a), for an inductor with a ferromagnetic core in Fig. 11.20(b), for a tapped coil in Fig. 11.20(c), and for a variable inductor in Fig. 11.20(d).

Practical Equivalent Inductor

Inductors, like capacitors, are not ideal. Associated with every inductor is a resistance determined by the resistance of the turns of wire and by core losses. Both elements are included in the equivalent circuit in Fig. 11.24. For most applications in this text, the capacitance can be ignored, resulting in the equivalent model in Fig. 11.25.

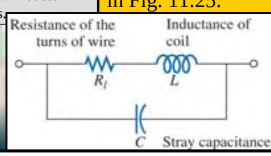


Figure 11.24 Complete equivalent model for an inductor.

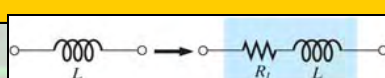


Figure 11.25 Practical equivalent model for an inductor.

Induced Voltage v_L

Faraday's law of electromagnetic induction is one of the most important in this field because it enables us to establish ac and dc voltages with generator. If we move a conductor through a magnetic field so that it cuts magnetic lines of flux as shown in Fig. 11.28. If we go a step further and move a coil of N turns through the magnetic field as shown in Fig. 11.29, a voltage will be induced across the coil as determined by **Faraday's law**:

$$e = N\frac{d\phi}{dt}\quad (volts, V)$$

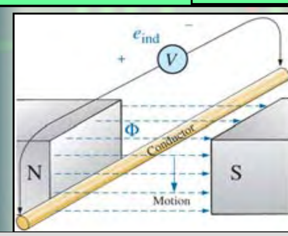


Figure 11.28 Generating an induced voltage by moving a conductor through a magnetic field.

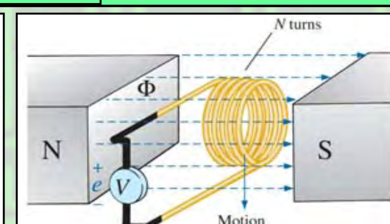


Figure 11.29 Demonstrating Faraday's law.

The polarity of the induced voltage across the coil is such that it opposes the increasing level of current in the coil as shown in Fig. 11.30. In other words, the changing current through the coil induces a voltage across the coil that is opposing the applied voltage that establishes the increase in current to the first place. The quicker the change in current through the coil, the greater the opposing induced voltage to squelch the attempt of the current to increase in magnitude. This effect is a result of an important law referred to as Lenz's law, which states that

an induced effect is always such as to oppose the cause that produced it.

$$L = N\frac{d\phi}{di_L} \quad (henries,\ H)$$

If the inductance level is very small, there will be almost no change in flux linking the coil, and the induced voltage across the coil will be very small. That is

$$e = N\frac{d\phi}{dt} = \left(N\frac{d\phi}{di_L}\right)\left(\frac{di_L}{dt}\right) \quad and \quad e_L = L\frac{di_L}{dt} \quad (volts,\ V)$$

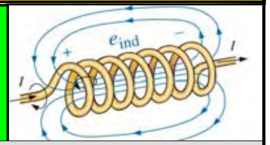


Figure 11.30 Demonstrating the effect of Lenz's law.

In network analysis, the voltage induced across an inductor will always have a polarity that opposes the applied voltage. Therefore, the following notation is used for the induced voltage across an inductor:

the larger the inductance and/or the more rapid the change in current through a coil, the larger will the induced voltage across the coil

$$V_L = L\frac{di_L}{dt} \quad (volts,\ V)$$

R-L Transients – The Storage Phase

A great number of similarities exist between the analyses of inductive and capacitive networks. The storage waveforms have the same shape, and time constants are defined for each configuration. The circuit in Fig. 11.31 is used to describe the charging phase of capacitors, with a simple replacement of the capacitor by an ideal inductor. It is important to remember that the energy is stored in the form of an electric field between the plates of a capacitor. ***For inductors, on the other hand, energy is stored in the form of magnetic field linking the coil.***

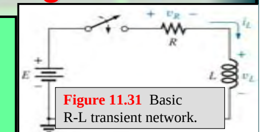


Figure 11.31 Basic R-L transient network.

At the instant the switch is closed, the choking action of the coil prevents an instantaneous change in current through the coil, resulting in $i_L = 0\ A$ as shown in Fig. 11.32(a). The absence of a current through the across the resistor as determined by $v_R = i_R R = i_L R = (0\ A)R = 0\ V$, as shown in Fig. 11.32(c). Applying KVL around the closed loop results in E volts across the coil at the instant the switch is closed, as shown in Fig. 11.32(b).

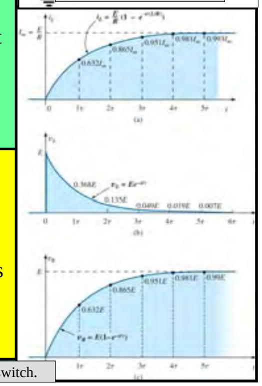


Figure 11.32 i_L, v_L, and v_R for the circuit in Fig. 11.31 following the closing of the switch.

The equation for the transient response of the current through an inductor if the following:

$$i_L = \frac{E}{R}\left(1 - e^{-t/\tau}\right) \quad (amperes,\ A)$$

with the time constant now defined by

$$\tau = \frac{L}{R} \quad (seconds,\ s)$$

If we keep R constant and increases as shown in Fig. 11.33 for increasing levels of L. The change in transient response is expected because the higher the inductance level, the greater the choking action on the changing current level, and longer it will take to reach-steady state conditions.

The equation for the voltage across the coil is the following:

$$v_L = Ee^{-t/\tau} \quad (volts,\ V)$$

and for the voltage across resistor:

$$v_R = E\left(1 - e^{-t/\tau}\right) \quad (volts,\ V)$$

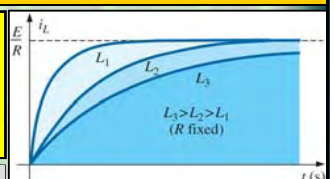


Figure 11.33 Effect of L on the shape of the i_L storage waveform.

Since the waveforms are similar to those obtained for capacitive networks, we will assume that

the storage phase passed and steady-state conditions have been established once a period of time equal to five time constants has occurred.

In addition, since $\tau = L/R$ will always have some numerical value, even though it may be very small at times, the transient period of 5τ will always have some numerical value. Therefore,

the current cannot change instantaneously in an inductive network.

If we examine the conditions that exist at the instant the switch is closed, we find that the voltage across the coil is E volts, although the current is zero amperes as shown in Fig. 11.34. In essence, therefore,

the inductor takes on the characteristics of an open circuit at the instant the switch is closed.

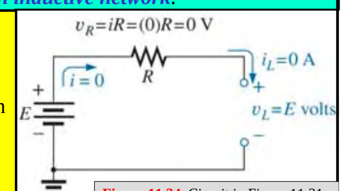


Figure 11.34 Circuit in Figure 11.31 the instant the switch is closed.

However, if we consider the conditions that exist when steady-state conditions have been established, we find that the voltage across the coil is zero volts and the current is a maximum value of E/R ampere as shown in Fig. 11.35. In essence, therefore,

the inductor takes on the characteristics of a short circuit when steady-state conditions have been established.

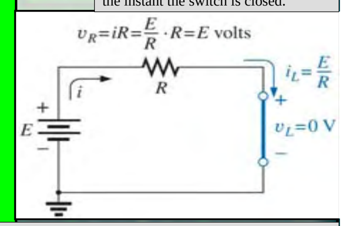


Figure 11.35 Circuit in Figure 11.31 under steady-state conditions.

Ex. 11-3 Find the mathematical expressions for the transient behavior of i_L and v_L for the circuit in Fig. 11.36 if the switch is closed at $t = 0\ s$. Sketch the resulting curves.

First, the time constant is determined :

$$\tau = \frac{L}{R_1} = \frac{4\ H}{2\ k\Omega} = 2\ ms$$

Then the maximum or steady – state current is

$$I_m = \frac{E}{R_1} = \frac{50V}{2\ k\Omega} = 25 \times 10^{-3}\ A = 25\ mA$$

Substituting into Eq.(11.13) :

$$i_L = 25\ mA(1 - e^{-t/2ms})$$

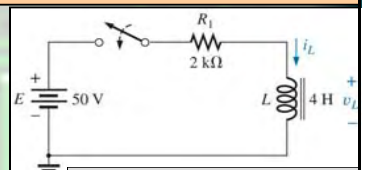


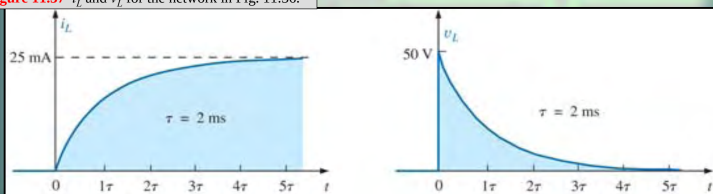
Figure 11.36 R-L circuit for Example 11.3.

Using Eq.(11.15) :

$$v_L = 50Ve^{-t/2ms}$$

The resulting waveform appear in Fig.11.37.

Figure 11.37 i_L and v_L for the network in Fig. 11.36.



Initial Conditions

Since the current through a coil cannot change instantaneously, the current through a coil begins the transient phase at the initial value established by the network (note Fig. 11.38) before the switch was closed. It then passes through the transient phase until it reaches the steady-state level after about five time constants. The steady-state level of the inductor current can be found by substituting its short-circuit equivalent and finding the resulting current through the element.

Using the transient equation developed in the previous discussion, an equation for the current i_L can be written for the entire time interval in Fig. 11.38; that is

$$i_L = I_f + (I_i - I_f)e^{-t/\tau}$$

with $(I_f - I_i)$ representing the total change during the transient phase. However, by multiplying through and rearranging terms:

$$i_L = I_i + I_f - I_f e^{-t/\tau} - I_i + I_i e^{-t/\tau}$$
$$= I_f - I_f e^{-t/\tau} + I_i e^{-t/\tau}$$

we find

$$i_L = I_i + (I_f - I_i)(1 - e^{-t/\tau})$$

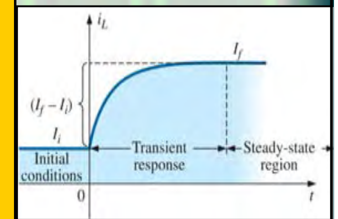


Figure 11.38 Defining the three phases of a transient waveform.

R-L Transients – The Release Phase

In the analysis of R-C circuits, we found that the capacitor could hold its charge and store energy in the form of an electric field for a period of time determined by the leakage factors. In R-L circuits, the energy is stored in the form of a magnetic field established by the current through the coil. Unlike the capacitor, however, an isolated inductor cannot continue to store energy, because the absence of a closed path causes the current to drop to zero, releasing the energy stored in the form of a magnetic field.

If the series R-L circuit in Fig. 11.41 reaches steady-state conditions and the switch is quickly opened, a spark will occur across the contacts due to the rapid change in current di/dt of the equation $v_L = L(di/dt)$ establishes a high voltage v_L across the coil that, in conjunction with the applied voltage E, appears across the points of the switch. This is the same mechanism used in the ignition system of a car to ignite the fuel in the cylinder.

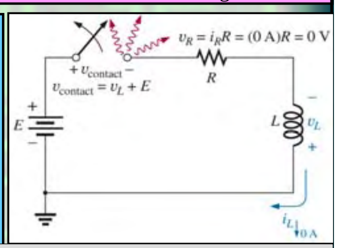


Figure 11.41 Demonstrating the effect of opening a switch in series with an inductor with a steady-state current.

Thevenin Equivalent – $\tau = L/T_{Th}$

Ex. 11-6 For the network in Fig. 11.46:

a. Find the mathematical expression for the transient behavior of the current i_L and the voltage v_L after the closing of the switch ($I_i = 0\ mA$).

b. Draw the resultant waveform for each.

a. Applying thevenin's theorem to the 80 mH inductor (Fig.11.47) yields

$$T_{Th} = \frac{R}{N} = \frac{20\ k\Omega}{2} = 10\ k\Omega$$

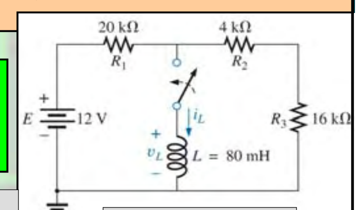
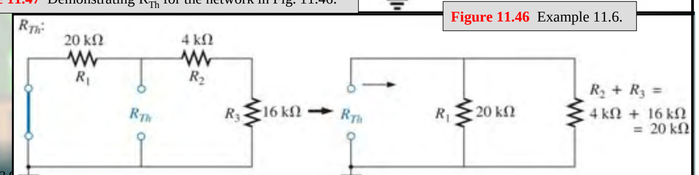


Figure 11.46 Example 11.6.

Figure 11.47 Demonstrating R_{Th} for the network in Fig. 11.46.



Applying the voltage divider rule (Fig. 11.48), we obtain

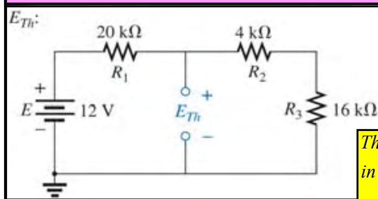


Figure 11.48 Determining E_{th} for the network in Fig. 11.46.

$$E_{Th} = \frac{(R_2+R_3)E}{R_1+R_2+R_3} = \frac{(4k\Omega+16k\Omega)(12\,V)}{20k\Omega+4k\Omega+16k\Omega} = \frac{(20k\Omega)(12\,V)}{40k\Omega} = 6\,V$$

The Thevenin equivalent circuit is shown in Fig. 11.49. Using Eq. (11.13)

$$i_L = \frac{E_{Th}}{R}\left(1-e^{-t/\tau}\right)$$

$$\tau = \frac{L}{R_{Th}} = \frac{80\times10^{-3}\,H}{10\times10^{3}\,\Omega} = 8\times10^{-6}\,s = 8\,\mu s$$

$$I_m = \frac{E_{Th}}{R_{Th}} = \frac{6\,V}{10\times10^{3}\,\Omega} = 0.6\times10^{-3}\,A = 0.6\,mA$$

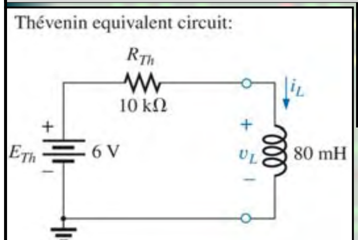


Figure 11.49 The resulting Thevenin equivalent circuit for the network in Fig. 11.46.

and $i_L = 0.6\,mA(1-e^{-t/8\mu s})$

Using Eq. (11.15):

$$v_L = E_{Th}e^{-t/\tau}$$

so that $v_L = 6Ve^{-t/8\mu s}$

b. See Fig. 11.50.

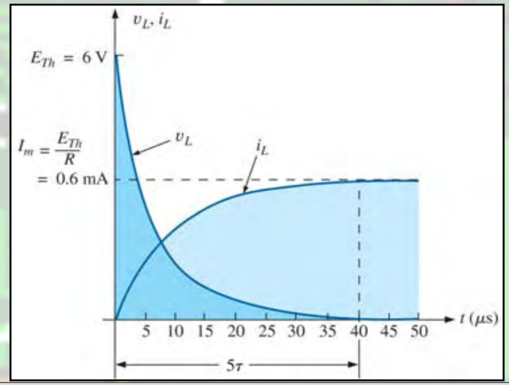


Figure 11.50 The resulting waveforms for i_L and v_L for the network in Fig. 11.46.

HW 11-22 For Fig. 11.94:

a. Determine the mathematical expressions for i_L and v_L following the closing of the switch.

b. Determine i_L and v_L after one time constant.

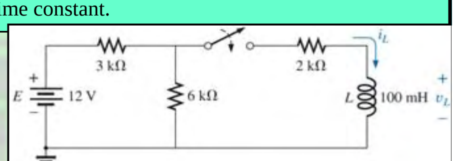


Figure 11.94 Problem 22.

a. $R_{Th} = 2\text{ k}\Omega + 3\text{ k}\Omega \,\|\, 6\text{ k}\Omega = 2\text{ k}\Omega + 2\text{ k}\Omega = 4\text{ k}\Omega$

$E_{Th} = \dfrac{6\text{ k}\Omega(12\text{ V})}{6\text{ k}\Omega+3\text{ k}\Omega} = 8\text{ V}$, $\tau = \dfrac{L}{R} = \dfrac{100\text{ mH}}{4\text{ k}\Omega} = 25\ \mu s$

$I_f = \dfrac{E_{Th}}{R_{Th}} = \dfrac{8\text{ V}}{4\text{ k}\Omega} = 2\text{ mA}$

$\mathbf{i_L = 2\ mA(1 - e^{-t/25\mu s})}$

$\mathbf{v_L = 8\ Ve^{-t/25\mu s}}$

b. $i_L = 2\text{ mA}(1 - e^{-1}) = \mathbf{1.26\ mA}$

$v_L = 8\text{ V}e^{-1} = \mathbf{2.94\ V}$

Homework 11: 2, 4, 8, 10, 12, 14, 22

EET1122/ET162 Circuit Analysis

Magnetic Circuits

Electrical and Telecommunications Engineering Technology Department

Professor Jang

Prepared by textbook based on "Introduction to Circuit Analysis" by Robert Boylestad, Prentice Hall, 11th edition.

OUTLINES

- **Introduction to Magnetic Field**
- **Reluctance**
- **Ohm's Law for the Magnetic Circuits**
- **Magnetizing Force and Flux Φ**
- **Hysteresis**
- **Ampere's Circuital Law**
- **Series Magnetic Circuits**

Key Words: Magnetic field, Reluctance, Flux, Hysteresis, Ampere's Circuital Law

Introduction to Magnetic Field

The magnetic field distribution around a permanent magnet or **electromagnet** was covered in previous module. The **flux density** is defined by Eq. 12.1.

$$B = \frac{\Phi}{A}$$

B = Wb/m^2 = teslas (F)
Φ = webers (Wb)
A = m^2

The "pressure" on the system to establish magnetic lines of force is determined by the applied magnetomotive force which is directly related to the number of turns and current of the magnetizing coil as appearing in Eq. 12.2.

$$F = N\,I$$

$F = ampere-turns\ (At)$
$N = turns\ (t)$
$I = ampere\ (A)$

The level of magnetic flux established in a ferromagnetic core is a direction function of the **permeability** of the material. **Ferromagnetic materials** have a very high level of permeability while non-magnetic material such as air and wood have very low levels. The radio of the permeability of the material to that of air is called the **relative permeability** and is defined by Eq. 12.3.

$$\mu_r = \frac{\mu}{\mu_o} \qquad \mu_o = 4\pi\times10^{-7}\ Wb/A\cdot m$$

Reluctance

The resistance of a material to the flow of charge is determined for electric circuits by the equation

$$R = \rho\frac{l}{A} \quad (ohms,\ \Omega)$$

The reluctance of a material to the setting up of magnetic flux lines in the material is determined by the following equation:

$$\Re = \frac{l}{\mu A} \quad (rels,\ or\ At/Wb)$$

Where ℜ is the reluctance, *l* is the length of the magnetic path, and A is the cross-sectioned area. The t in the units At/Wb is the number of turns of the applied winding. Note that the resistance and reluctance are inversely proportional to the area, indicating that an increase in area results in a reduction in each and an increase in the desired result: current and flux. For an increase in length, the opposite is true, and the desired effect is reduced. The reluctance, however, is inversely proportional to the permeability, while the resistance is directly proportional to the resistivity.

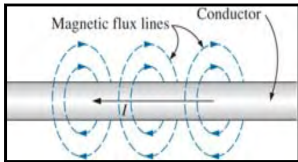


Figure 11.6 Magnetic flux lines around a current-carrying conductor.

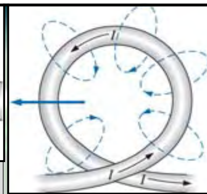


Figure 11.7 Flux distribution of a single-turn coil.

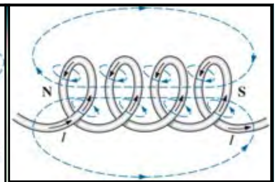


Figure 11.8 Flux distribution of a current carrying coil.

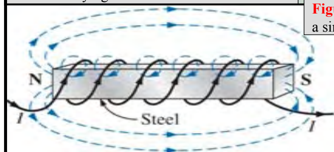


Figure 11.9 Electromagnet.

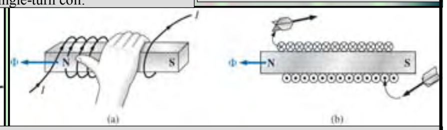


Figure 11.10 Determining the direction of flux for an electromagnet: (a) method; (b) notation.

In the SI system of units, magnetic flux is measured in webers (WB). The applied symbol is Φ. The number of flux lines per unit area, called the flux density, is denoted by B and is measured in teslas (T).

$$B = \frac{\Phi}{A}$$

$$B = Wb/m^2 = teslas\,(T)$$
$$\Phi = webers\,(Wb)$$
$$A = m^2$$

$$1\ tesla = 1T = 1Wb/m^2$$

Ohm's Law for the Magnetic Circuits

For the magnetic circuits, the effect desired is the flux Φ. The cause is the **magnetomotive force (mmf)** $\mathcal{F}$, which is the external force (or "pressure") required to set up the **magnetic flux lines** within the magnetic material. The opposition to the setting of the flux Φ is the reluctance $\Re$.

Substituting, we have
$$\Phi = \frac{\mathcal{F}}{\Re}$$

Since $\mathcal{F}$ = NI, above equation clearly reveals that an increase in the number of turns or the current through the wire in Fig. 12.1 results in an increased "pressure" on the system to establish the flux lines through the core.

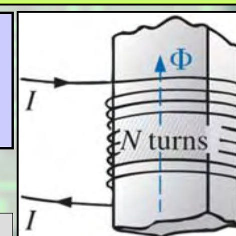


Figure 12.1 Defining the components of a magnetomotive force.

Magnetizing Force

The magnetomotive force per unit length is called the magnetizing force (H). In equation form,

$$H = \frac{\mathcal{F}}{l} \qquad (At/m)$$

Substituting for the magnetomotive force results in

$$H = \frac{NI}{l} \qquad (At/m)$$

For the magnetic circuit in Fig. 12.2, if NI = 40 At and l = 0.2 m, then

$$H = \frac{NI}{l} = \frac{40\ At}{0.2\ m} = 200\ At/m$$

Note in Fig. 12.2 that the direction of the flux Φ can be determined by placing the fingers of your right hand in the direction of the thumb. It is interesting to realize that ***the magnetizing force is independent of the type of core material***–it is determined solely by the number of turns, the current, and the length of the core.

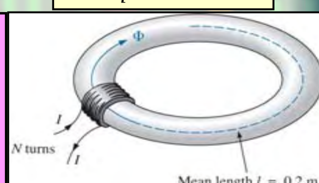


Figure 12.2 Defining the magnetizing force of a magnetic circuit.

The flux density and the magnetizing force are related by the following equations :

$$B = \mu H$$

Hysteresis

A curve of the flux density B versus the magnetizing force H of a material is of particular important to the engineer. A typical B-H curve force a ferromagnetic material such as can be derived using the setup in Fig. 12.4.

The core is initially unmagnetized, and the current *I = 0*. If the current I is increased to some value above zero, the magnetizing force H increases to a value determined by

$$H\uparrow = \frac{NI\uparrow}{l}$$

Figure 12.4 Series magnetic circuit used to define the hysteresis curve.

The flux Φ and the flux density B (B = Φ/A) also increase with the current I (or H). If the material has no residual magnetism, and the magnetizing force H is increased from zero to some value H_a, the B-H curve follows the path shown in Fig. 12.5 between *0* to *a*. If the magnetizing force H is increased until saturation (H_s) occurs, the curve continues as shown in the figure to point *b*. When saturation occurs, the flux density has, for all practical purposes, reached its maximum value.

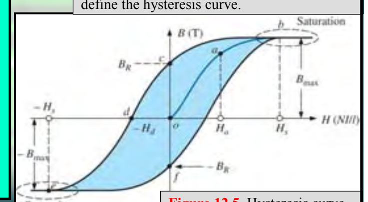


Figure 12.5 Hysteresis curve.

If the magnetizing force is reduced to zero by letting I decrease to zero, the curve follows the path of the curve follows the path of the curve between b and c. The flux density B_R, which remains when the magnetizing force is zero, is called the residual flux density. It is this residual flux density that makes it possible to create permanent magnets. If the current I is reserved, developing a magnetizing force, –H, the flux density when $-H_d$ is reached. The magnetizing force $-H_d$ required to "coerce" the flux density to reduce its level to zero is called the coercive force. As the force –H is increased until saturation again occurs and is then reserved and brought back to zero, the path *def* results. If the magnetizing force is increased in the positive direction (+H), the curve traces the path shown from f to b. the entire curve represented by *bcdefb* is called the **hysteresis** curve.

If the entire cycle is repeated, the curve obtained for the same core will be determined by the maximum H applied. Three hysteresis loops for the same material for maximum values of H less than the saturation value are shown in Fig. 12.6. In addition, the saturation curve is repeated for comparison purposes.

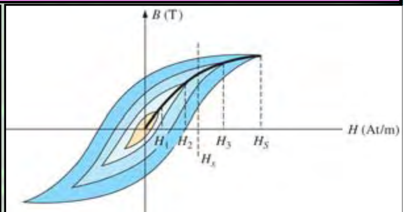


Figure 12.4 Defining the normal magnetization curve.

Ampere's Circuital Law

The similarity between the analyses of electric and magnetic circuits has been demonstrated to some extent for the quantities in Table 12.1.

If we apply the "cause" analogy to KCL ($\sum_{\circlearrowright} V = 0$), we obtain the following:

$\sum_{\circlearrowright} \mathcal{F} = 0$ (for magnetic Circuits)

which, in words, states that the algebraic sum of the rises and drops of the mmf around a closed loop of a magnetic circuits is equal to zero.

Table 12.1

	Electric Circuit	Magnetic Circuits
Cause	E	$\mathcal{F}$
Effect	I	Φ
Opposition	R	$\Re$

Above equation is referred as Ampere's circuital law. When it is applied to magnetic circuits, sources of mmf are expressed by the equation

$\mathcal{F} = NI$ (At)

The equation for the mmf drop across a portion of a magnetic circuit can be found by applying the relationships listed in Table 12.1; that is, for electric circuits,

$V = IR$

Resulting in the following for magnetic circuits

$\mathcal{F} = \Phi\Re$ (At)

Where Φ is the flux passing through a section of the magnetic circuit and R is the reluctance of that section. A more practical equation for the mmf drop is,

$\mathcal{F} = H\,l$ (At)

where H is the magnetizing force on a section of a magnetic circuit and *l* is the length of the section.

Flux Φ

If we continue to apply the relationships described in the previous module to KCL, we find that the sum of the fluxes entering a junction is equal to the sum of the fluxes leaving a junction; that is, for the circuit in Fig. 12.11,

$$\Phi_a = \Phi_b + \Phi_c \quad (at\ \ junction\ a)$$
$$or \quad \Phi_b + \Phi_c = \Phi_a \quad (at\ \ junction\ b)$$

both of which are equivalent.

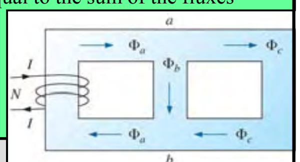


Figure 12.11 Flux distribution of a series-parallel magnetic network.

Series Magnetic Circuits – Determining NI

In one type, Φ is given, and the impressed mmf NI must be computed. This is the type of problem encountered in the design of motors, generators, and transformers. ***In the other type, NI is given, and the flux Φ of the magnetic circuit must be found***. This type of problem is encountered primarily in the design of magnetic amplifiers. This section considers only series magnetic circuits in which the flux Φ is the same throughout.

Ex. 12-1 For the series magnetic circuit in Fig. 12.12:

a. Find the value of I required to develop a magnetic flux of $\Phi = 4\times10^{-4}$ Wb.
b. Determine μ and μ_r for the material under these conditions.

The magnetic circuit can be represented by the system shown in Fig. 12.13(a). The electric circuit analogy is shown in Fig. 12.13(b). Analogies of this type can be very helpful in the solution of magnetic circuits. Table 12.2 is for part (a) of this problem.

a. The flux density B is

$$B = \frac{\phi}{A} = \frac{4\times10^{-4}\,Wb}{2\times10^{-3}\,m^2} = 2\times10^{-1}\,T = 0.2\,T$$

Using the B-H curves in Fig. 12.8, we can determine the magnetizing force H:

H (cast steel) = 170 At/m

Applying Ampere's circuital law yields

$$NI = Hl$$

and $$I = \frac{Hl}{N} = \frac{(170At/m)(0.16m)}{400t} = 68\,mA$$

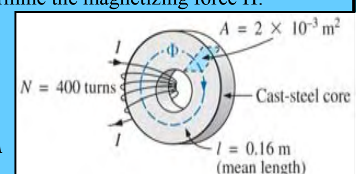


Figure 12.12 Example 12.1.

b. *The permeability of the material can be found*

$$\mu = \frac{B}{H} = \frac{0.2\,T}{170\,At/m} = 1.176 \times 10^{-3}\ Wb/A \cdot m$$

and the relative permeability is

$$\mu_r = \frac{\mu}{\mu_o} = \frac{1.176 \times 10^{-3}}{4\pi \times 10^{-7}} = 935.83$$

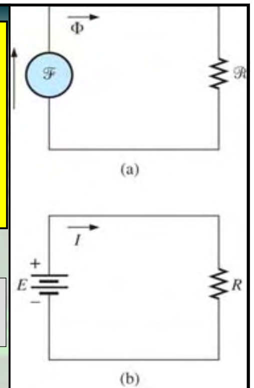


Figure 12.13 (a) magnetic circuit equivalent and (b) electric circuit analogy.

Table 12.1

Section	Φ (Wb)	A(m²)	B(T)	H(At/m)	*l*(m)	*Hl*(At)
One continuous section	4×10^{-4}	2×10^{-3}			0.16	

Air Gaps

Let's consider the effects that an air gap has on a magnetic circuit. Note the presence of air gaps in the magnetic circuits of the motor and meter in Fig. 11.15. The spreading of the flux lines outside the common area of the core for the air gap in Fig. 12.18(a) is known as ***fringing***. For our purposes, we shall ignore this effect and assume the flux distribution to be as in Fig. 12.18(b).

The flux density of the air gap in Fig. 12.18(b) is given by

$$B_g = \frac{\Phi_g}{A_g}$$

where, for our purposes, $\Phi_g = \Phi_{core}$

and $A_g = A_{core}$

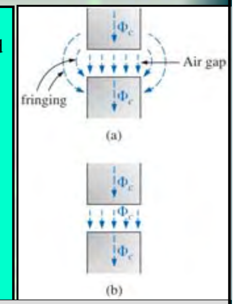


Figure 12.18 Air gap: (a) with fringing; (b) ideal.

For the most practical applications, the permeability of air is taken to be equal to that of free space. The magnetizing force of the air gap is then determined by

$$H_g = \frac{B_g}{\mu_o}$$

and the mmf drop across the air gap is equal to H_gL_g. An equation for H_g is as follows:

$$H_g = \frac{B_g}{\mu_o} = \frac{B_g}{4\pi \times 10^{-7}}$$

$$and \quad H_g = (7.96 \times 10^5)B_g \quad (At/m)$$

Ex. 12-4 Find the value of I required to establish a magnetic flux of Φ = 0.75×10^{-4} Wb in the series magnetic circuit in Fig. 12.19.

An equivalent magnetic circuit and its electric circuit analogy are shown in Fig. 12.20.

The flux density for each section is

$$B = \frac{\Phi}{A} = \frac{0.75 \times 10^{-4}\ Wb}{1.5 \times 10^{-4}\ m^2} = 0.5\ T$$

From the B-H curves in Fig. 12.28,

$$H(cast\ steel) \cong 280\ At/m$$

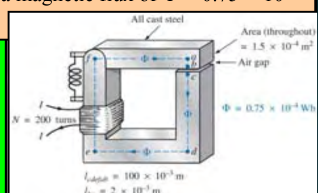


Figure 12.19 Relay for Example 12.4.

Applying Equation,

$$H_g = (7.96 \times 10^5)B_g = (7.96 \times 10^5)(0.5T) = 3.98 \times 10^5\ At/m$$

The mmf drops are

$$H_{core}I_{core} = (280At/m)(100 \times 10^{-3}m) = 28\ At$$

$$H_gI_g = (3.98 \times 10^5\ At/m)(2 \times 10^{-3}m) = 796\ At$$

Applying Ampere's circuital law,

$$NI = H_{core}I_{core} + H_gI_g = 28At + 796At$$

$$(200)I = 824At \quad and \quad I = 4.12\ A$$

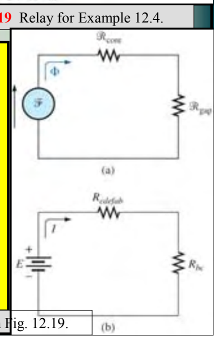


Figure 12.20 (a) Magnetie circuit equivalent and (b) electric circuit analogy for the relay in Fig. 12.19.

HW 12-6 Repeat Problem 5 for Φ = 72,000 maxwells and an impressed mmf of 120 gilberts.

$$\Re = \frac{\Im}{\Phi} = \frac{120\ gilberts}{72{,}000\ \max wells} = 1.67 \times 10^{-3}\ rels\,(CGS)$$

Homework 12: 2, 4, 6, 12-14.